CSE 332 Autumn 2024
Lecture 21: Parallel Prefix

Nathan Brunelle

http://www.cs.uw.edu/332

Which Data Structures are “Suitable” for Parallelism?

- For each data structure, can we write a parallel algorithm to sum all of its values that’s *more efficient* than a sequential one?
  - Array
  - Linked List
  - Binary Tree

ForkJoin Framework

- This strategy is common enough that Java (and C++, and C#, and…) provides a library to do it for you!

What you would do in Threads	What to instead in ForkJoin
Subclass **Thread**	Subclass **RecursiveTask<V>**
Override **run**	Override **compute**
Store the answer in a field	Return a V from compute
Call **start**	Call **fork**
join synchronizes only	**join** synchronizes and returns the answer
Call **run** to execute sequentially	Call **compute** to execute sequentially
Have a topmost thread and call **run**	Create a pool and call **invoke**

Divide and Conquer with ForkJoin

```
class SumTask extends RecursiveTask<Integer> {
        int lo; int hi; int[] arr; // fields to know what to do
        SumTask(int[] a, int l, int h) { … }
        protected Integer compute(){// return answer
                if(hi – lo < SEQUENTIAL_CUTOFF) {  // base case
                        int ans = 0; // local var, not a field
                        for(int i=lo; i < hi; i++) {
                                ans += arr[i]; return ans; }
                else {
                        SumTask left = new SumTask(arr,lo,(hi+lo)/2); // divide
                        SumTask right= new SumTask(arr,(hi+lo)/2,hi); // divide
                        left.fork(); // fork a thread and calls compute (conquer)
                        int rightAns = right.compute(); //call compute directly (conquer)
                        int leftAns = left.join(); // get result from left
                        return leftAns + rightAns; // combine
                }
        }
}
```

Divide and Conquer with ForkJoin (continued)

```
static final ForkJoinPool POOL = new ForkJoinPool();
static int parallelSum(int[] arr){
        SumTask task = new SumTask(arr,0,arr.length)
        return POOL.invoke(task); // invoke returns the value compute returns
}
```

Find Max with ForkJoin

```
class MaxTask extends RecursiveTask<Integer> {
        int lo; int hi; int[] arr; // fields to know what to do
        SumTask(int[] a, int l, int h) { … }
        protected Integer compute(){// return answer
                if(hi – lo < SEQUENTIAL_CUTOFF) {  // base case
                        int ans = Integer.MIN_VALUE; // local var, not a field
                        for(int i=lo; i < hi; i++) {
                                ans = Math.max(ans, arr[i]);}
                        return ans;
                else {
                        MaxTask left = new MaxTask(arr,lo,(hi+lo)/2); // divide
                        MaxTask right= new MaxTask(arr,(hi+lo)/2,hi); // divide
                        left.fork(); // fork a thread and calls compute (conquer)
                        int rightAns = right.compute(); //call compute directly (conquer)
                        int leftAns = left.join(); // get result from left
                        return Math.max(rightAns, leftAns); // combine
                }
        }
}
```

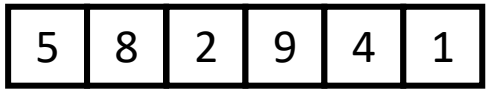
Other Problems that can be solved similarly

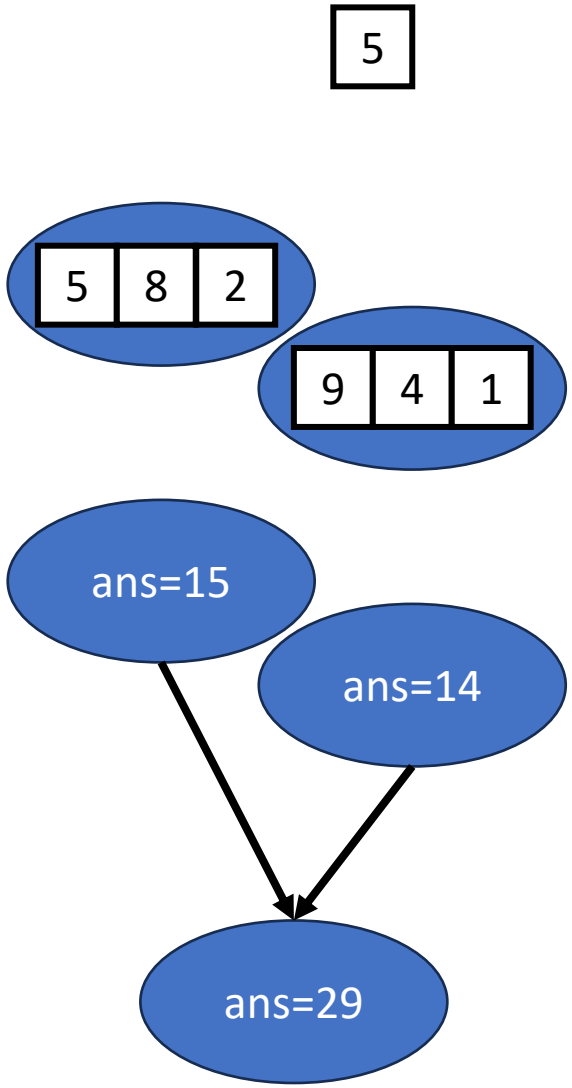
- Element Search
  - Is the value 17 in the array?
- Counting items with a certain property
  - How many elements of the array are divisible by 5?
- Checking if the array is sorted
- Find the smallest rectangle that covers all points in the array
- Find the first thing that satisfies a property
  - What is the leftmost item that is divisible by 20?

