CSE 332
Data Structures & Parallelism

Priority Queues

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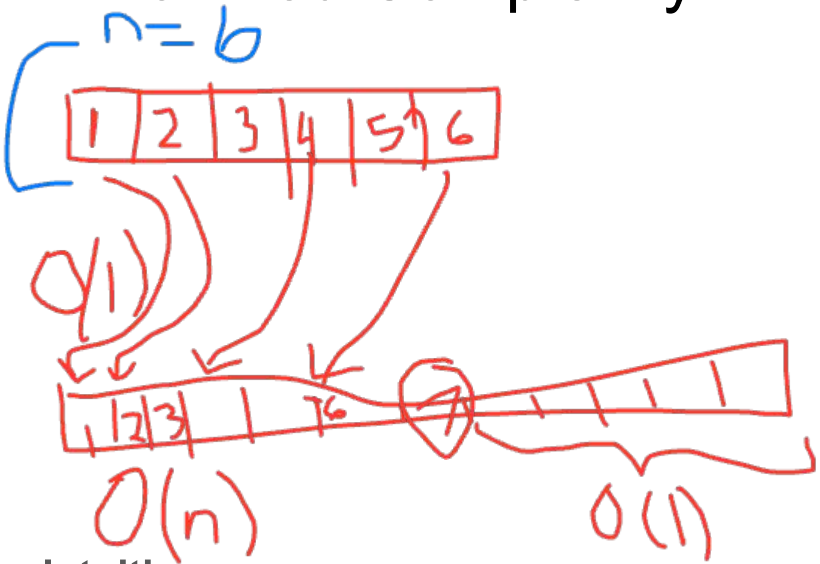
Spring 2024

Amortized Complexity



- “Cost” of first n inserts is $1 per insert
- “Cost” of the (n+1)th insert is $(n+1)
- How many “cheap” ($1) inserts can we do before we encounter another “expensive” insert?

Intuition:

(n * $1) + (1 * $(n+1)) + ((n-1) * $1) = **$(3n)** - cost of entire sequence of 2n inserts

$3n / 2n = $3/2 = O(1) average cost per operation given this sequence

Chapter 11 - a lot more complicated to “prove” that this

Today & Next Time - Priority Queues / Heaps

- What is a Priority Queue?
- Introduction to the heap
- Heap operations
- Heap implementation
- Building a heap

Scenario

What is the difference between waiting for service at a pharmacy versus an ER?

- Pharmacies usually follow the rule: First Come, First Served
- Emergency Rooms assign priorities based on each individual's need

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  Queue (FIFO)

- Emergency Rooms assign priorities based on each individual's need

  Priority Queue

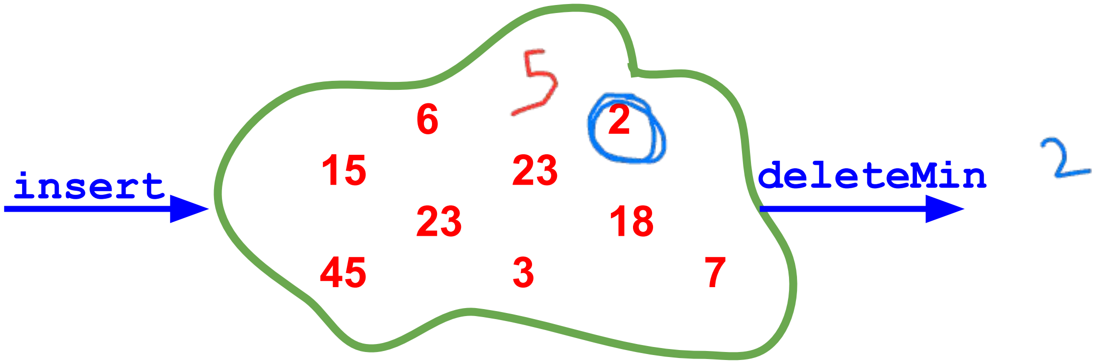
A new ADT: Priority Queue

- Textbook Chapter 6
    - We will go back to binary search trees (ch4) and hash tables (ch5) later
    - Nice to see a new and surprising data structure first
- A **priority queue** holds *compare-able* data
    - Unlike stacks and queues need to compare items
    - Given **x** and **y**, is **x** less than, equal to, or greater than **y**
    - What this means can depend on your data
    - Much of course will require comparable data: e.g. sorting
- Integers are comparable, so will use them in examples
    - But the priority queue ADT is much more general
    - Typically two fields, the *priority* and the *data*

