CSE 332 Autumn 2024
Lecture 17: Graphs

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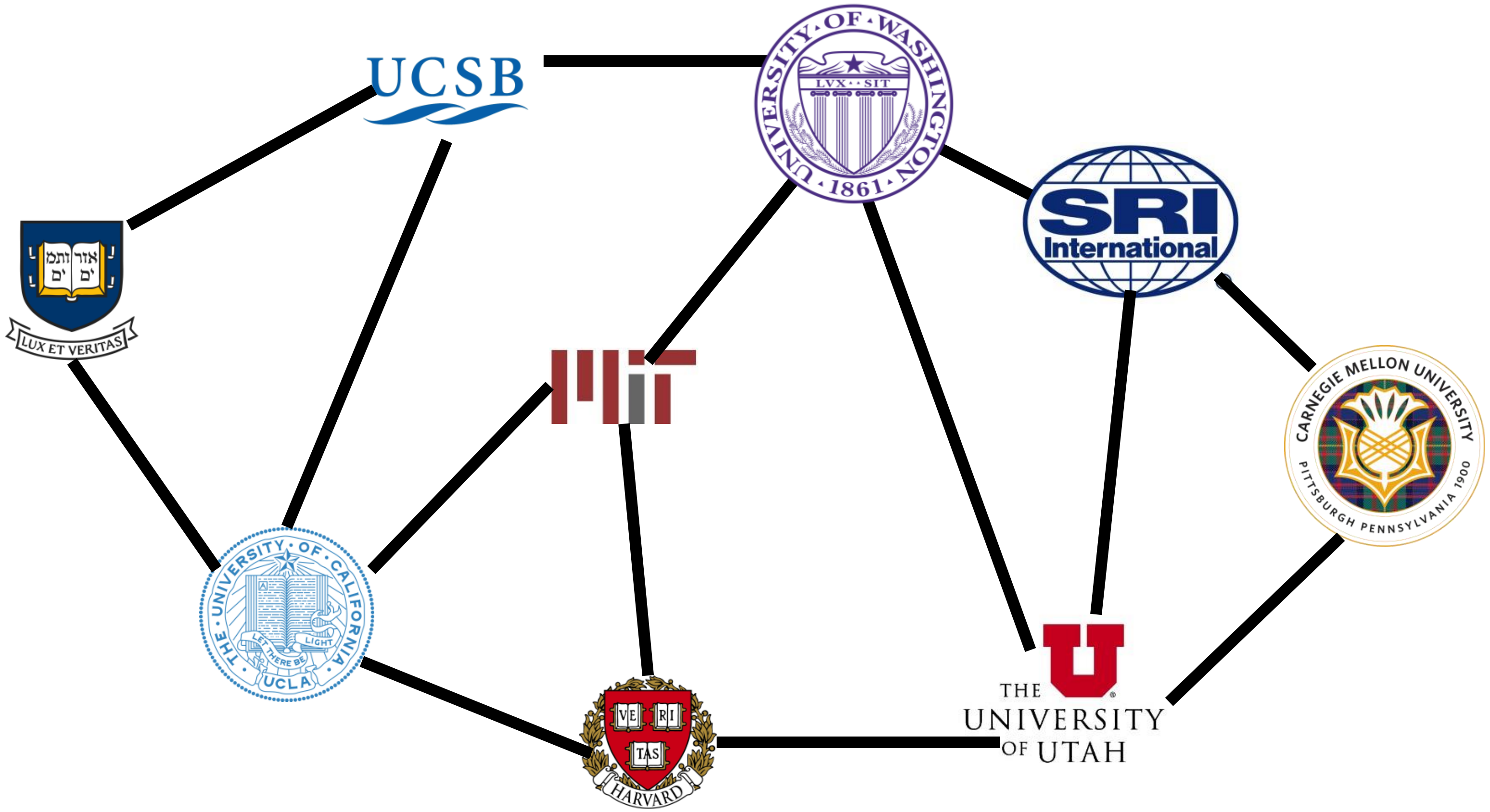
http://www.cs.uw.edu/332