Reduction/Fold

- All examples of a category of computation called a reduction (or fold)
  - We “reduce” all elements in an array to a single item
  - Requires operation done among elements is associative
    - $(x + y) + z = x + (y + z)$
  - The “single item” can itself be complex
    - E.g. create a histogram of results from an array of trials

Reduction (sum an array)



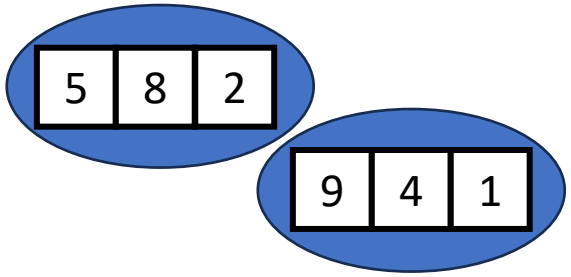


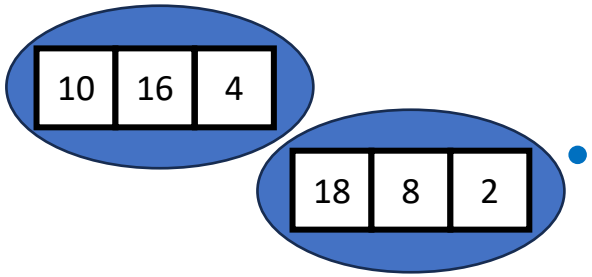
- **Base Case**:
  - If the list's length is smaller than the Sequential Cutoff, reduce things sequentially
- **Divide**:
  - Split the list into two "sublists" of (roughly) equal length, create a thread to reduce each sublist.
- **Conquer**:
  - Call **start()** for each thread
- **Combine**:
  - Reduce the answers from each thread

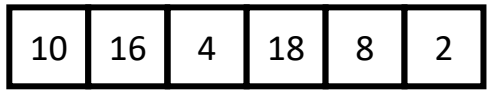
Map

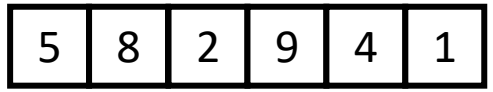
- Perform an operation on each item in an array to create a new array of the same size
- Examples:
  - Vector addition:
    - sum[i] = arr1[i] + arr2[i]
  - Function application:
    - out[i] = f(arr[i]);

Map (double each value)









- **Base Case**:
  - If the list's length is smaller than the Sequential Cutoff, convert each thing sequentially
- **Divide**:
  - Split the list into two "sublists" of (roughly) equal length, create a thread to map each sublist.
- **Conquer**:
  - Call **start()** for each thread
- **Combine**:
  - No additional work necessary

Map with ForkJoin

```
class AddTask extends RecursiveAction {
        int lo; int hi; int[] arr; // fields to know what to do
        AddTask(int[] a, int[] b, int[] sum, int l, int h) { … }
        protected void compute(){// return answer
                if(hi – lo < SEQUENTIAL_CUTOFF) {  // base case
                        for(int i=lo; i < hi; i++) {
                                sum[i] = a[i] + b[i];}
                else {
                        AddTask left = new AddTask(a,b,sum,lo,(hi+lo)/2); // divide
                        AddTask right= new AddTask(a,b,sum,(hi+lo)/2,hi); // divide
                        left.fork(); // fork a thread and calls compute (conquer)
                        right.compute(); //call compute directly (conquer)
                        left.join(); // get result from left
                        return; // combine
                }
        }
}
```

Map with ForkJoin (continued)

```
static final ForkJoinPool POOL = new ForkJoinPool();
Int[] add(int[] a, int[] b){
        ans = new int[a.length];
        AddTask task = new AddTask(a, b, ans, 0, a.length)
        POOL.invoke(task);
        return ans;
}
```

Maps and Reductions

- “Workhorse” constructs in parallel programming
- Many problems can be written in terms of maps and reductions
- With practice, writing them will become second nature
  - Like how over time for loops and if statements have gotten easier

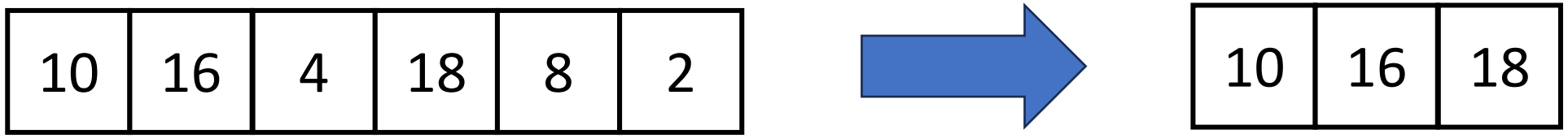
Map/Reduction Example

- Multiply together the lengths of all of the odd-length strings in a given array
  - First, do a map to covert the array of strings into an array of their lengths
  - Then do a map on that array so each value maps to 1 if it's even and itself if it's odd
  - Then do a reduction to multiply together that final result

- Note: You could do this in a single ForkJoin RecursiveTask, but it's worthwhile to recognize how to "deconstruct" it since some programming languages designed specifically for parallelism have Map/Reduce built in.
  - Map and Reduce are two from a trio, with Pack/Filter being the third

Pack/Filter

- Given an array of values and a Boolean function, return a new array which contains only elements that were “true”

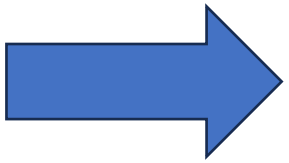


$f(x) = x > 9$

Prefix Sum

- Given an array, compute a new array where each index i is the sum of all values up to i

10	16	4	18	8	2



10	26	30	48	56	58

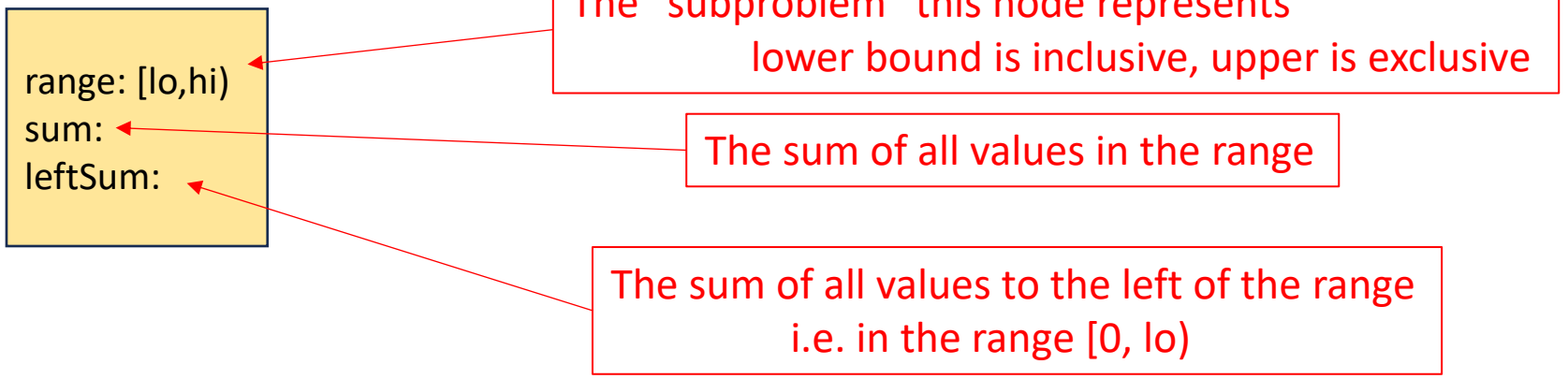
```
int[] prefixSum(int[] arr){
        int[] output = new int[arr.length];
        output[0] = arr[0];
        for (int i = 1; I < arr.length, i++)
                output[i] = output[i-1] + arr[i];
        return output;
}
```

