CSE 332: Data Structures & Parallelism

Lecture 20b: Graph Traversals

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Graph Traversals

Next problem: For an arbitrary graph and a starting node **v**, find all nodes *reachable* (i.e., there exists a path) from **v**

- Possibly "do something" for each node (an iterator!)
  - E.g. Print to output, set some field, etc.

Related Questions:

- Is an undirected graph connected?
- Is a directed graph weakly / strongly connected?
  - For strongly, need a cycle back to starting node

Basic idea:

- Keep following nodes
- But "mark" nodes after visiting them, so the traversal terminates and processes each reachable node exactly once

Graph Traversal: Abstract Idea

```
traverseGraph(Node start) {
    Set pending = emptySet();
    pending.add(start)
   mark start as visited
   while(pending is not empty) {
     next = pending.remove()
     for each node u adjacent to next
        if(u is not marked) {
          mark u
          pending.add(u)
        }
    }
}
```

Running time and options

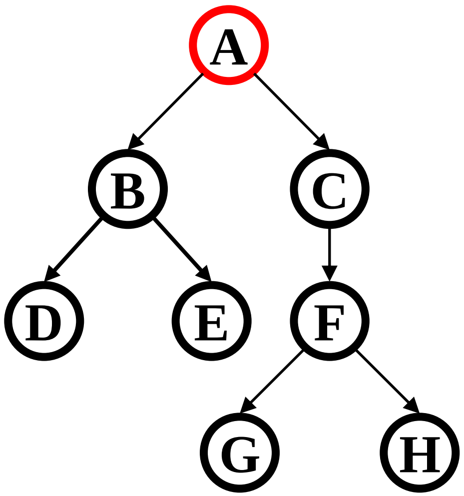
- Assuming add and remove are $O(1)$, entire traversal is $O(|E|)$
  - Use an adjacency list representation

- The order we traverse depends entirely on how add and remove work/are implemented
  - Depth-first graph search (DFS): a stack
  - Breadth-first graph search (BFS): a queue

- DFS and BFS are “big ideas” in computer science
  - Depth: recursively explore one part before going back to the other parts not yet explored
  - Breadth: Explore areas closer to the start node first

Recursive DFS, Example : trees

- A tree is a graph and DFS and BFS are particularly easy to "see"

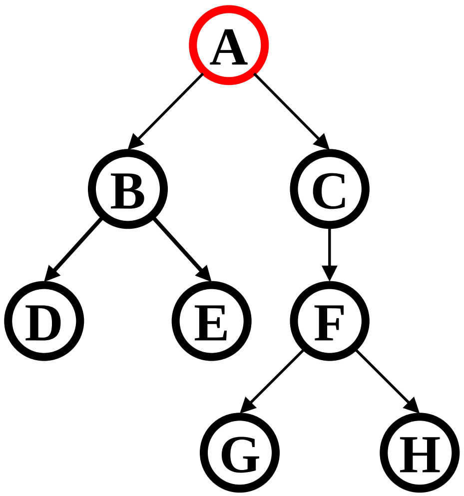



```
DFS(Node start) {
  mark and "process"(e.g. print) start
  for each node u adjacent to start
    if u is not marked
      DFS(u)
}
```

Order processed: A, B, D, E, C, F, G, H

- Exactly what we called a "pre-order traversal" for trees
- The marking is not needed here, but we need it to support arbitrary graphs , we need a way to process each node exactly once

DFS with a stack, Example: trees

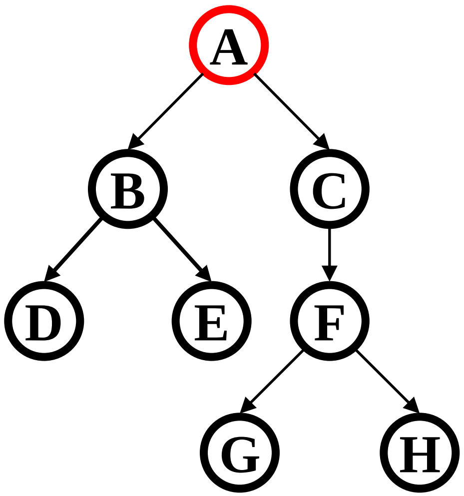



```
DFS2(Node start) {
  initialize stack s to hold start
  mark start as visited
  while(s is not empty) {
    next = s.pop() // and "process"
    for each node u adjacent to next
     if(u is not marked)
       mark u and push onto s
  }
}
```

Order processed:

- A different but perfectly fine traversal

BFS with a queue, Example: trees




```
BFS(Node start) {
  initialize queue q to hold start
  mark start as visited
  while(q is not empty) {
    next = q.dequeue()// and "process"
    for each node u adjacent to next
     if(u is not marked)
       mark u and enqueue onto q
  }
}
```

Order processed:

- A "level-order" traversal

DFS/BFS Comparison

Breadth-first search:

- Always finds shortest paths, i.e., “optimal solutions
  - Better for “what is the shortest path from **`x`** to **`y`**”
- Queue may hold *O*(|V|) nodes (e.g. at the bottom level of binary tree of height h, 2^h nodes in queue)

Depth-first search:

- Can use less space in finding a path
  - If *longest path* in the graph is **`p`** and highest out-degree is **`d`** then DFS stack never has more than **`d*p`** elements

A third approach: *Iterative deepening (IDDFS)*:

  - Try DFS but don’t allow recursion more than **`K`** levels deep.
  - If that fails, increment **`K`** and start the entire search over
- Like BFS, finds shortest paths. Like DFS, less space.

