CSE 332
Data Structures & Parallelism

Graph Traversals

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Graph Traversals

Next problem: For an arbitrary graph and a starting node **`v`**, find all nodes *reachable* (i.e., there exists a path) from **`v`**

- Possibly “do something” for each node (an iterator!)
  - E.g. Print to output, set some field, etc.

Related Questions:

- Is an undirected graph connected?
- Is a directed graph weakly / strongly connected?
  - For strongly, need a cycle back to starting node

Basic idea:

- Keep following nodes
- But “mark” nodes after visiting them, so the traversal terminates and processes each reachable node exactly once

Graph Traversal: Abstract Idea

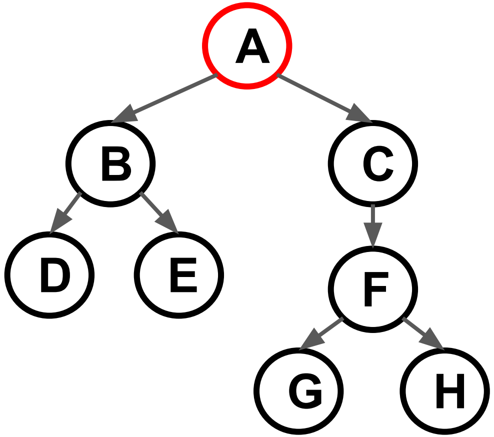
```
traverseGraph(Node start) {
  Set pending = emptySet();
  pending.add(start)
  mark start as visited
  while(pending is not empty) {
    next = pending.remove()
    for each node u adjacent to next
      if(u is not marked) {
        mark u
        pending.add(u)
      }
  }
}
```

Running time and options

- Assuming add and remove are O(1), entire traversal is O(|E|)
    - Use an adjacency list representation
- The order we traverse depends entirely on how add and remove work/are implemented
    - Depth-first graph search (DFS): a **stack**
    - Breadth-first graph search (BFS): a **queue**
- DFS and BFS are “big ideas” in computer science
    - **Depth**: recursively explore one part before going back to the other parts not yet explored
    - **Breadth**: Explore areas closer to the start node first

Recursive DFS, Example: trees

A tree is a graph and DFS and BFS are particularly easy to "see"



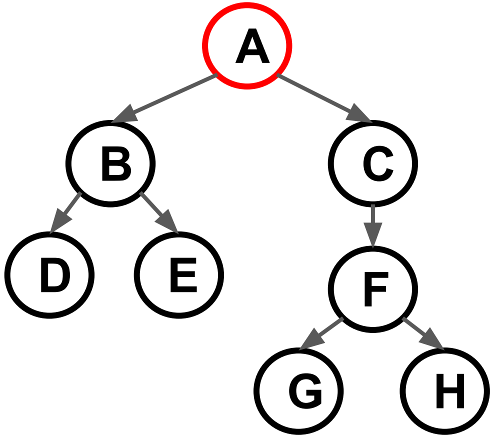
```
DFS(Node start) {
  mark and "process" (eg print) start
  for each node u adjacent to start
    if u is not marked
      DFS(u)
}
```

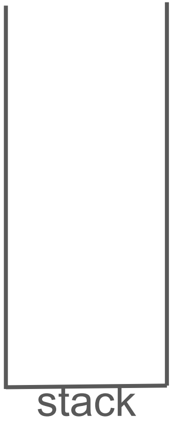
Order processed: A, B, D, E, C, F, G, H

- Exactly what we called a "pre-order traversal" for trees
- The marking is not needed here, but we need it to support arbitrary graphs , we need a way to process each node exactly once

DFS with a stack, Example: trees

A tree is a graph and DFS and BFS are particularly easy to “see”





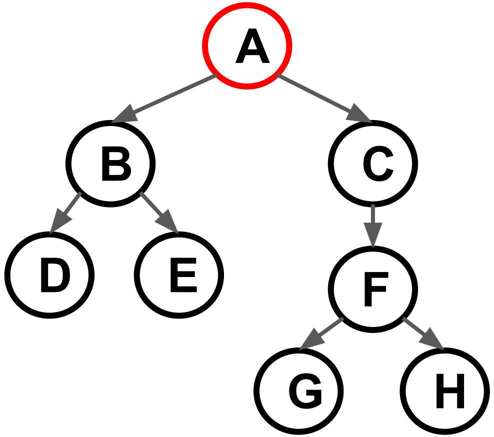
```
DFS2(Node start) {
  initialize stack s to hold start
  mark start as visited
  while(s is not empty) {
    next = s.pop() // and “process”
    for each node u adjacent to next
      if(u is not marked)
        mark u and push onto s
  }
}
```