ARPANET


UCSB
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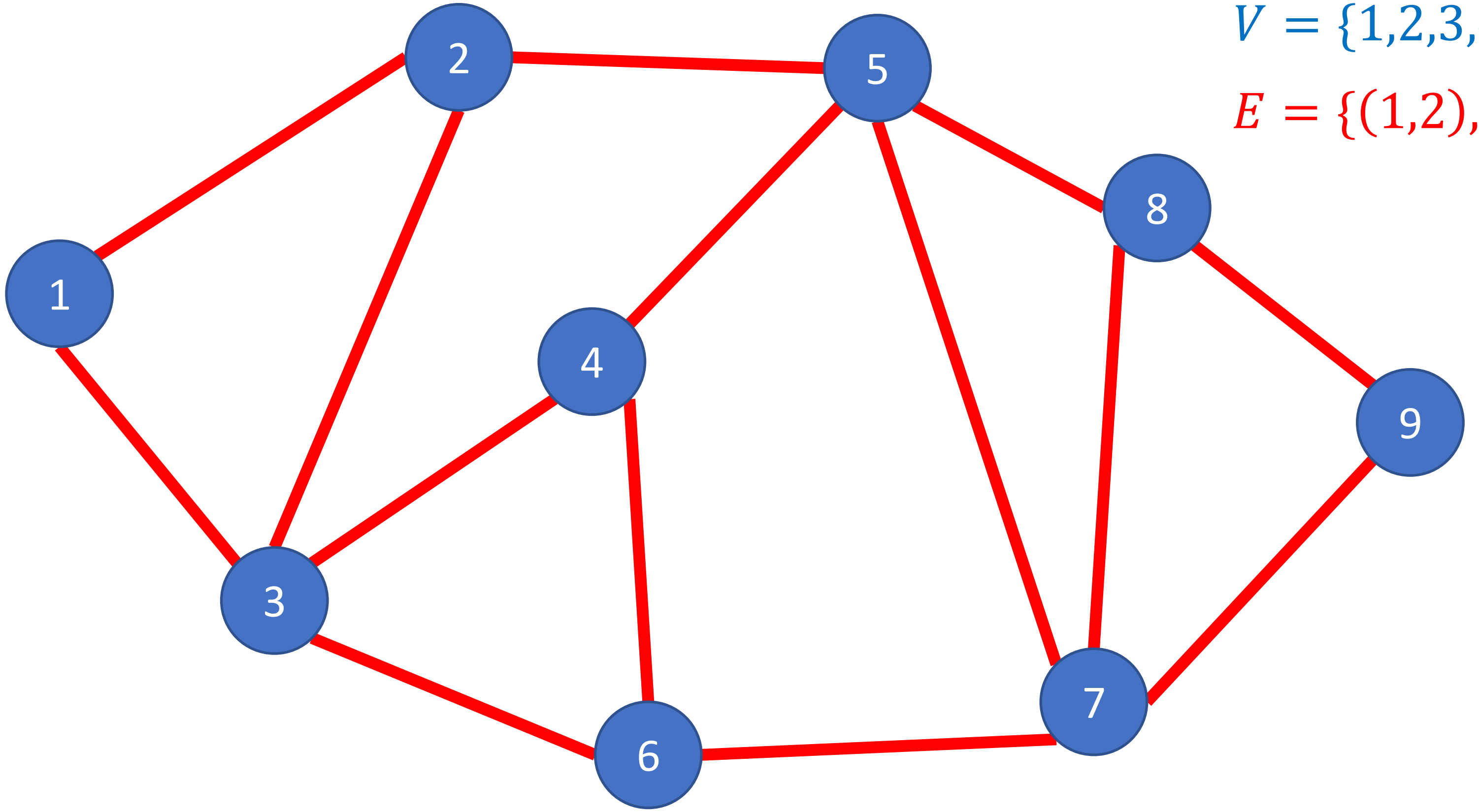
Undirected Graphs