Parallel Prefix Sum

Richard Ladner
Allen School Faculty

- Algorithm will have two major parallel steps
  - Called a “two pass” parallel algorithm
- First step:
  - Create a tree data structure
- Second Step:
  - Use the tree to fill in the output array

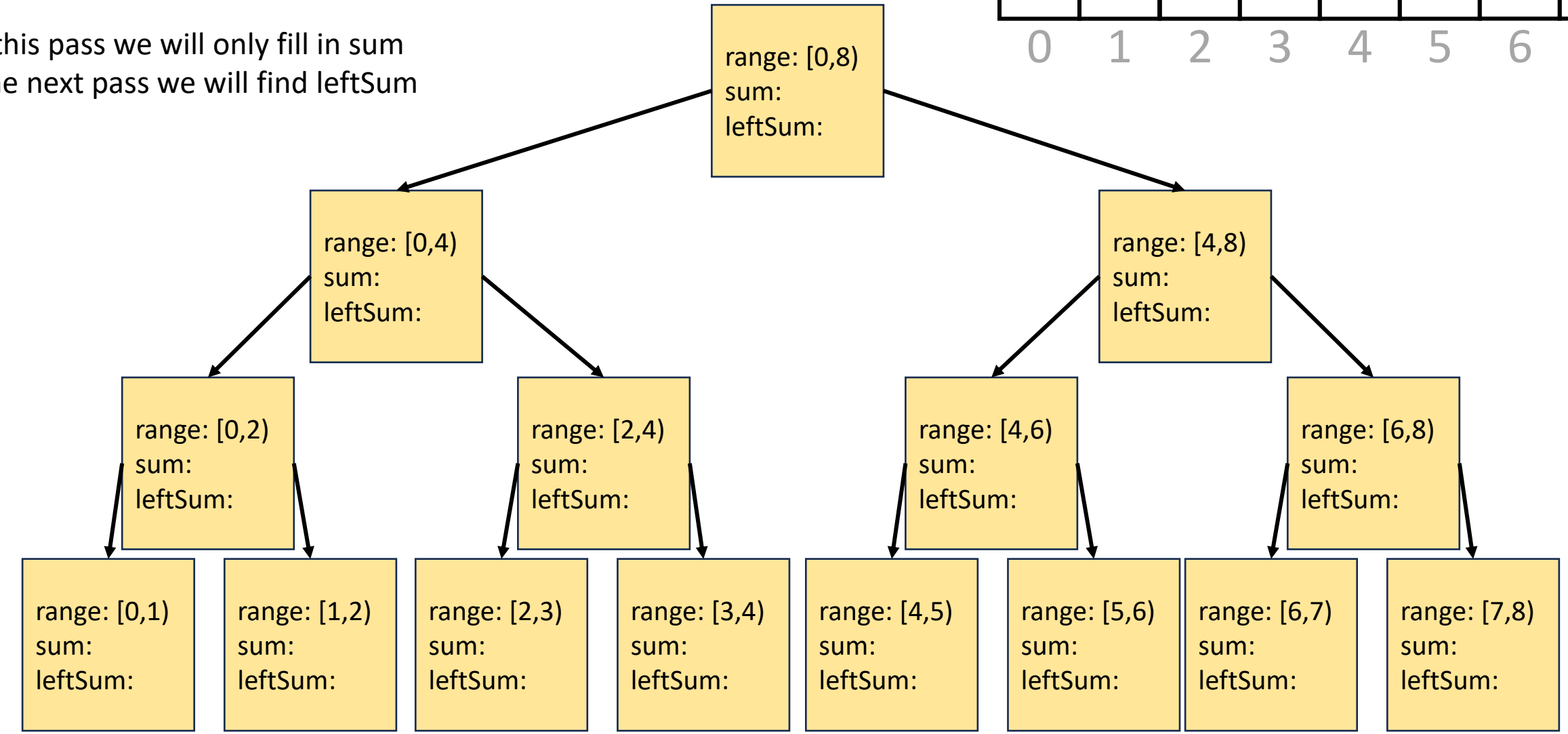
Tree Node:



Step 1: Using D&C
Create a Tree, Fill in sum

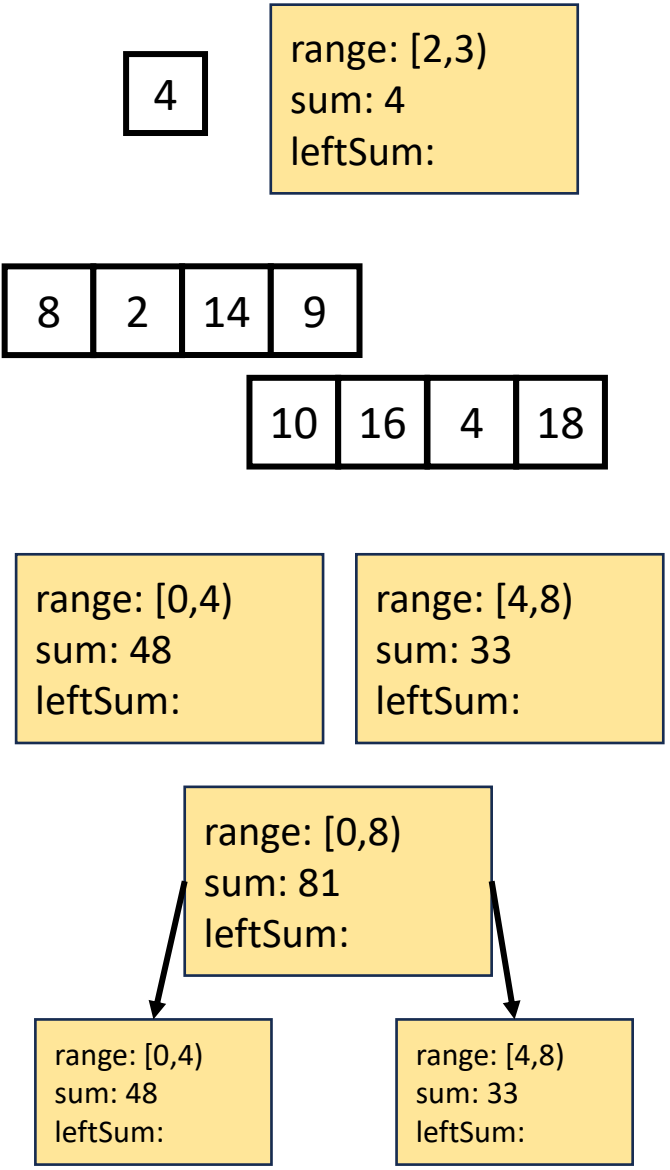
Input:	10	16	4	18	8	2	14	9
Output:								
	0	1	2	3	4	5	6	7

For this pass we will only fill in sum
In the next pass we will find leftSum



Step 1: Create a Tree, Fill in sum

10	16	4	18	8	2	14	9



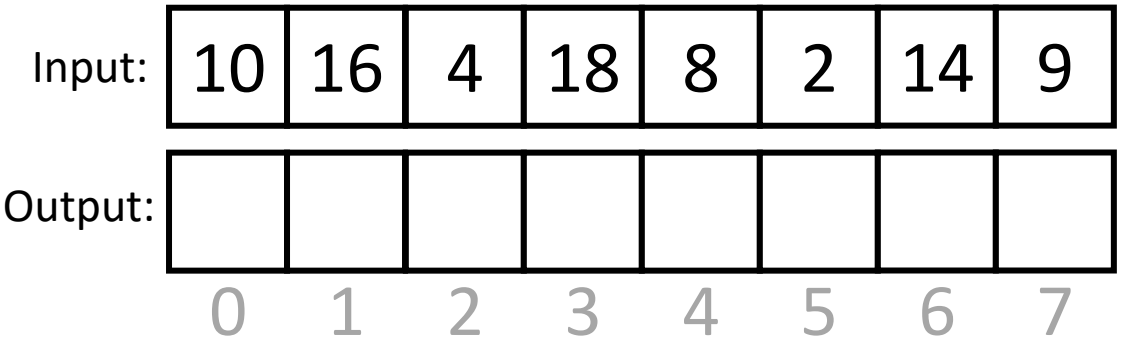
- **Base Case:**
  - If the rand is smaller than the Sequential Cutoff, create a node for that range and find the sum sequentially
- **Divide:**
  - Split the list into two "sublists" of (roughly) equal length, create a thread for each sublist.
- **Conquer:**
  - Call **start()** for each thread to compute the left and right subtrees
- **Combine:**
  - Create parent node, connect to children, fill in sum

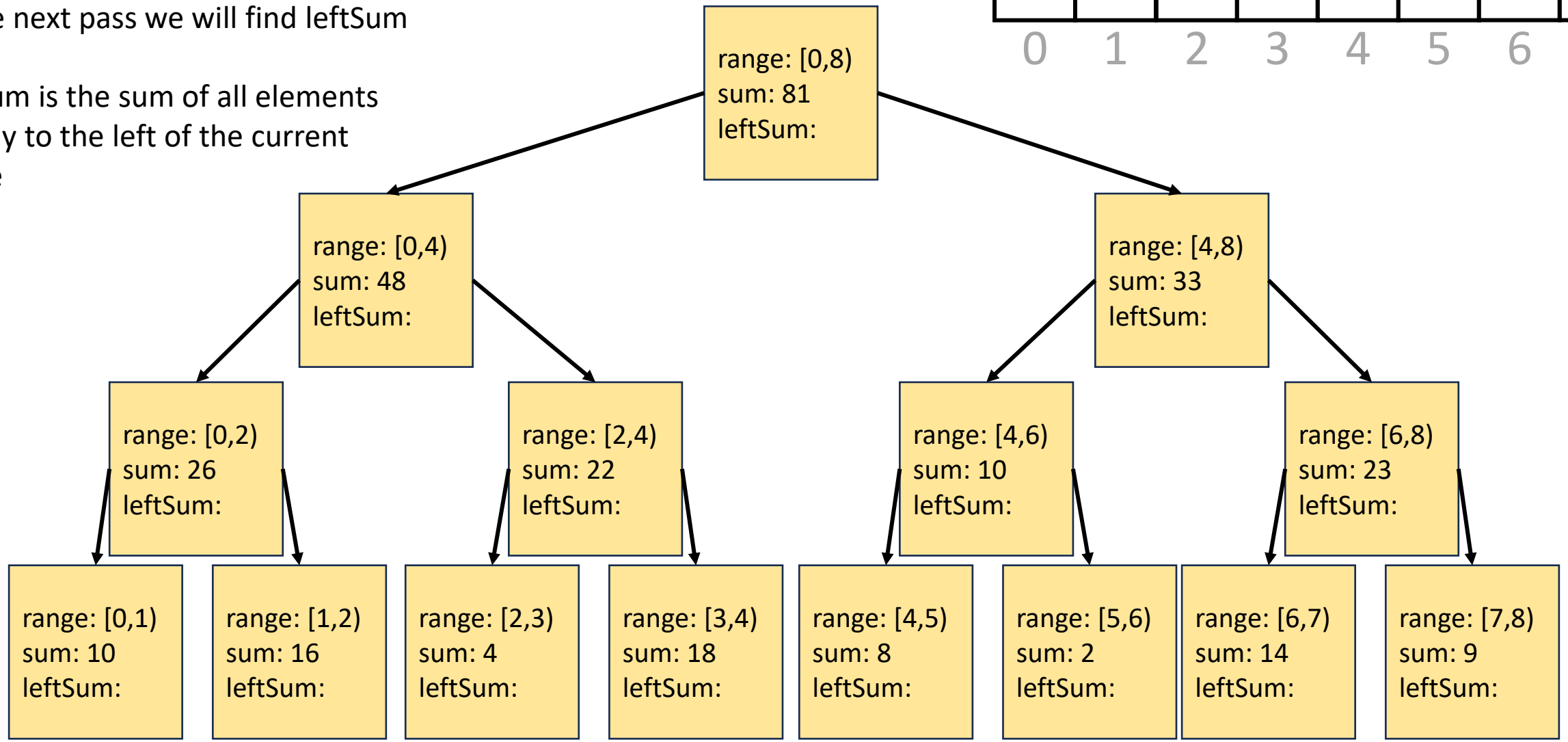
```
class BuildTree extends RecursiveTask<PrefixSumNode> {
        protected PrefixSumNode compute(){
                if(hi – lo < SEQUENTIAL_CUTOFF) {  // base case
                        int ans = 0; // local var, not a field
                        for(int i=lo; i < hi; i++)
                                ans += arr[i];
                        return new PrefixSumNode(lo, hi, ans); }
                else {
                        BuildTree left = new BuildTree(arr,lo,(hi+lo)/2);
                        BuildTree right= new BuildTree(arr,(hi+lo)/2,hi);
                        left.fork();
                         PrefixSumNode rightChild = right.compute();
                         PrefixSumNode leftChild = left.join();
                        int ans = rightChild.sum + leftChild.sum;
                        parent = new PrefixSumNode(lo, hi, ans);
                        parent.left = leftChild;
                        parent.right = rightChild;
                        return parent; }
        }
}
```