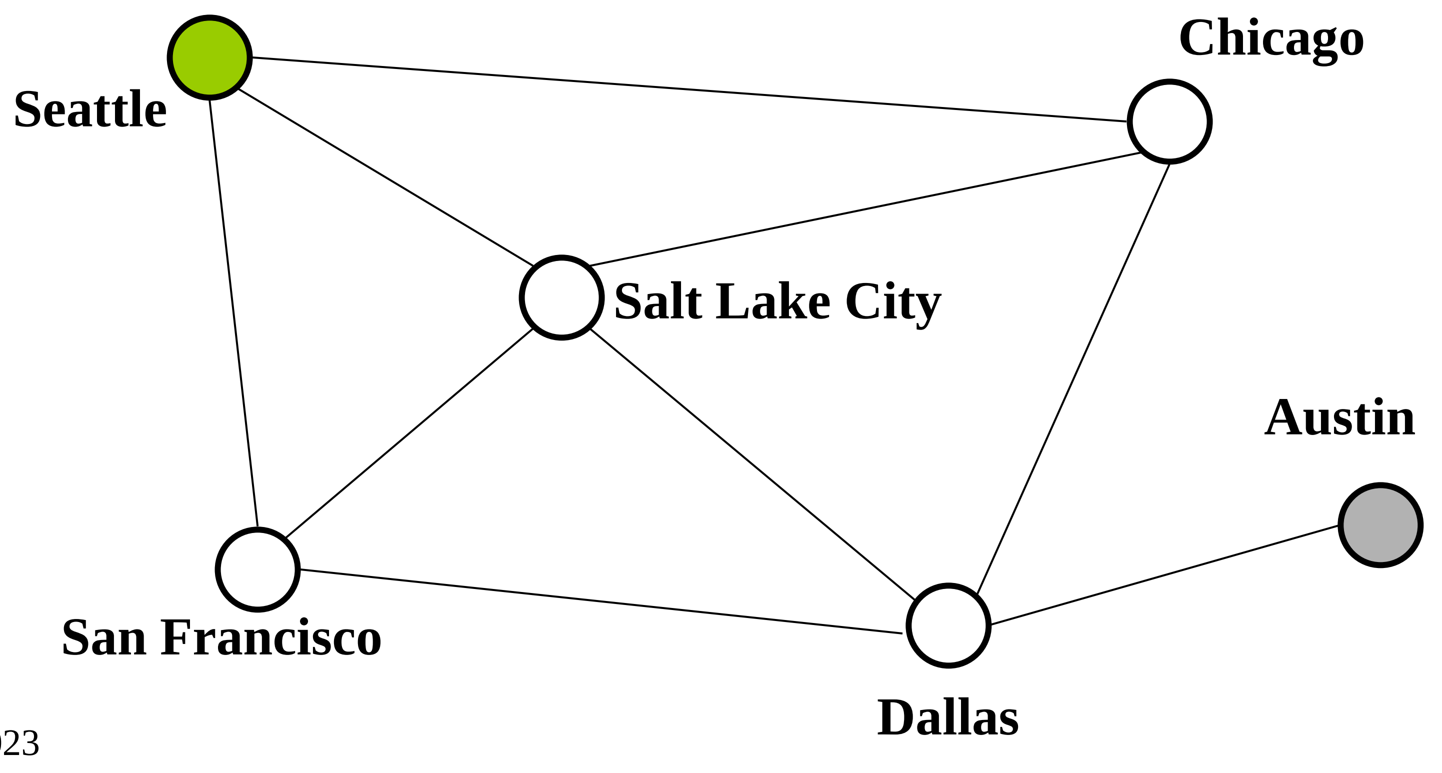
Saving the path

- Our graph traversals can answer the "reachability question":
  - "***Is there*** a path from node x to node y?"
- Q: But what if we want to ***output the actual path***?
  - Like getting driving directions rather than just knowing it's possible to get there!
- A: Like this:
  - Instead of just "marking" a node, store the **previous node** along the path (when processing **`u`** causes us to add **`v`** to the search, set **`v.path`** field to be **`u`**)
  - When you reach the goal, follow **`path`** fields backwards to where you started (and then reverse the answer)
  - If just wanted path *length*, could put the integer distance at each node instead

Example using BFS

What is a path from Seattle to Austin

- Remember marked nodes are not re-enqueued
- Note shortest paths may not be unique



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