Vertices/Nodes

Definition: $G = (V, E)$

Edges



$V = \{1,2,3,4,5,6,7,8,9\}$

$E = \{(1,2), (2,3), (1,3), \ldots\}$

Directed Graphs

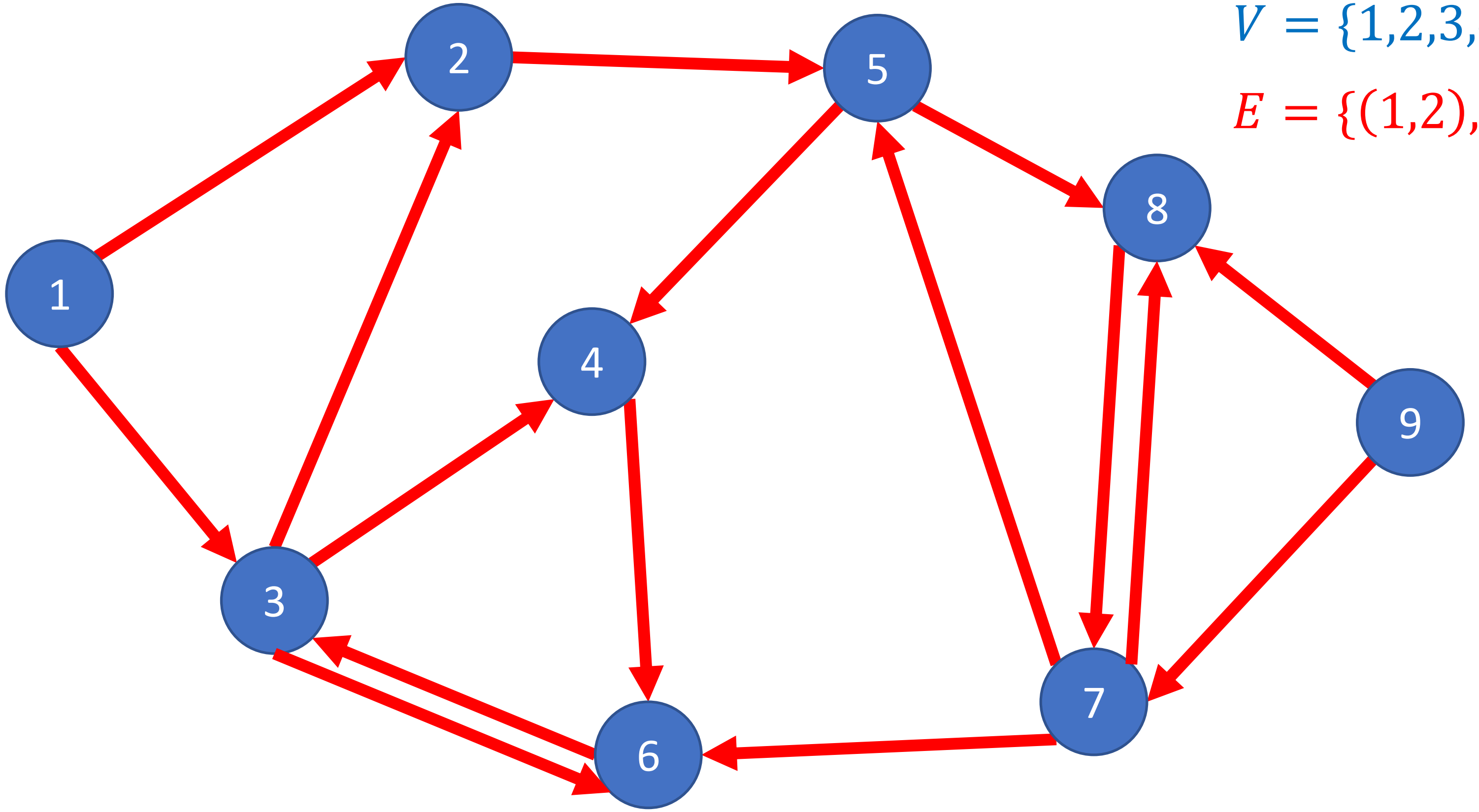
Vertices/Nodes

Definition: $G = (V, E)$

Edges

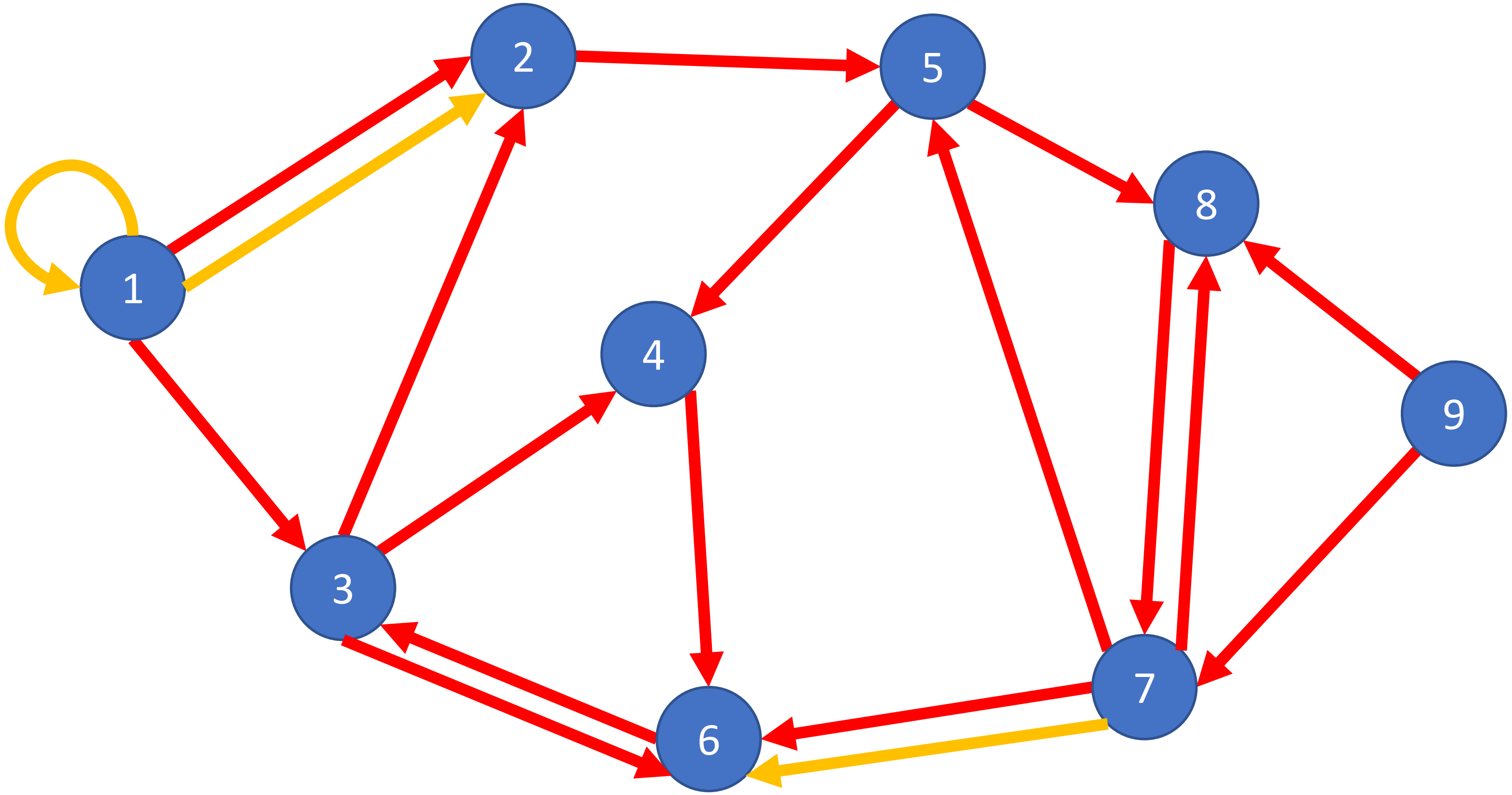
$V = \{1,2,3,4,5,6,7,8,9\}$

$E = \{(1,2), (2,3), (1,3), \dots\}$



Self-Edges and Duplicate Edges

Some graphs may have duplicate edges (e.g. here we have the edge (1,2) twice).
Some may also have self-edges (e.g. here there is an edge from 1 to 1).
Graph with Neither self-edges nor duplicate edges are called **simple graphs**



