CSE 332 Autumn 2024
Lecture 12: hashing

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Next topic: Hash Tables

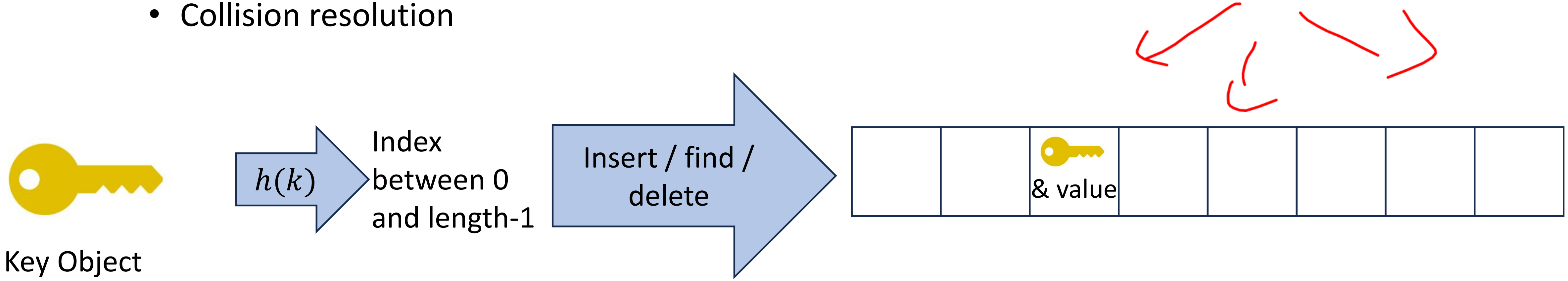
Data Structure	Time to insert	Time to find	Time to delete
Unsorted Array	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Unsorted Linked List	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Sorted Array	$\Theta(n)$	$\Theta(\log n)$	$\Theta(n)$
Sorted Linked List	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Binary Search Tree	$\Theta(\text{height})$	$\Theta(\text{height})$	$\Theta(\text{height})$
AVL Tree	$\Theta(\log n)$	$\Theta(\log n)$	$\Theta(\log n)$
Hash Table (Worst case)	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Hash Table (Average)	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$

Dictionary (Map) ADT

- Contents:
  - Sets of key+value pairs
  - ~~Keys must be comparable~~
- Operations:
  - insert(key, value)
    - Adds the (key,value) pair into the dictionary
    - If the key already has a value, overwrite the old value
      - Consequence: Keys cannot be repeated
  - find(key)
    - Returns the value associated with the given key
  - delete(key)
    - Remove the key (and its associated value)

Hash Tables

- Idea:
  - Have a small array to store information
  - Use a **hash function** to convert the key into an index
    - Hash function should "scatter" the keys, behave as if it randomly assigned keys to indices
  - Store key at the index given by the hash function
  - Do something if two keys map to the same place (should be very rare)
    - Collision resolution



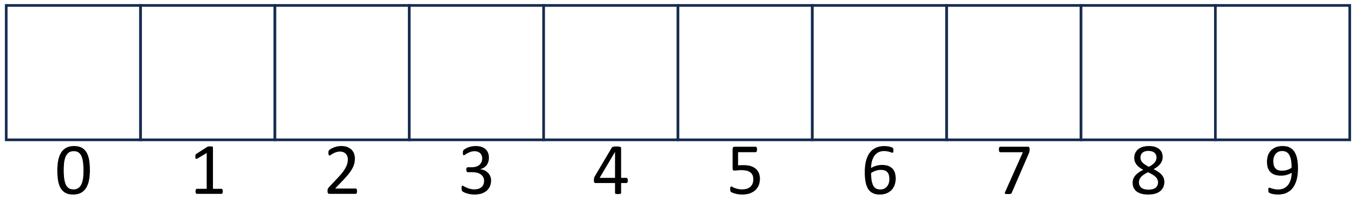
Properties of a "Good" Hash

- Definition: A hash function maps objects to integers

- Should be very efficient
  - Time to calculate the hash should be negligible
- Should "randomly" scatter objects
  - Even similar objects should hash to arbitrarily different values
- Should use the entire table
  - There should not be any indices in the table that nothing can hash to
  - Picking a table size that is prime helps with this
- Should use things needed to "identify" the object
  - Use only fields you would check for a .equals method be included in calculating the hash
    - $\{\text{fields used for hashing}\} \subseteq \{\text{fields used for .equals}\}$
  - More fields typically leads to fewer collisions, but less efficient calculation