Order processed:

- A different but perfectly fine traversal

BFS with a queue, Example: trees

A tree is a graph and DFS and BFS are particularly easy to “see”



queue

```
BFS(Node start) {
  initialize stack s to hold start
  mark start as visited
  while(s is not empty) {
    next = s.pop() // and “process”
    for each node u adjacent to next
      if(u is not marked)
        mark u and push onto s
  }
}
```

Order processed:

- A “level-order” traversal

DFS/BFS Comparison

Breadth-first search:

- Always finds shortest paths, i.e., “optimal solutions
  - Better for “what is the shortest path from **x** to **y**”
- Queue may hold O(|V|) nodes (e.g. at the bottom level of binary tree of height h, 2^h nodes in queue)

Depth-first search:

- Can use less space in finding a path
  - If *longest path* in the graph is **p** and highest out-degree is **d** then DFS stack never has more than **d*p** elements

A third approach: *Iterative deepening (IDDFS)*:

- Try DFS but don’t allow recursion more than **K** levels deep.
  - If that fails, increment **K** and start the entire search over
- Like BFS, finds shortest paths. Like DFS, less space.

Saving the path

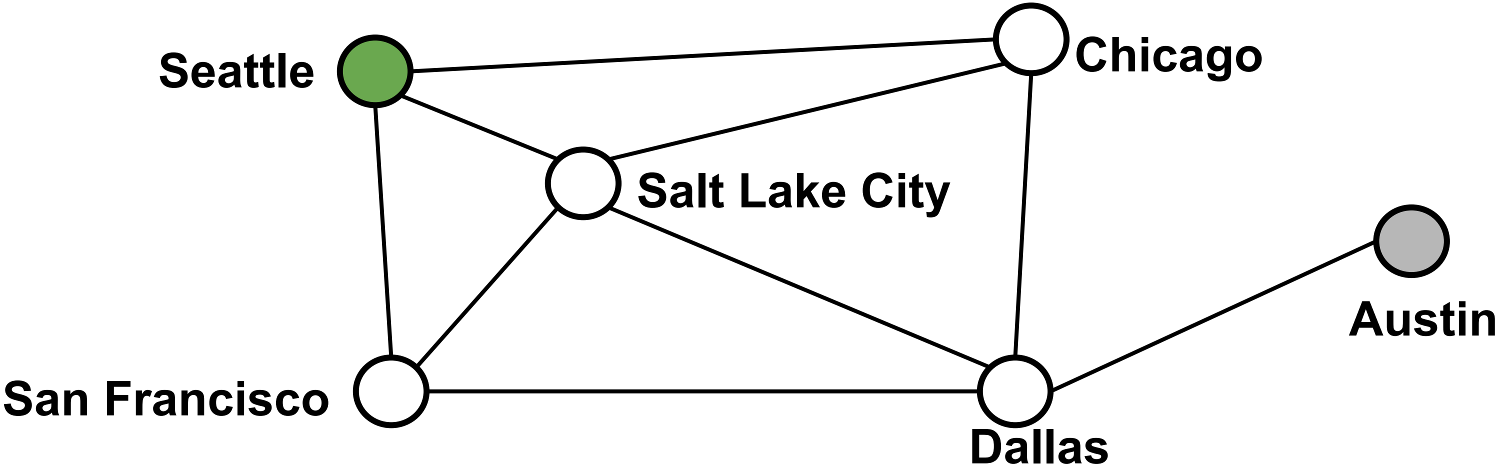
- Our graph traversals can answer the "reachability question":
  - "***Is there*** a path from node x to node y?"
- Q: But what if we want to ***output the actual path***?
  - Like getting driving directions rather than just knowing it's possible to get there!
- A: Like this:
  - Instead of just "marking" a node, store the **previous node** along the path (when processing **u** causes us to add **v** to the search, set **v.pred** field to be **u**)
  - When you reach the goal, follow **pred** fields backwards to where you started (and then reverse the answer)
  - If just wanted path *length*, could put the integer distance at each node instead

Queue

Example using BFS

What is a path from Seattle to Austin?

- Remember marked nodes are not re-enqueued
- Note shortest paths may not be unique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Austin
San Francisco
Dallas