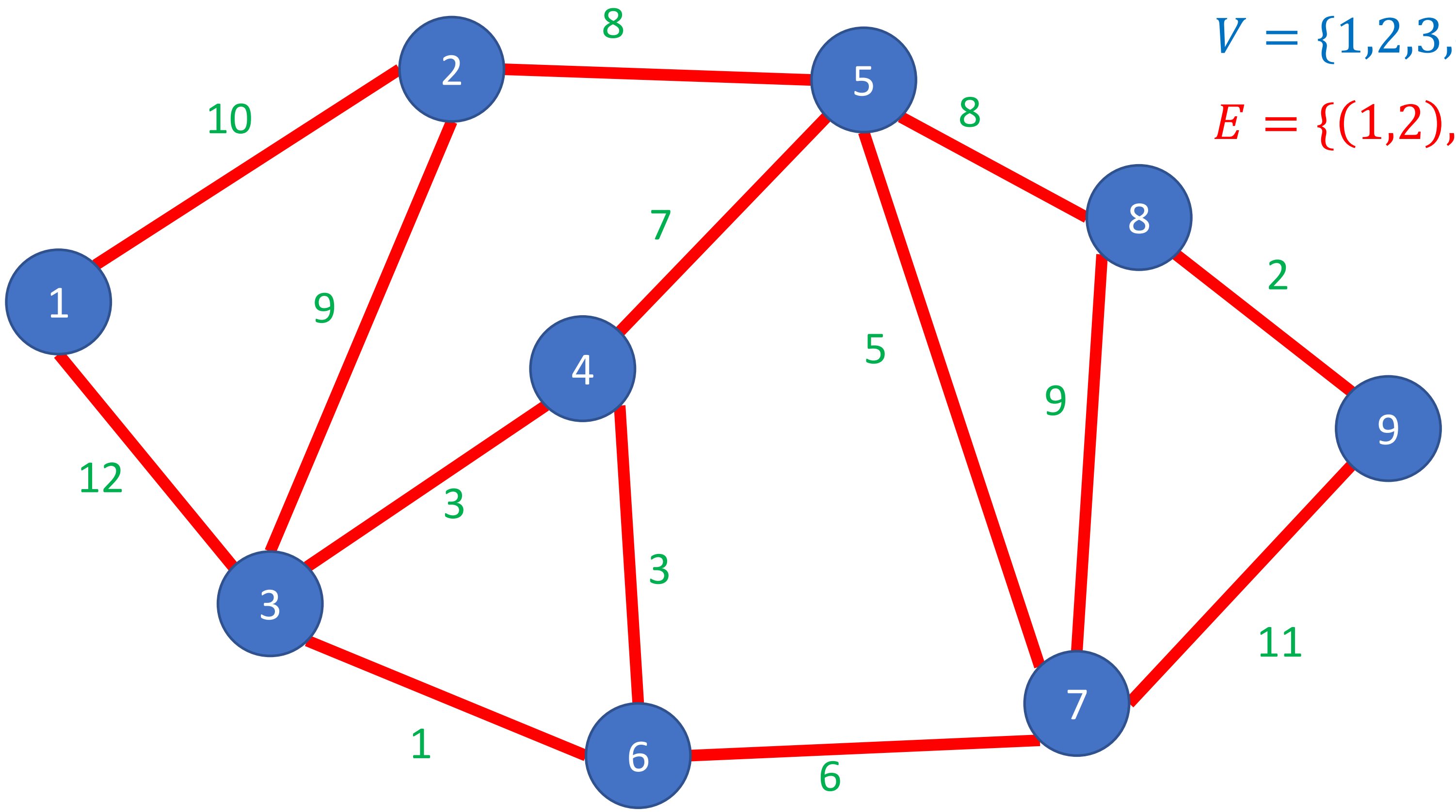
Weighted Graphs

Vertices/Nodes

Definition: $G = (V, E)$

Edges

$w(e)$ = weight of edge e



$V = \{1,2,3,4,5,6,7,8,9\}$

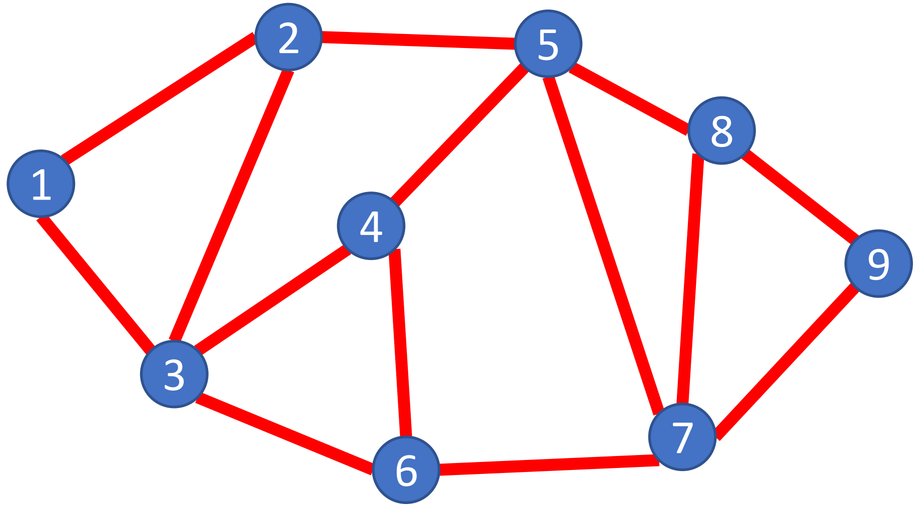
$E = \{(1,2), (2,3), (1,3), \ldots\}$

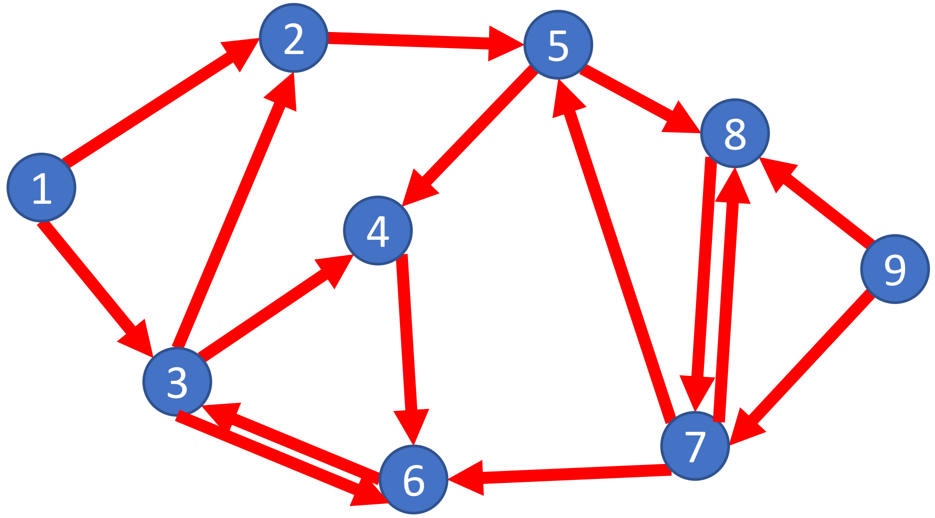
Graph Applications

- For each application below, consider:
  - What are the nodes, what are the edges?
  - Is the graph directed?
  - Is the graph simple?
  - Is the graph weighted?
- LinkedIn Connections
- Twitter Followers
- Java Inheritance