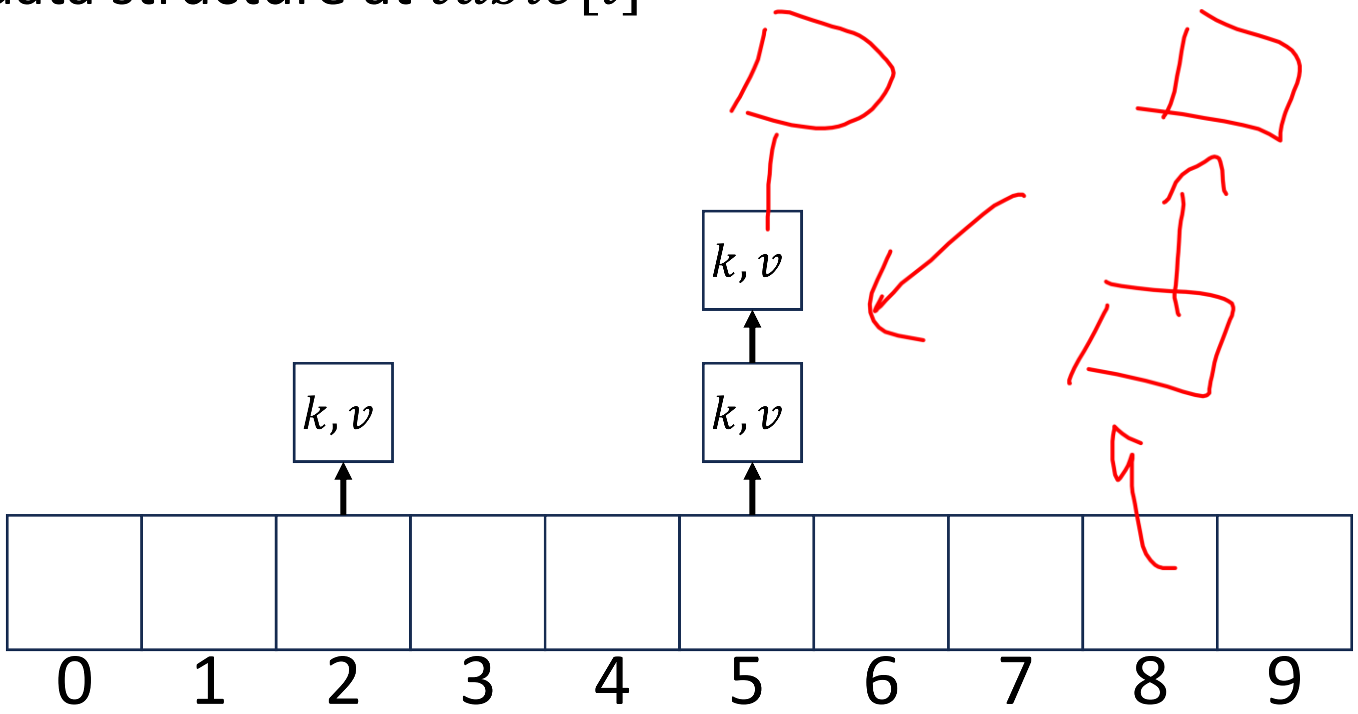
Collision Resolution

- A Collision occurs when we want to insert something into an already-occupied position in the hash table
- 2 main strategies:
  - Separate Chaining
    - Use a secondary data structure to contain the items
      - E.g. each index in the hash table is itself a linked list
  - Open Addressing
    - Use a different spot in the table instead
      - Linear Probing
      - Quadratic Probing
      - Double Hashing



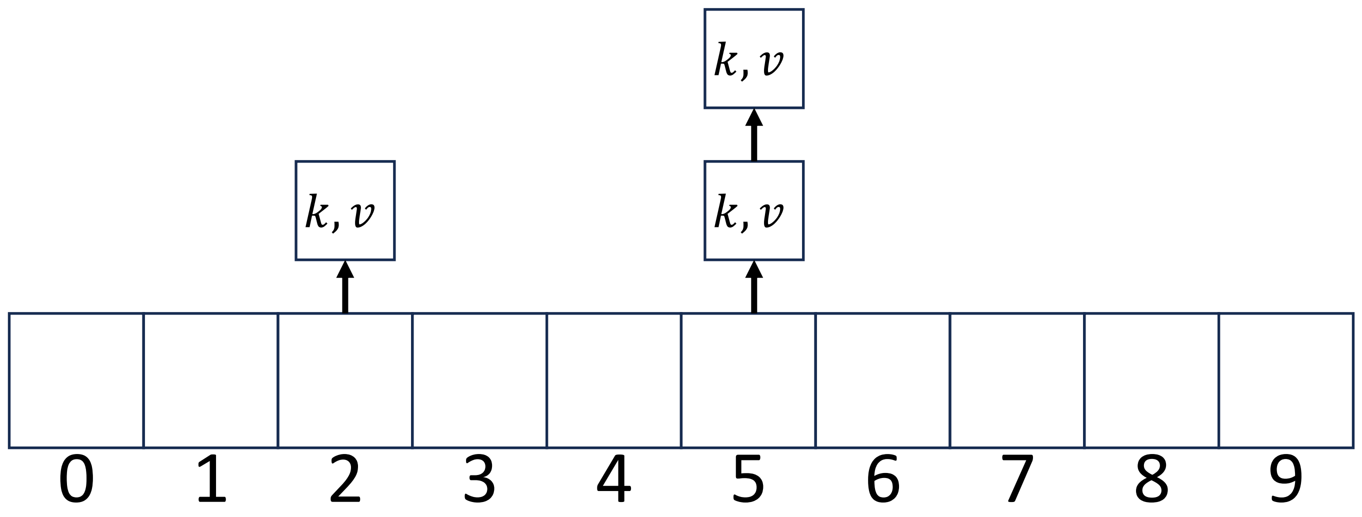
Separate Chaining Insert

- To insert k, v:
  - Compute the index using $i = h(k) \ \% \ length$
  - Add the key-value pair to the data structure at $table[i]$