After Step 1

All sums filled in per node
In the next pass we will find leftSum

leftSum is the sum of all elements strictly to the left of the current range

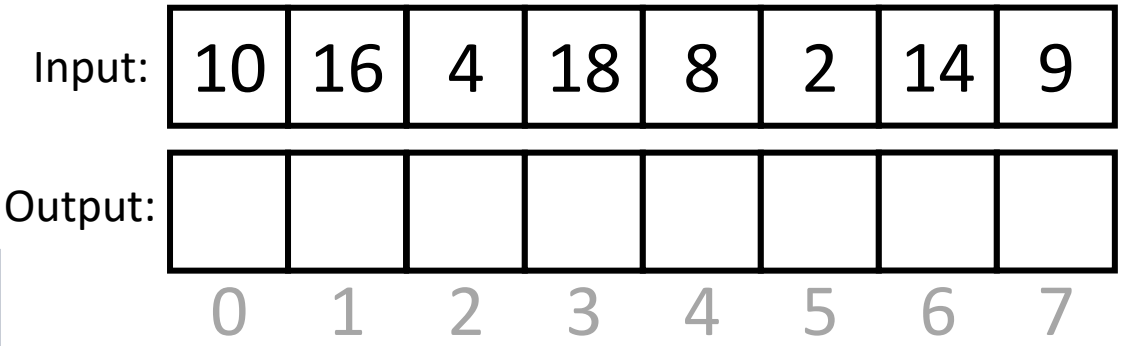


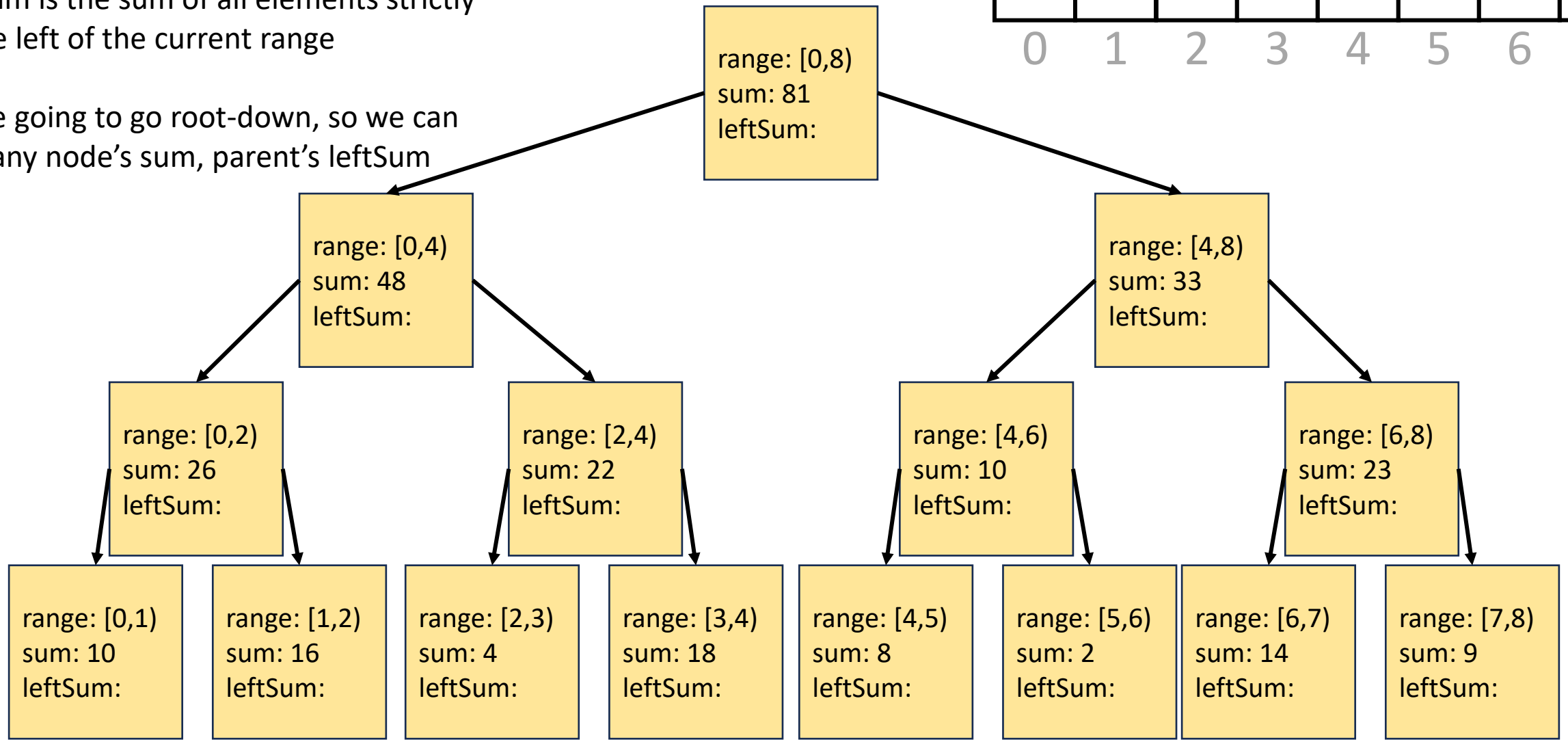


Step 2: fill in leftSum and Output