- Airline Routes
- Course Prerequisites

Some Graph Terms

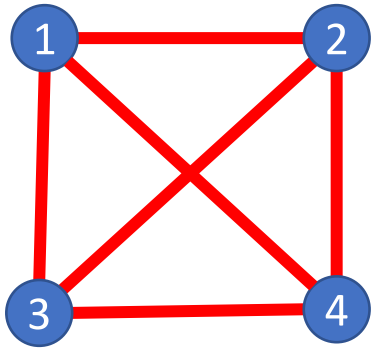
- Adjacent/Neighbors
  - Nodes are adjacent/neighbors if they share an edge
- Degree
  - Number of edges “touching” a vertex
- Indegree
  - Number of incoming edges
- Outdegree
  - Number of outgoing edges



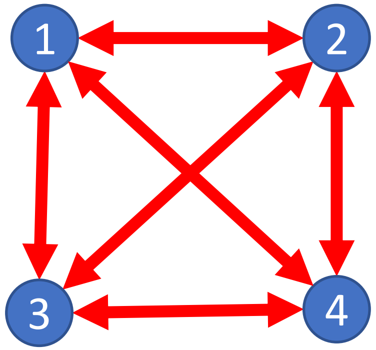


Definition: Complete Graph

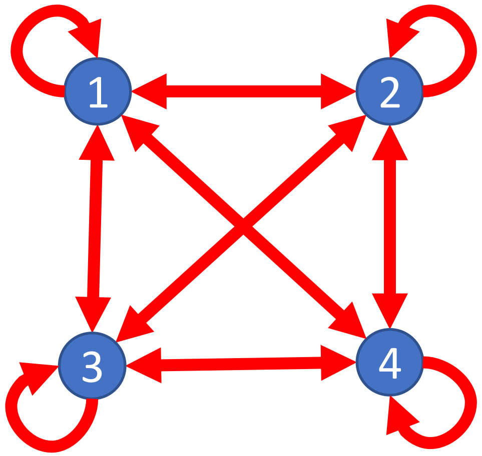
A Graph $G = (V, E)$ s.t. for any pair of nodes $v_1, v_2 \in V$ there is an edge from v_1 to v_2