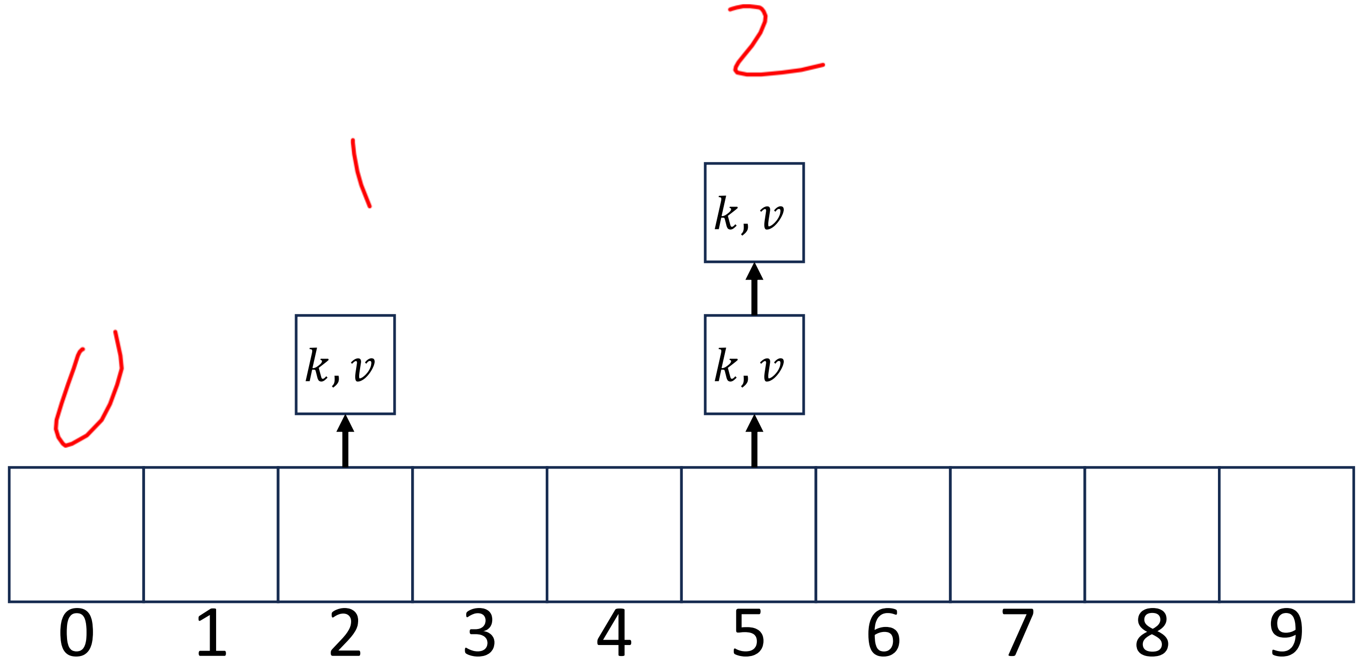
Separate Chaining Find

- To find k:
  - Compute the index using $i = h(k)\ \%\ length$
  - Call find with the key on the data structure at $table[i]$



Separate Chaining Delete

- To delete k:
  - Compute the index using $i = h(k) \% length$
  - Call delete with the key on the data structure at $table[i]$



Formal Running Time Analysis

- The **load factor** of a hash table represents the average number of items per "bucket"
  - $\lambda = \frac{n}{length}$
- Assume we have a hash table that uses a linked-list for separate chaining
  - What is the expected number of comparisons needed in an unsuccessful find?
    - Procedure: Apply the hash function to the given key. This provides an index. Next, do a find on the linked list found at that index.
      - Time to run the hash function (small) + time to do a find on the LL, which is λ
  - What is the expected number of comparisons needed in a successful find?
    - $\frac{\lambda}{2}$
- How can we make the expected running time $\Theta(1)$?
  - Right now: $\Theta(\lambda)$
  - $\lambda \leq c$ meaning $length \leq c \cdot n$

Formal Running Time Analysis

- The **load factor** of a hash table represents the average number of items per "bucket"
  - $\lambda = \frac{n}{length}$
- Assume we have a hash table that uses a linked-list for separate chaining
  - What is the expected number of comparisons needed in an unsuccessful find?
    - Will hash to an index, then compare to all items in that separate chain
      - λ
  - What is the expected number of comparisons needed in a successful find?
    - Will hash to an index, then compare to half of the items in that separate chain.
      - $\frac{\lambda}{2}$
- How can we make the expected running time $\Theta(1)$?
  - Make $length \leq c \cdot n$ so that $\lambda \leq c$