Priority Queue ADT

- Assume each item has a “priority”
  - The lesser item is the one with the greater priority
  - So “priority 1” is more important than “priority 4”
  - Just a convention, could also do a maximum priority

- Main Operations:
  - `insert(int)`
  - `deleteMin()`



- Key property: **`deleteMin`** returns and deletes from the queue the item with greatest priority (lowest priority value)
  - Can resolve ties arbitrarily

Aside: ints as data *and* priority

- For simplicity in lecture, we'll often suppose items are just ints and the int is also the priority
- So an operation sequence could be

```
insert 6
insert 5
x = deleteMin // Now x = 5
```

- `int` priorities are common, but really just need comparable
- Not having "other data" is very rare
  - Example: print job has a priority and the file to print is the data

Priority Queue Example

insert a with priority **5**
insert b with priority **3**
insert c with priority **4**
w = **deleteMin**
x = **deleteMin**
insert d with priority **2**
insert e with priority **6**
y = **deleteMin**
z = **deleteMin**

after execution:

w = b
x = c
y = d
z = a

To simplify our examples, we will just use the priority values from now on
Analogy: insert is like enqueue, deleteMin is like dequeue
But the whole point is to use priorities instead of FIFO

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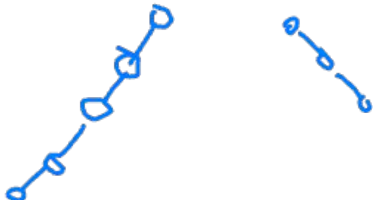
Applications of Priority Queues

- Like all good ADTs, the priority queue arises often
    - Sometimes “directly”, sometimes less obvious
- Run multiple programs in the operating system
    - “critical” before “interactive” before “compute-intensive”
    - Maybe let users set priority level
- Treat hospital patients in order of severity (or triage)
- Select print jobs in order of decreasing length?
- Forward network packets in order of urgency
- Select most frequent symbols for data compression (cf. CSE123)
- Sort: insert all, then repeatedly deleteMin

Preliminary Implementations of Priority Queue ADT

	insert	deleteMin
Unsorted Array	1	n
Unsorted Linked List	1	n
Sorted Circular Array	n	1
Sorted Linked List	n	1
Binary Search Tree (BST)	n	n

Note: Worst case, assume arrays have enough space



Preliminary Implementations of Priority Queue ADT

	insert	deleteMin
Unsorted Array	θ(1)	θ(n)
Unsorted Linked List	θ(1)	θ(n)
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Our Data Structure: The Heap

Or more specifically, a "binary min heap"

- Worst case: O(log n) for insert
- Worst case: O(log n) for deleteMin
- If items arrive in random order, then the average-case of insert is O(1)
- Very good constant factors

Key idea: Only pay for functionality needed

- We need something better than scanning unsorted items
- But we do not need to maintain a full sorted list

- We will visualize our heap as a tree, so we need to review some tree terminology

Reviewing Some Tree Terminology

root(**T**): A

leaves(**T**):

children(**B**):

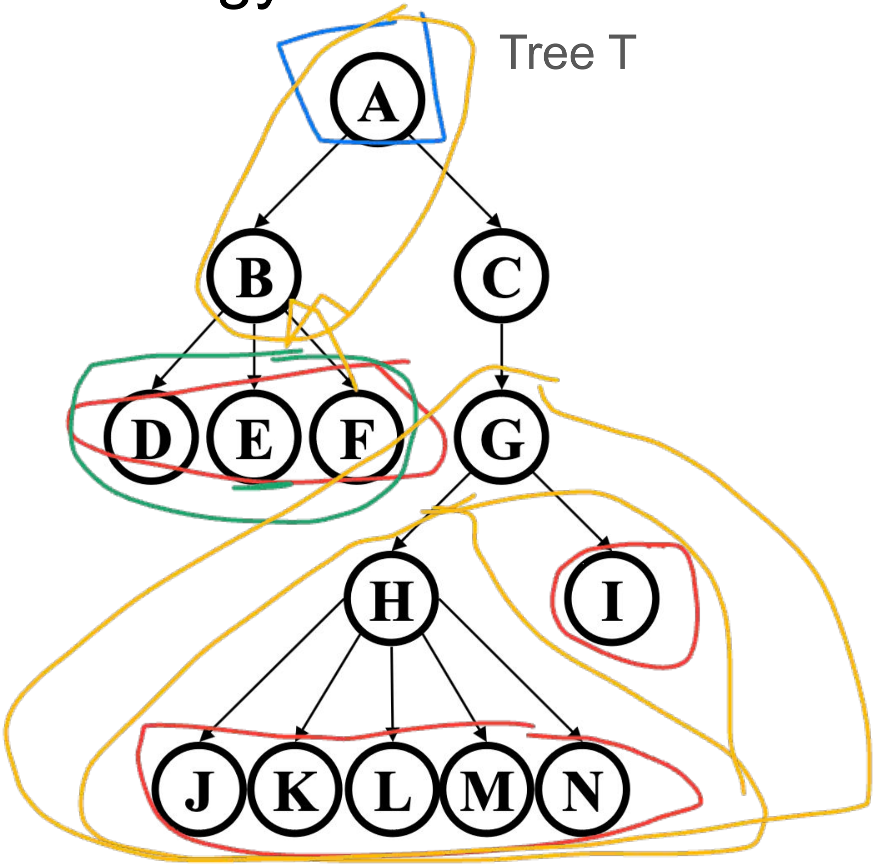
parent(**H**):