Complete
Undirected Graph



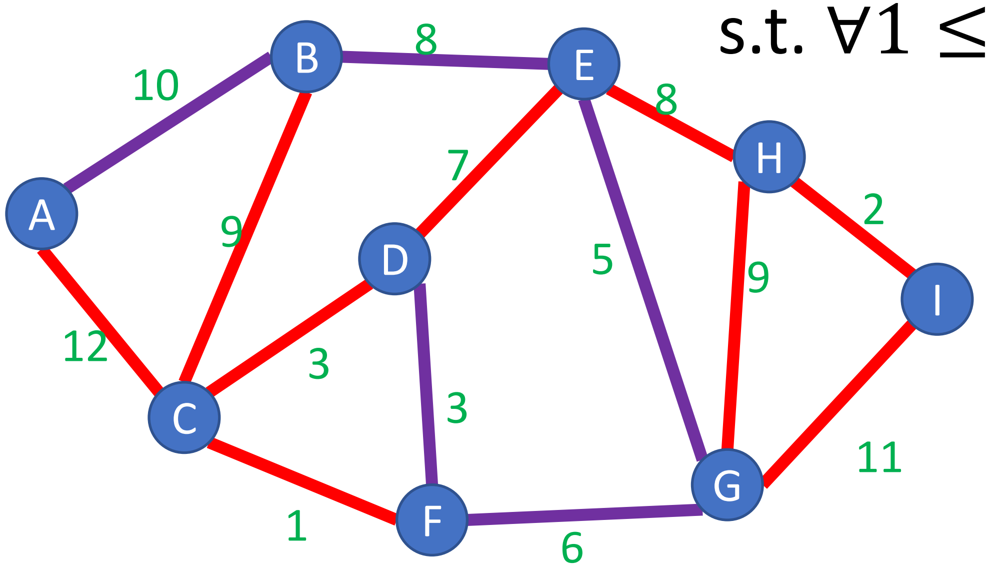
Complete
Directed Graph



Complete Directed
Non-simple Graph

Definition: Path

A sequence of nodes $(v_1, v_2, \dots, v_k)$
s.t. $\forall 1 \leq i \leq k-1, (v_i, v_{i+1}) \in E$

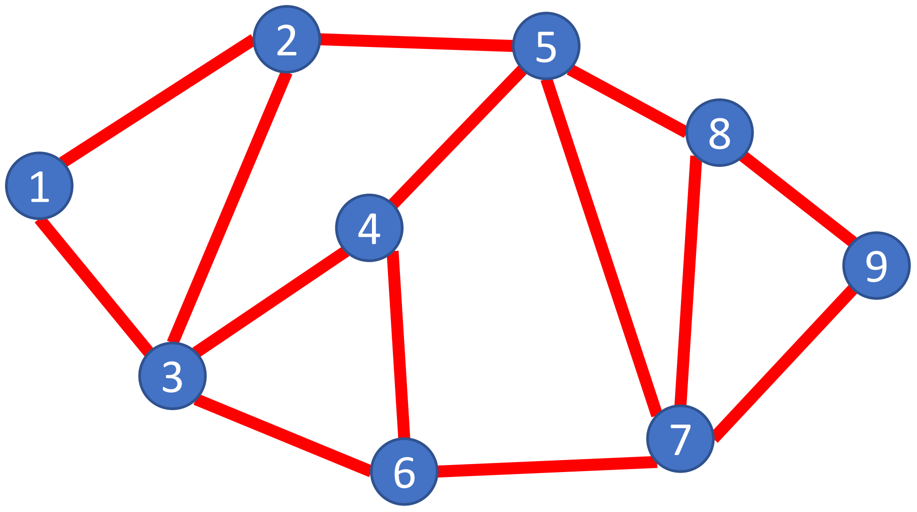


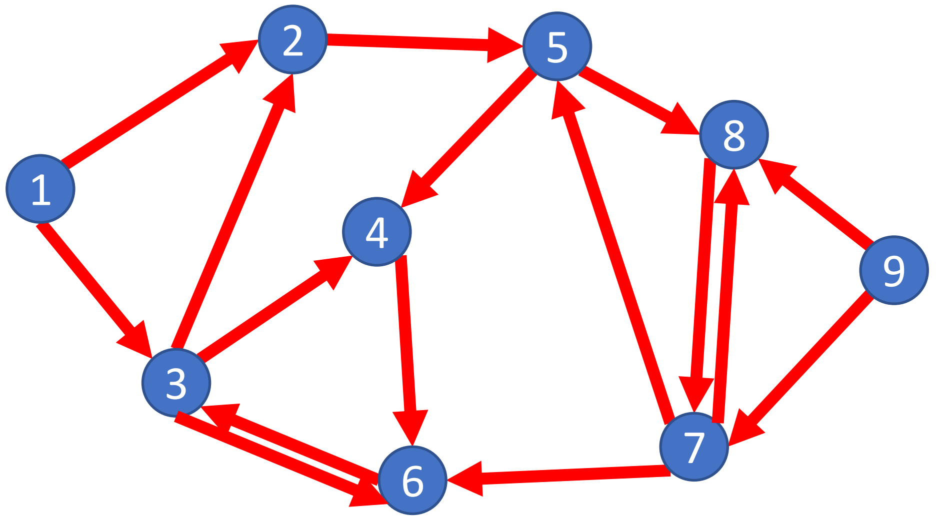
Simple Path:
A path in which each node appears at most once