Rehashing

- If your load factor λ gets too large, copy everything over to a larger hash table
  - To do this: make a new, larger array
  - Re-insert all items into the new hash table by reapplying the hash function
    - We need to reapply the hash function because items should map to a different index
  - New array should be "roughly" double the length (but probably still want it to be prime)
- What does "too large" mean?
  - For separate chaining, typically we want $\lambda < 2$
  - For open addressing, typically we want $\lambda < \frac{1}{2}$

Hash Tables Running Time

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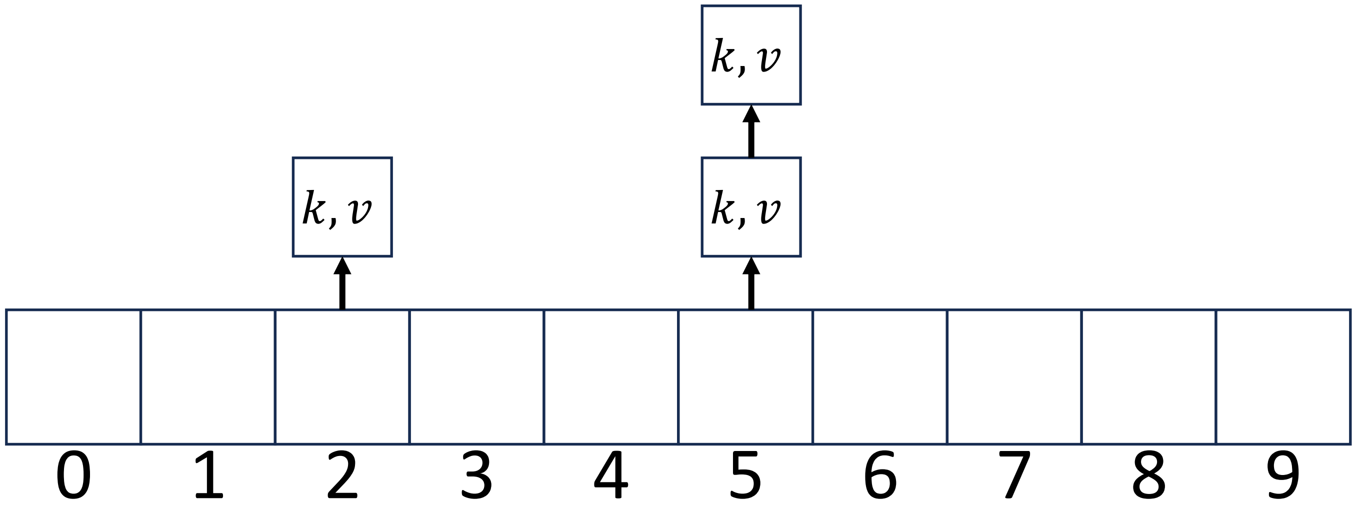
Hash Table (Expected and Amortized)	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$

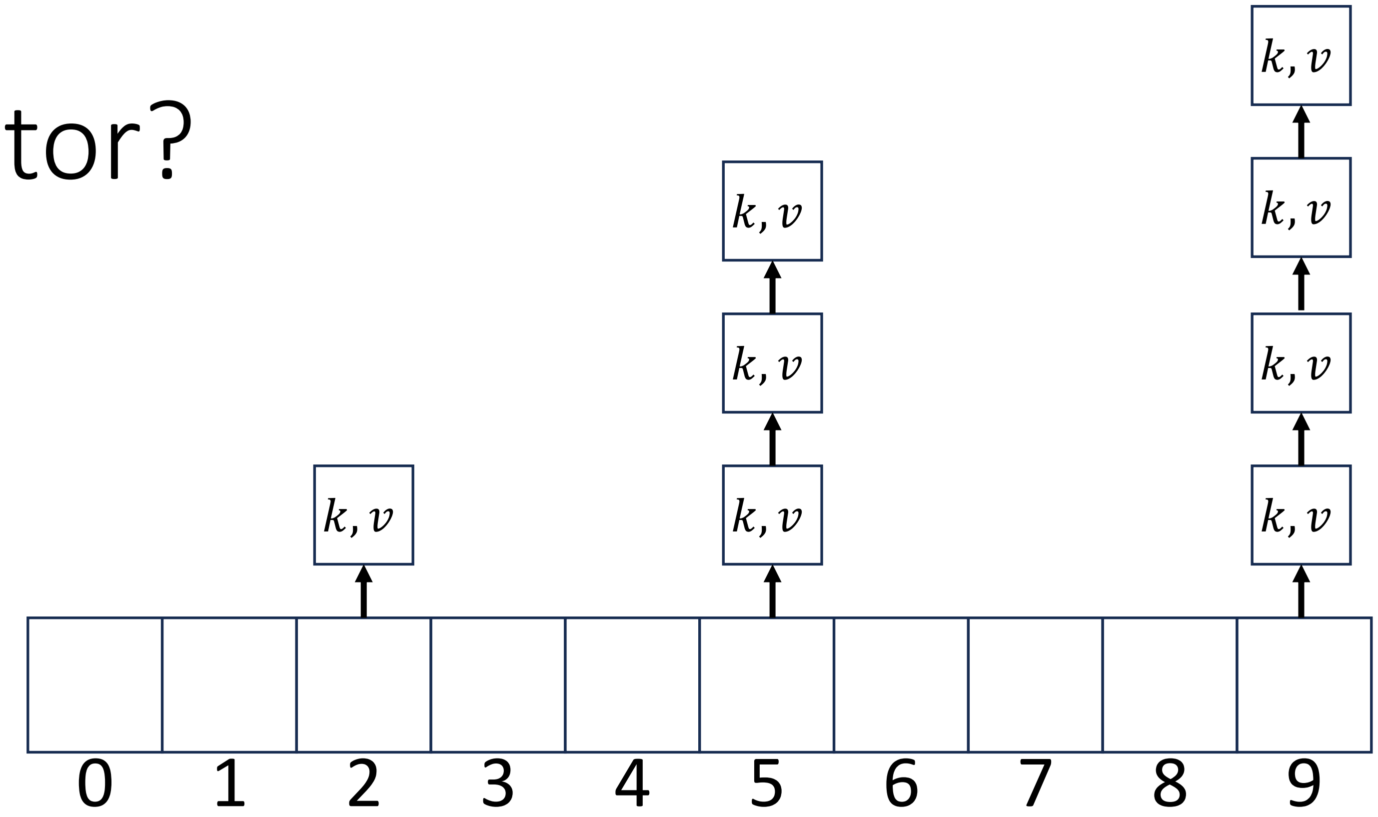
Load Factor?