siblings(**E**):

ancestors(**F**):

descendents(**G**):

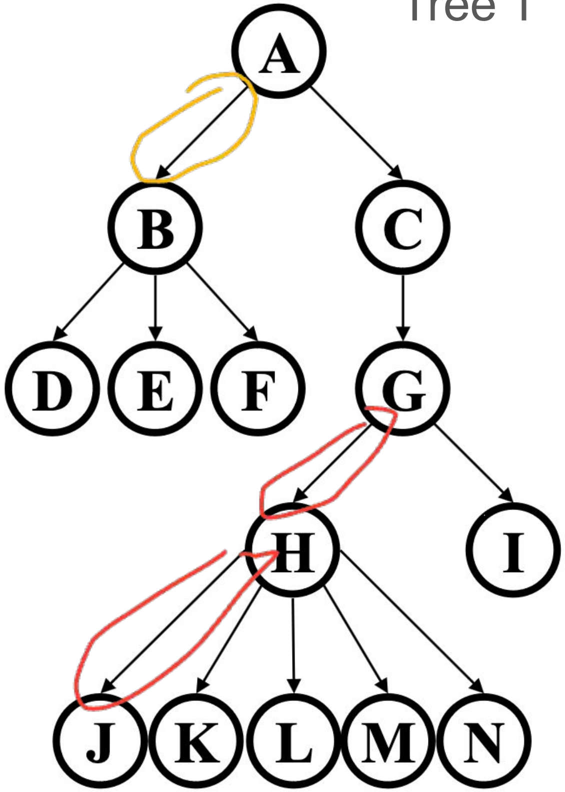
subtree(**G**):



Some More Tree Terminology

depth(**B**): 1
height(**G**): 2
height(**T**): 4
~~degree(**B**):~~
~~branching factor(**T**):~~

Tree T



height – number of edges in path fron node to deepest descendent

depth – number of edges in path from node to root

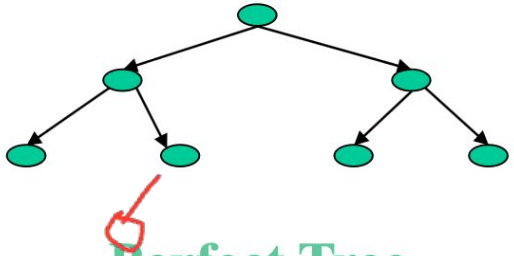
Types of Trees

Binary tree: Every node has ≤2 children

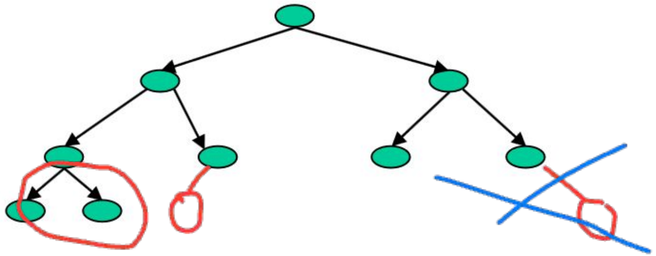
n-ary tree: Every node has ≤n children

Perfect tree: Every row is completely full

Complete tree: All rows except possibly the bottom are completely full, and it is filled from left to right