Cycle:
A path which starts and ends in the same place

Definition: (Strongly) Connected Graph

A Graph $G = (V, E)$ s.t. for any pair of nodes $v_1, v_2 \in V$ there is a path from v_1 to v_2





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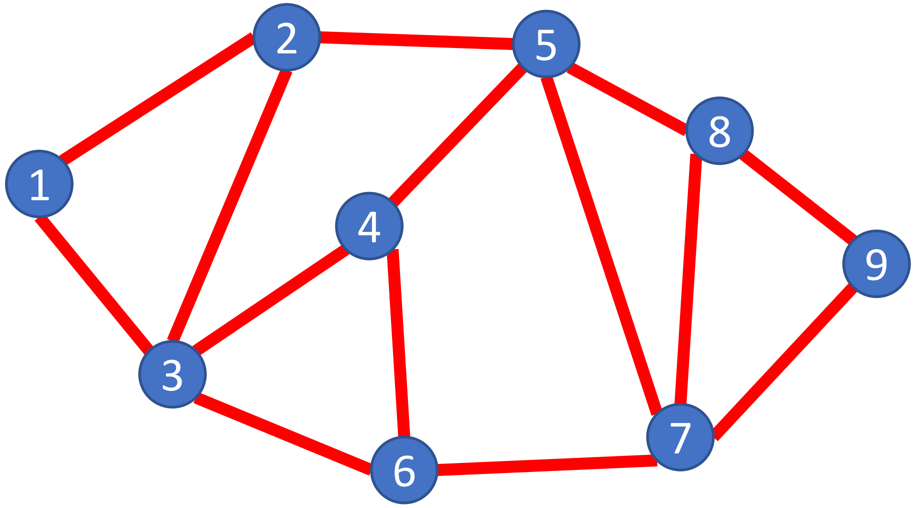
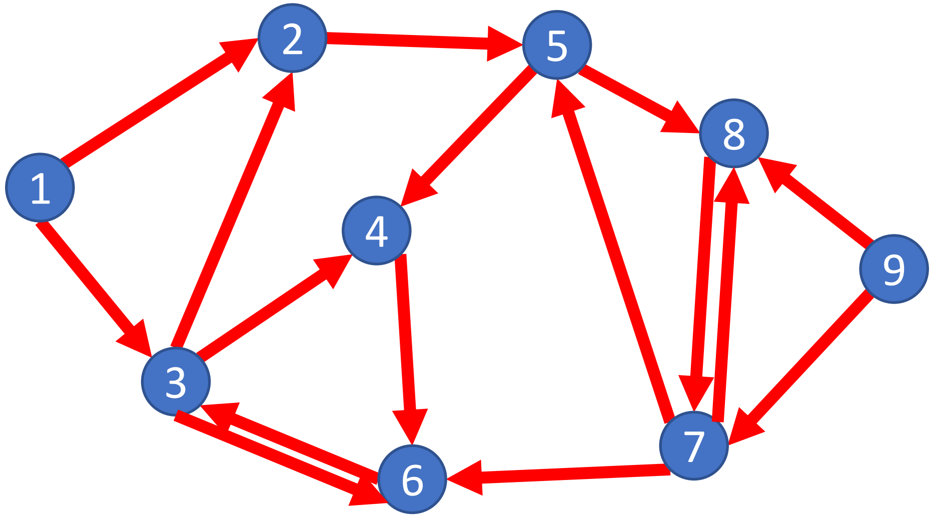