$$\frac{n}{len}$$



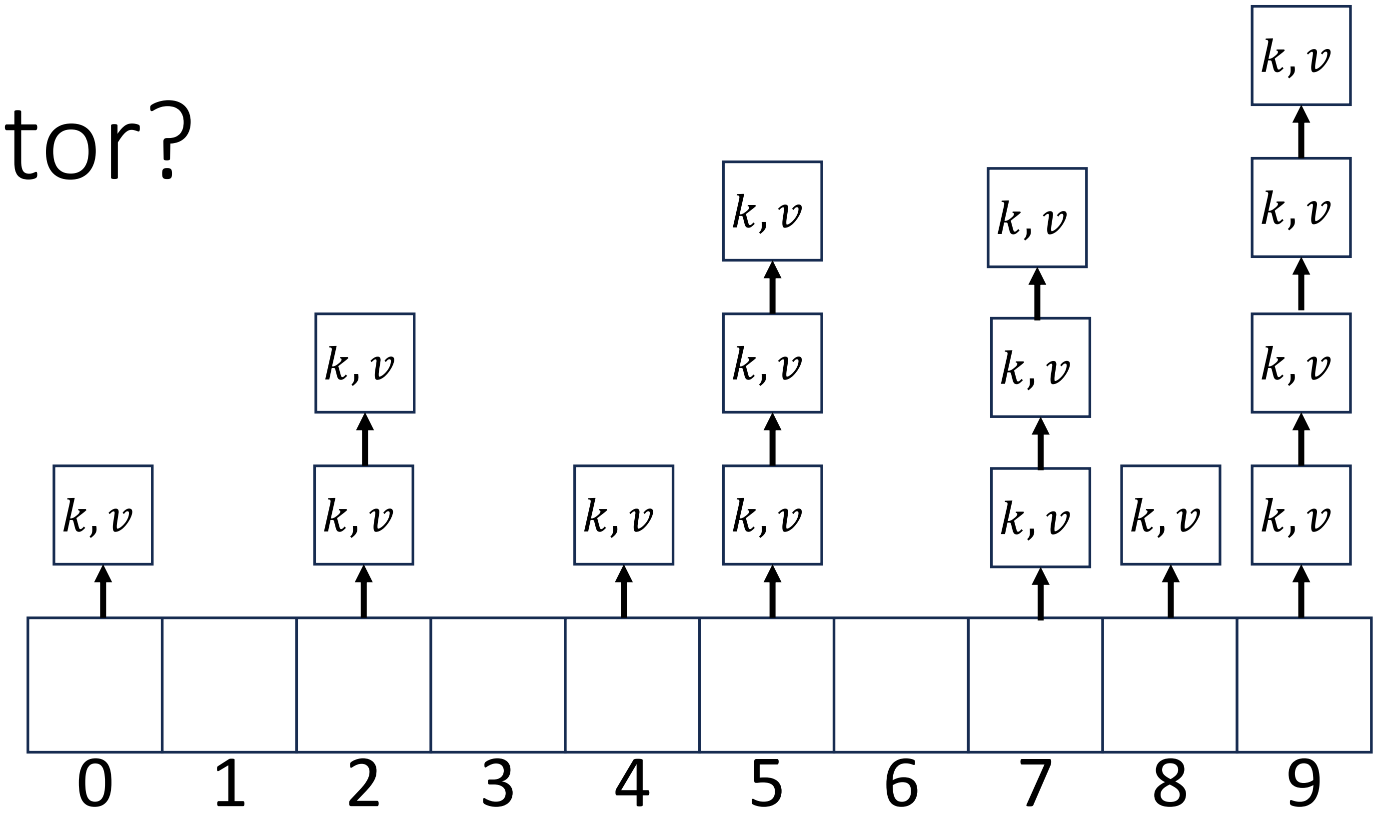
$$\frac{3}{10}$$

Load Factor?