leftSum is the sum of all elements strictly to the left of the current range

We're going to go root-down, so we can use: any node's sum, parent's leftSum





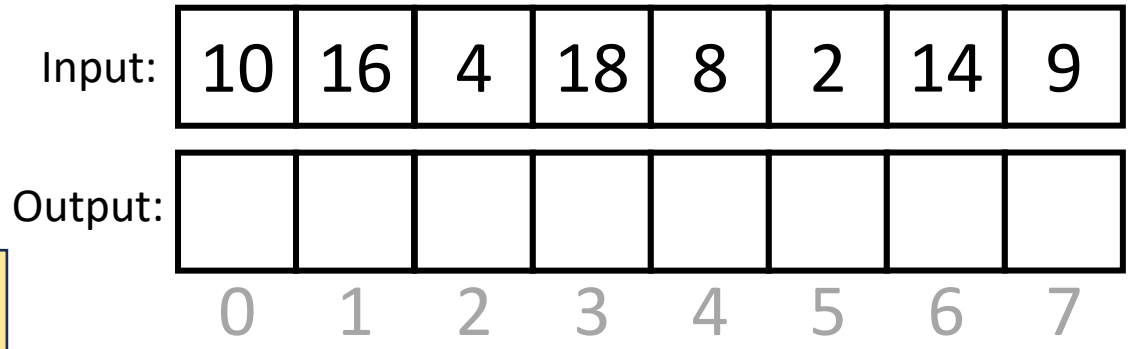
Step 2: fill in leftSum and Output

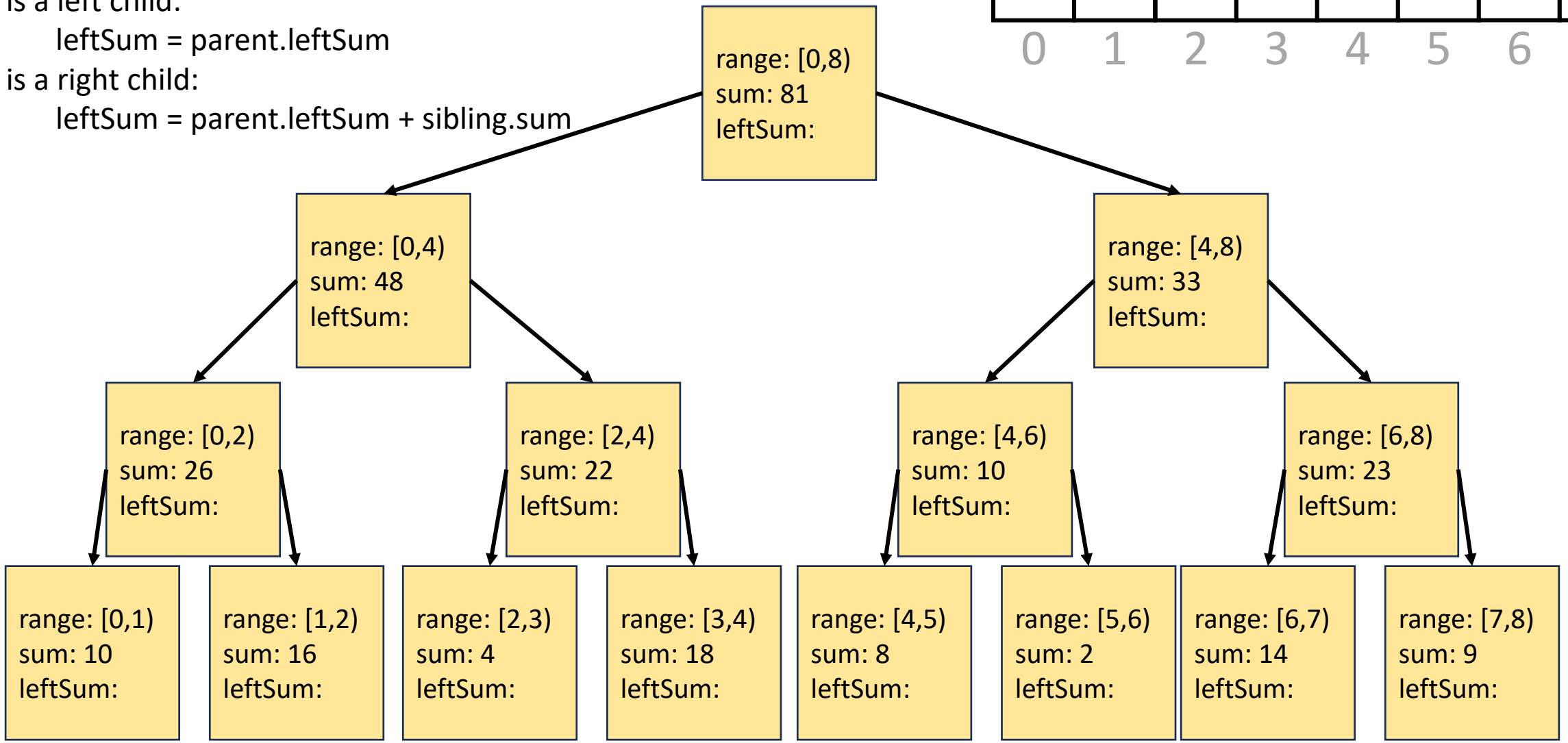
If this is a left child:

leftSum = parent.leftSum

If this is a right child:

leftSum = parent.leftSum + sibling.sum

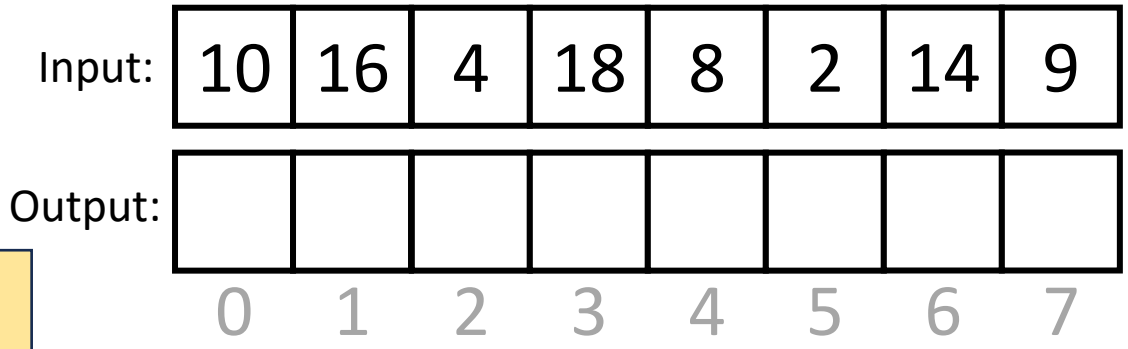


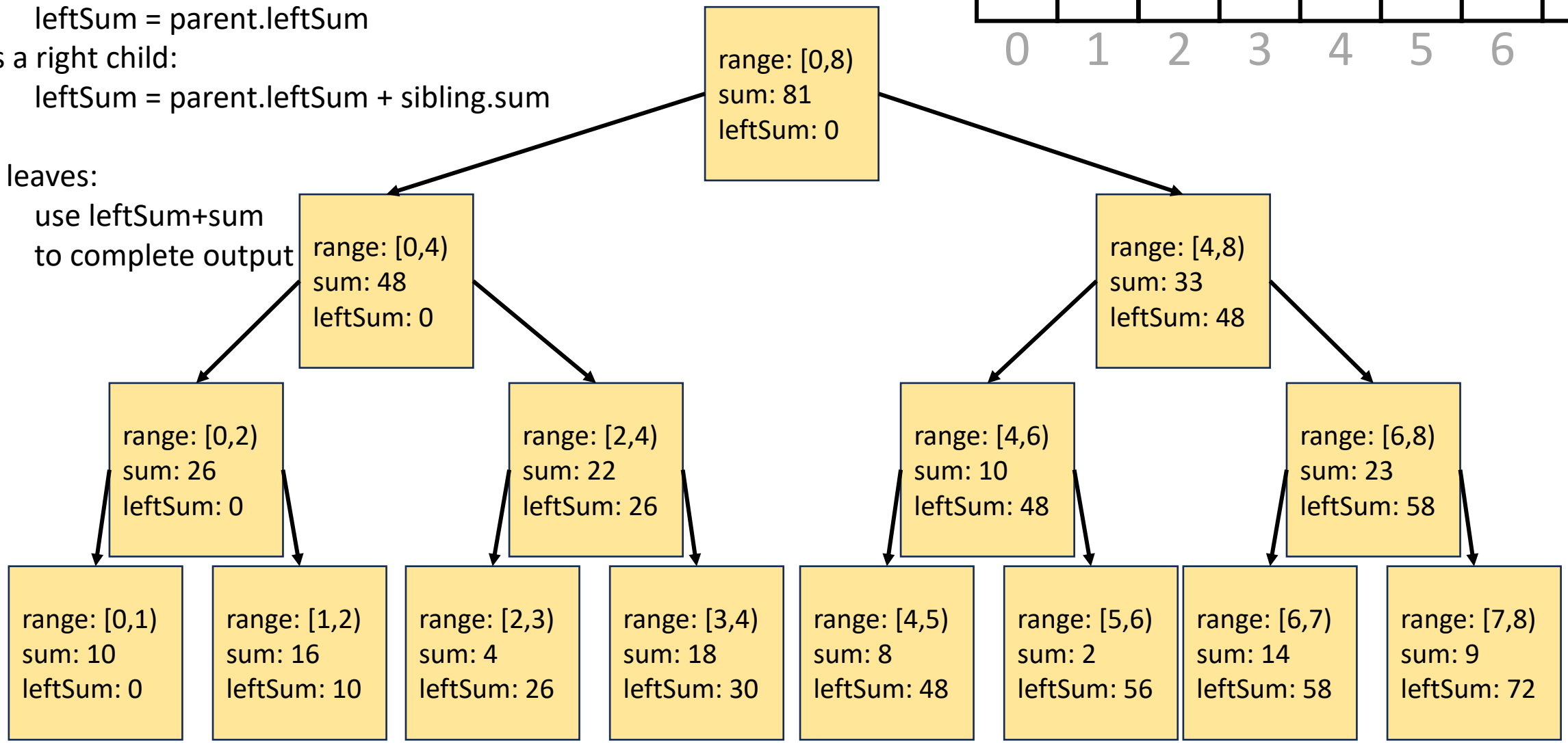


Step 2: fill in leftSum and Output

If this is a left child:
leftSum = parent.leftSum
If this is a right child:
leftSum = parent.leftSum + sibling.sum