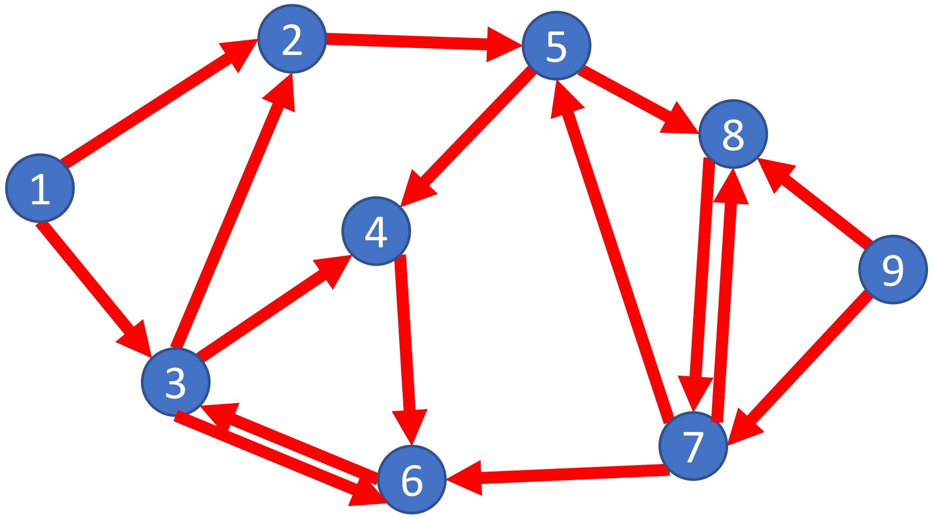
Connected



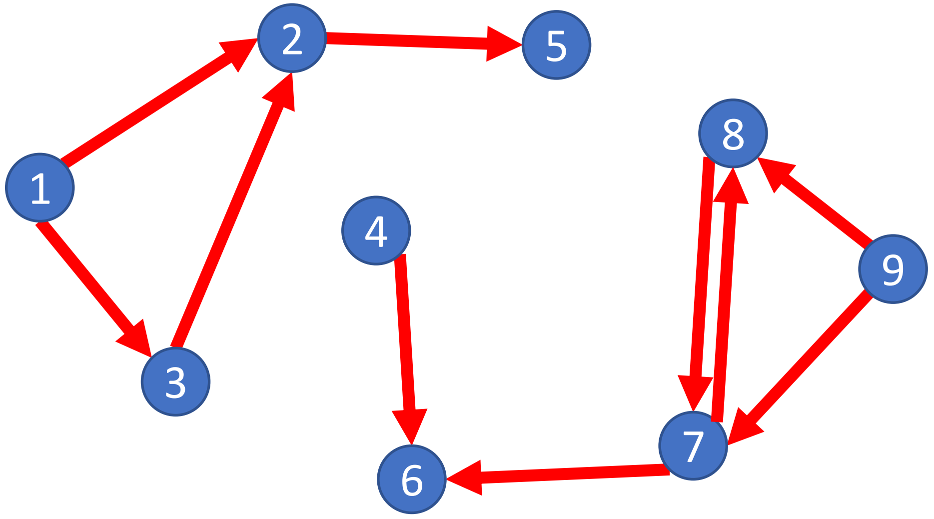
Not (strongly) Connected

Definition: Weakly Connected Graph

A Graph $G = (V, E)$ s.t. for any pair of nodes $v_1, v_2 \in V$ there is a path from v_1 to v_2 ignoring direction of edges



Weakly Connected



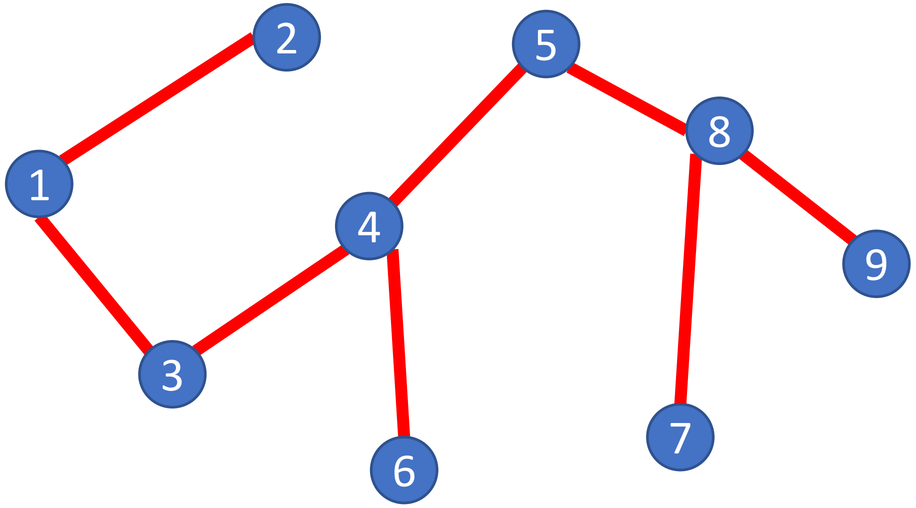
Not Weakly Connected

Graph Density, Data Structures, Efficiency

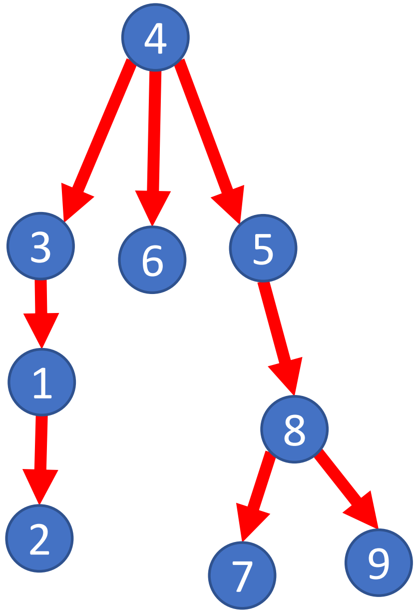
- The maximum number of edges in a graph is $\Theta(|V|^2)$:
  - Undirected and simple: $\frac{|V|(|V|-1)}{2}$
  - Directed and simple: $|V|(|V|-1)$
  - Direct and non-simple (but no duplicates): $|V|^2$
- If the graph is connected, the minimum number of edges is $|V|-1$
- If $|E| \in \Theta(|V|^2)$ we say the graph is **dense**
- If $|E| \in \Theta(|V|)$ we say the graph is **sparse**
- Because $|E|$ is not always near to $|V|^2$ we do not typically substitute $|V|^2$ for $|E|$ in running times, but leave it as a separate variable
  - However, $\log(|E|) \in \Theta(\log(|V|))$

Definition: Tree

A Graph $G = (V, E)$ is a tree if it is undirect, connected, and has no cycles (i.e. is acyclic). Often one node is identified as the "root"



A Tree



A Rooted Tree