$\frac{8}{10}$

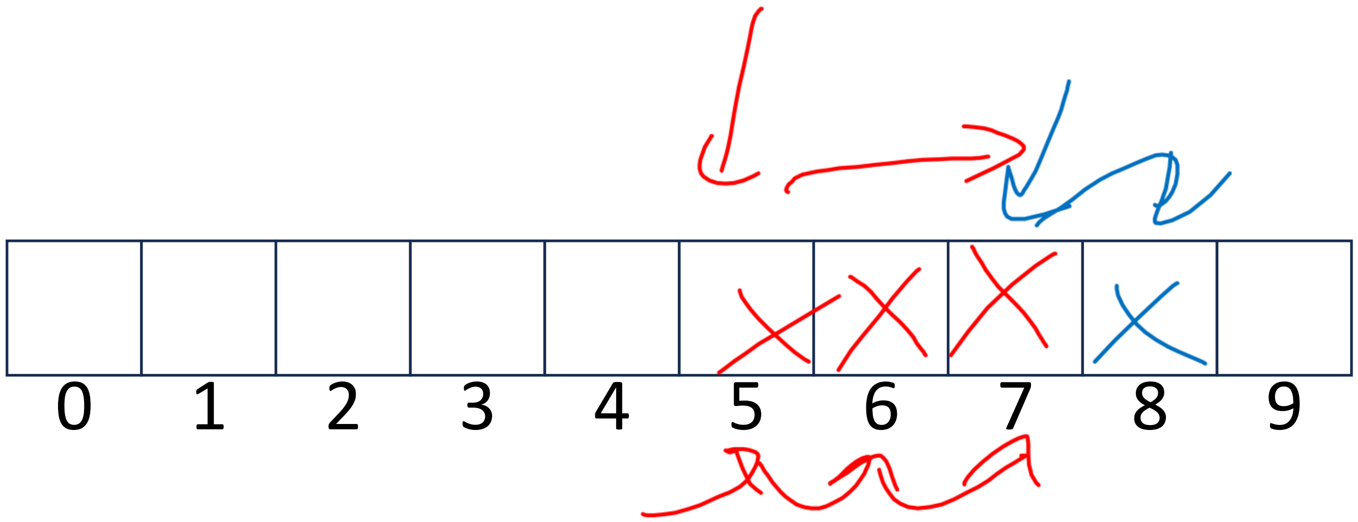
Load Factor?