For the leaves:
use leftSum+sum
to complete output



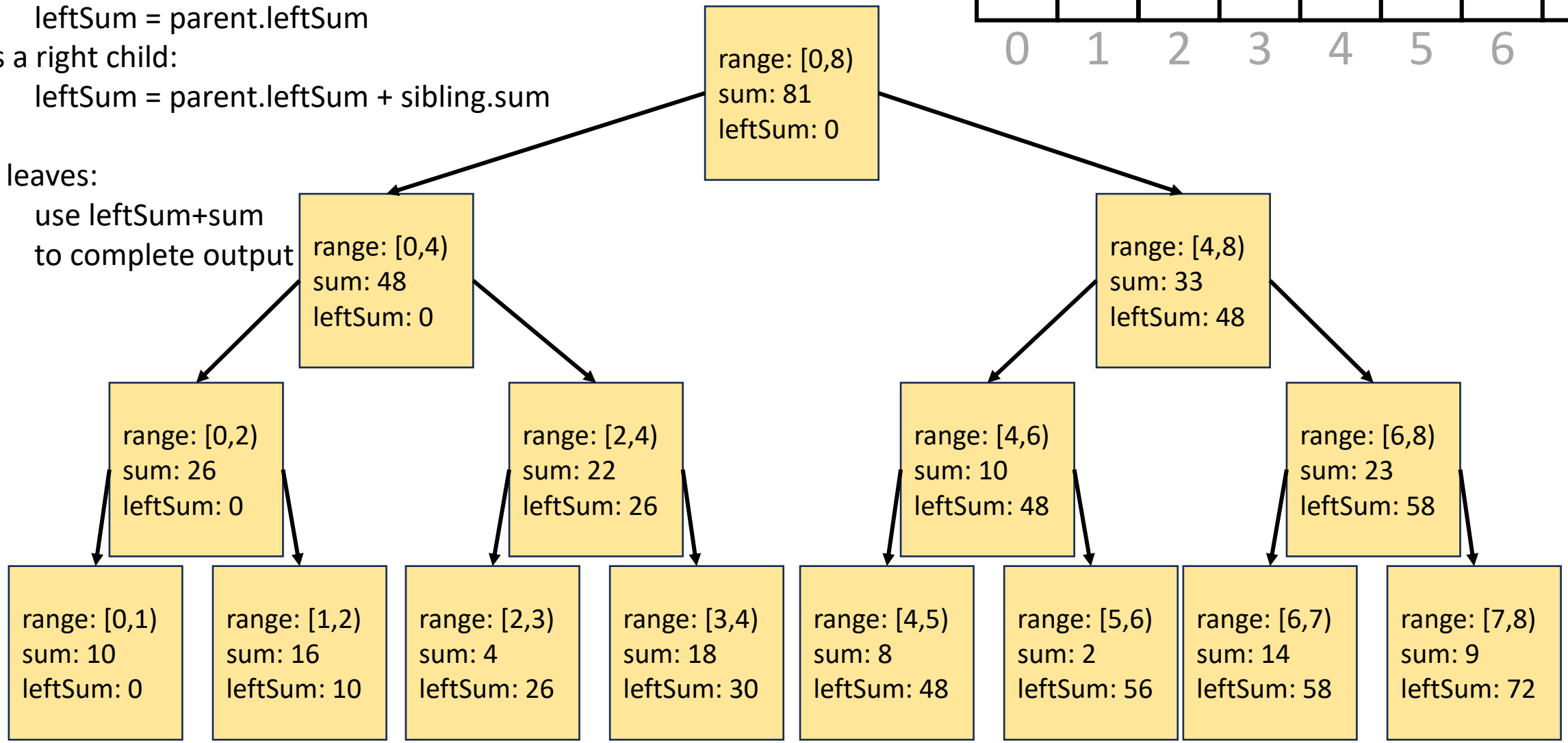


Step 2: fill in leftSum and Output

If this is a left child:
leftSum = parent.leftSum
If this is a right child:
leftSum = parent.leftSum + sibling.sum

For the leaves:
use leftSum+sum
to complete output

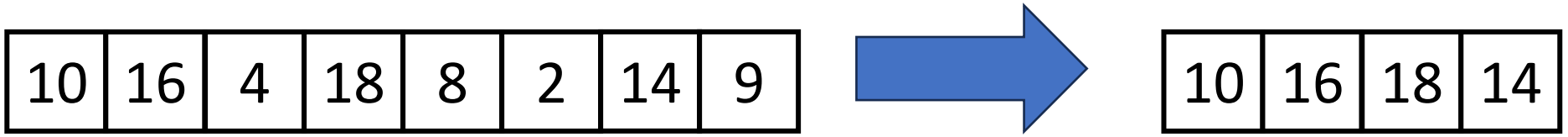
Input:	10	16	4	18	8	2	14	9
Output:	10	26	30	48	56	58	72	81
	0	1	2	3	4	5	6	7



```
class CompleteTree extends RecursiveAction {
        public CompleteTree(PrefixSumNode curr, PrefixSumNode parent, PrefixSumNode sibling, boolean isLeftChild, int[] output, int[] input){…}
        protected void compute(){
                if(isLeftChild)
                        curr.sumLeft = parent.sumLeft;
                else
                         curr.sumLeft = parent.sumLeft + sibling.sum;
                if (curr.leftChild != null && curr.rightChild != null){  // if this isn't a leaf
                        CompleteTree left = new CompleteTree(curr.leftChild, curr, curr.rightChild, true, output, input);
                        left.fork();
                        CompleteTree right = new CompleteTree(curr.rightChild, curr, curr.leftChild, false, output, input);
                        right.compute();
                        left.join();
                }
                else{
                        output[curr.lo] = curr.sumLeft + input[curr.lo];
                        for(int i = curr.lo+1; i < curr.hi; i++){
                                output[i] = output[i-1] + input[i]
                        }
                }
        }
}
```

Whew! Back to Pack/Filter

- Given an array of values and a Boolean function, return a new array which contains only elements that were “true



$f(x) = x > 9$

Parallel Pack

Input:

10	16	4	18	8	2	14	9

, $f(x) = x > 9$

Output:

10	16	18	14

0 1 2 3 4 5 6 7

1. Do a map to identify the true elements

1	1	0	1	0	0	1	0

2. Do prefix sum on the result of the map to identify the count of true elements seen to the left of each position

1	2	2	3	3	3	4	4

3. Do a map using the previous results fill in the output

10	16	18	14

3. Do a map using the result of the prefix sum to fill in the output

Input:	10	16	4	18	8	2	14	9
Map Result:	1	1	0	1	0	0	1	0
Prefix Result:	1	2	2	3	3	3	4	4
Output:								

- Because the last value in the prefix result is 4, the length of the output is 4
- Each time there is a 1 in the map result, we want to include that element in the output
- If element i should be included, its position matches prefixResult[i]-1

```
Int[] output = new int[prefixResult[input.length-1]];
FORALL(int i = 0; i < input.length; i++){
        if (mapResult[i] == 1)
                output[prefixResult[i]-1] = input[i];
}
```

Map/Reduction/Pack Example

- Multiply together the lengths of all of the odd-length strings in a given array
  - First, do a map to covert the array of strings into an array of their lengths
  - Then do a map on that array so each value maps to 1 if it's even and itself if it's odd
    - Alternatively, do a pack on the array to remove all even values
  - Then do a reduction to multiply together that final result