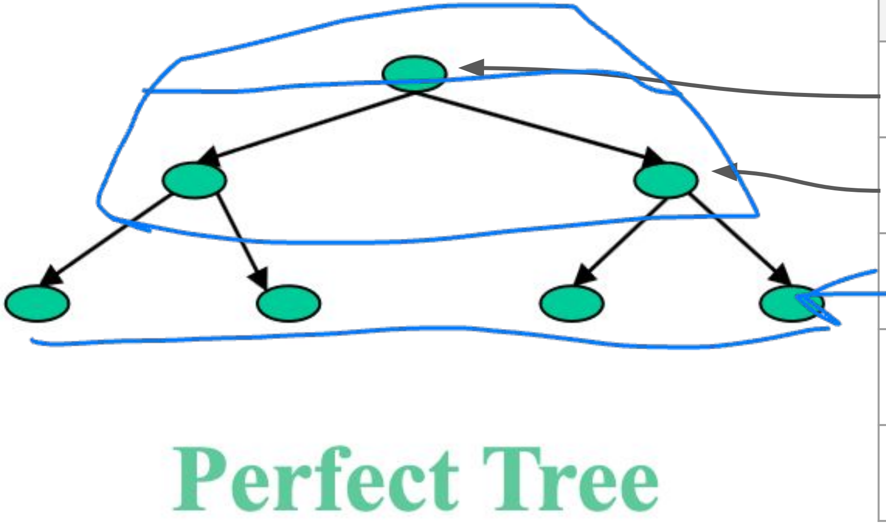
Perfect Tree



Complete Tree

More on Perfect Trees

Perfect tree: Every row is completely full



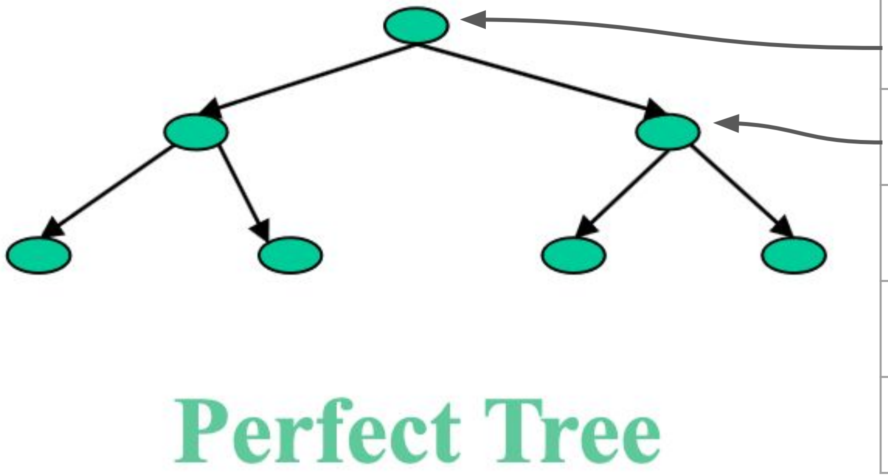
Height (h)	# nodes (n)	# leaves
0	1	1
1	3	2
2	7	4
3	15	8
h	$2^{h+1} - 1$	2^h

More on Perfect Trees

$$n = \sum_{i=0}^{h} 2^i = 2^{h+1} - 1$$

See Weiss 1.2.3 (p4)

Perfect tree: Every row is completely full



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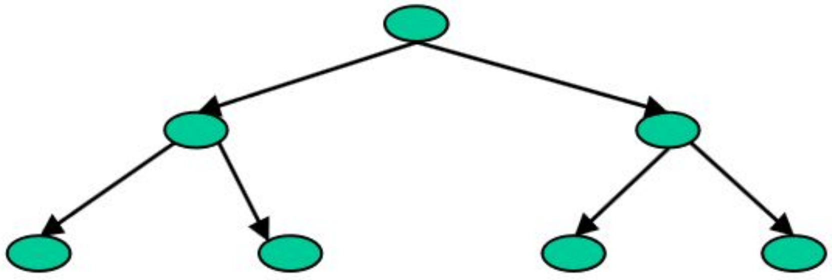

Perfect Tree

How does the height of a perfect tree relate to the number of nodes?

$2^{h+1} - 1 = n$
$2^{h+1} = n$
$h + 1 = \log n$
$h = O(\log n)$