$$\frac{3}{2} = \frac{15}{10}$$

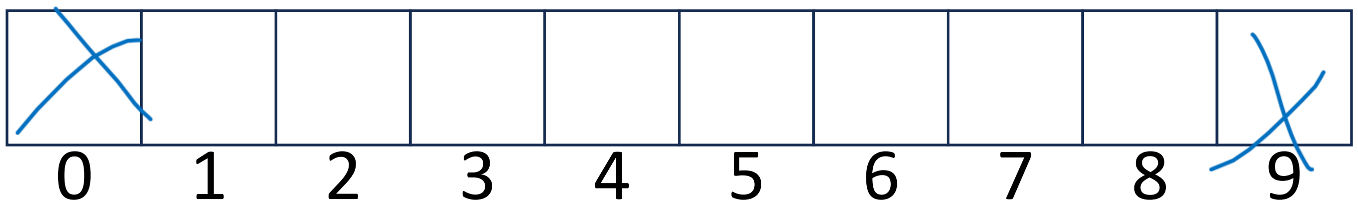
Collision Resolution: Linear Probing

- When there's a collision, use the next open space in the table

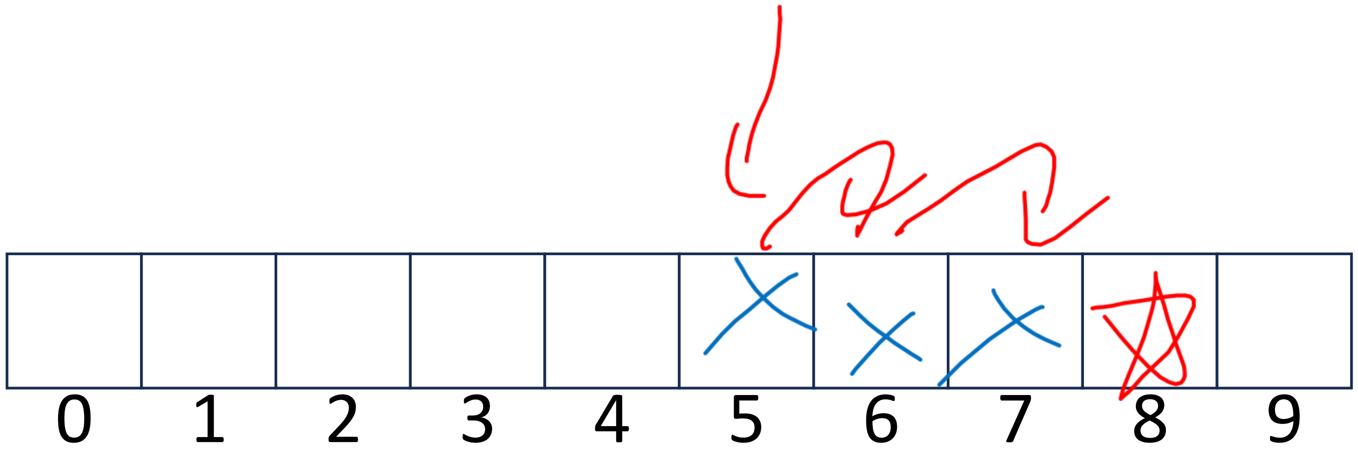


Linear Probing: Insert Procedure

- To insert k, v
  - Calculate $i = h(k)\ \%\ length$
  - If $table[i]$ is occupied then try $(i+1)\%\ length$
  - If that is occupied try $(i+2)\%\ length$
  - If that is occupied try $(i+3)\%\ length$
  - …

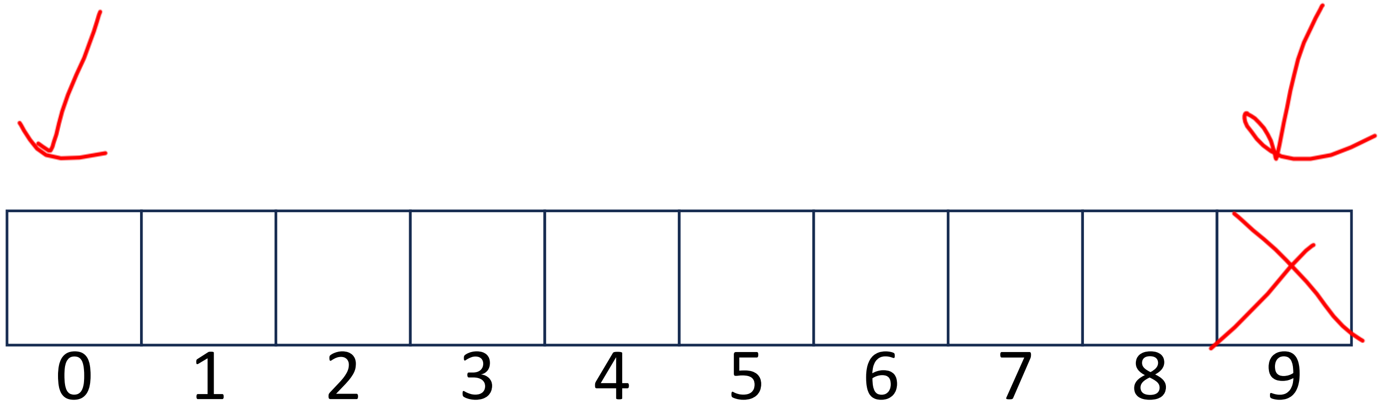


Linear Probing: Find



Linear Probing: Find

- To find key k
  - Calculate $i = h(k)\ \%\ length$
  - If $table[i]$ is occupied and does not contain k then look at $(i + 1)\ \%\ length$
  - If that is occupied and does not contain k then look at $(i + 2)\ \%\ length$
  - If that is occupied and does not contain k then look at $(i + 3)\ \%\ length$
  - Repeat until you either find k or else you reach an empty cell in the table



Linear Probing: Delete

- Suppose A, B, C, D, and E all hashed to 3
- Now let's delete B

Before:

0	1	2	3	4	5	6	7	8	9
			A	B	C	D	E		

After:

0	1	2	3	4	5	6	7	8	9
			A	C	D	E			

Linear Probing: Delete

- Suppose A, B, and E all hashed to 3, and C and D hashed to 5
- Now let's delete B

Before:

0	1	2	3	4	5	6	7	8	9
			A	B	C	D	E		

After:

0	1	2	3	4	5	6	7	8	9
			A						

Linear Probing: Delete

- Suppose A and E hashed to 3, and B,C, and D hashed to 4
- Now let's delete B

Before:

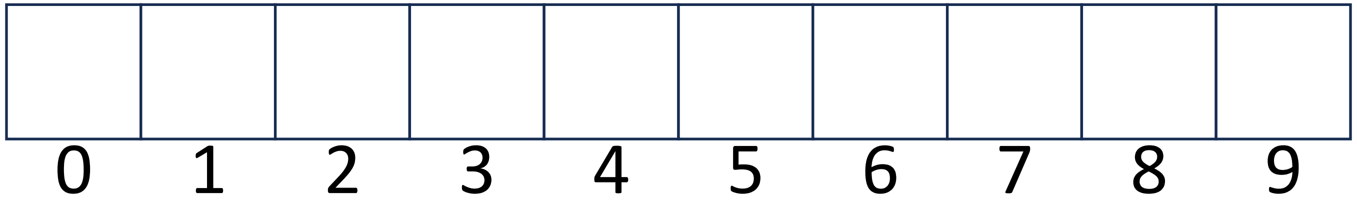
			A	B	C	D	E		
0	1	2	3	4	5	6	7	8	9

After:

0	1	2	3	4	5	6	7	8	9

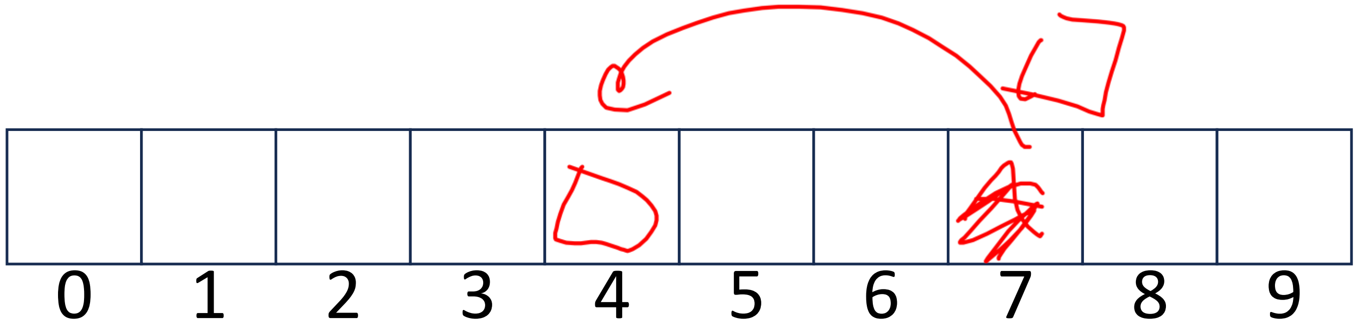
Linear Probing: Delete

- Let’s do this together!



Linear Probing: Delete

- To delete key k, where $h(k) = i$
  - Assume it is present
- Beginning at index i, probe until we find k (call this location index j)
- Mark j as empty (e.g. null), then continue probing while doing the following until you find another empty index
  - If you come across a key which hashes to a value $\leq j$ then move that item to index j and update j.



Linear Probing: Delete

- Option 1: Fill in with items that hashed to before the empty slot
- Option 2: “Tombstone” deletion. Leave a special object that indicates an object was deleted from there
  - The tombstone does not act as an open space when finding (so keep looking after its reached)
  - When inserting you can replace a tombstone with a new item

k, v		k, v	k, v		k, v				
0	1	2	3	4	5	6	7	8	9

Linear Probing + Tombstone: Find

- To find key k
  - Calculate $i = h(k) \ \% \ length$
  - While $table[i]$ has a tombstone or a key other than k, $i = (i + 1) \ \% \ length$
  - If you come across k return $table[i]$
  - If you come across an empty index, the find was unsuccessful

0	1	2	3	4	5	6	7	8	9

Linear Probing + Tombstone: Insert

- To insert k, v
  - Calculate $i = h(k)\ \%\ length$
  - While $table[i]$ has a key other than k, $i = (i + 1)\ \%\ length$
    - If $table[i]$ has a tombstone, set $x = i$
      - That is where we will insert if the find is unsuccessful
  - If you come across k, set $table[i] = k, v$
  - If you come across an empty index, the find was unsuccessful
    - Set $table[x] = k, v$ if we saw a tombstone
    - Set $table[i] = k, v$ otherwise

0	1	2	3	4	5	6	7	8	9

Downsides of Linear Probing

- What happens when λ approaches 1?
  - Get longer and longer contiguous blocks
  - A collision is guaranteed to grow a block
    - Larger blocks experience more collisions
    - Feedback loop!
- What happens when λ exceeds 1?
  - Impossible!
  - You can't insert more stuff

Quadratic Probing: Insert Procedure

- To insert k, v
  - Calculate $i = h(k)\ \%\ size$
  - If $table[i]$ is occupied then try $(i + 1^2)\%\ size$
  - If that is occupied try $(i + 2^2)\%\ size$
  - If that is occupied try $(i + 3^2)\%\ size$
  - If that is occupied try $(i + 4^2)\%\ size$
  - …

0	1	2	3	4	5	6	7	8	9

Quadratic Probing: Example

- Insert:
  - 76
  - 40
  - 48
  - 5
  - 55
  - 47

0	1	2	3	4	5	6

Using Quadratic Probing

- If you probe $tablesize$ times, you start repeating the same indices
- If $tablesize$ is prime and $\lambda < \frac{1}{2}$ then you're guaranteed to find an open spot in at most $tablesize/2$ probes

- Helps with the clustering problem of linear probing, but does not help if many things hash to the same value

Double Hashing: Insert Procedure

- Given h and g are both good hash functions
- To insert k, v
  - Calculate $i = h(k)\ \%\ size$
  - If $table[i]$ is occupied then try $\left(i + g(k)\right)\ \%\ size$
  - If that is occupied try $\left(i + 2 \cdot g(k)\right)\%\ size$
  - If that is occupied try $\left(i + 3 \cdot g(k)\right)\%\ size$
  - If that is occupied try $\left(i + 4 \cdot g(k)\right)\%\ size$
  - …