CSE 332: Data Structures & Parallelism

Lecture 3: Priority Queues

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Today

- Finish up Intro to Asymptotic Analysis
- New ADT! Priority Queues

Scenario

What is the difference between waiting for service at a pharmacy versus an ER?

Pharmacies usually follow the rule
First Come, First Served

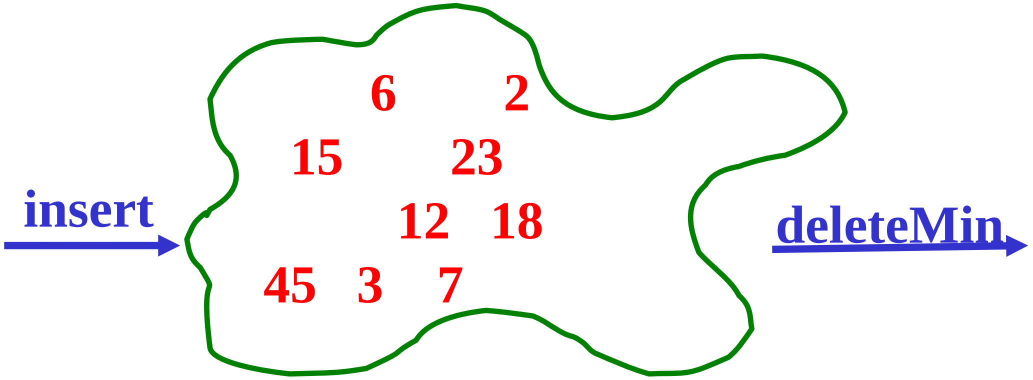
Emergency Rooms assign priorities
based on each individual's need

A new ADT: Priority Queue

- Textbook Chapter 6
  - We will go back to binary search trees (ch4) and hash tables (ch5) later
  - Nice to see a new and surprising data structure first
- A **priority queue** holds *compare-able data*
  - Unlike stacks and queues need to *compare items*
    - Given *x* and *y*, is *x* less than, equal to, or greater than *y*
    - What this means can depend on your data
    - Much of course will require comparable data: e.g. sorting
  - Integers are comparable, so will use them in examples
    - But the priority queue ADT is much more general
    - Typically two fields, the *priority* and the *data*

Priority Queue ADT

- Assume each item has a "priority"
  - The *lesser* item is the one with the *greater* priority
  - So "priority 1" is more important than "priority 4"
  - Just a convention, could also do a maximum priority



- Main Operations:
  - **`insert`**
  - **`deleteMin`**

- Key property: **`deleteMin`** returns and deletes from the queue the item with greatest priority (lowest priority value)
  - Can resolve ties arbitrarily

Aside: We will use ints as data and priority

For simplicity in lecture, we'll often suppose items are just **`int`**s and the **`int`** is also the priority

- So an operation sequence could be

```
insert 6
insert 5
x = deleteMin  // Now x = 5.
```

– **`int`** priorities are common, but really just need comparable

- Not having "other data" is very rare
  - Example: print job has a priority *and* the file to print is the data

Priority Queue Example

To simplify our examples, we will just use the priority values from now on

`insert` *a* with priority *5*

`insert` *b* with priority *3*

`insert` *c* with priority *4*

w = **`deleteMin`**

x = **`deleteMin`**

`insert` *d* with priority *2*

`insert` *e* with priority *6*

y = **`deleteMin`**

z = **`deleteMin`**

`after execution:`

Analogy: `insert` is like `enqueue`, `deleteMin` is like `dequeue`

But the whole point is to use priorities instead of FIFO

Priority Queue Example

To simplify our examples, we will just use the priority values from now on

`insert` *a* with priority *5*

`insert` *b* with priority *3*

`insert` *c* with priority *4*

w = **`deleteMin`**

x = **`deleteMin`**

`insert` *d* with priority *2*

`insert` *e* with priority *6*

y = **`deleteMin`**

z = **`deleteMin`**

`after execution:`

w = *b*

x = *c*

y = *d*

z = *a*

Analogy: `insert` is like `enqueue`, `deleteMin` is like `dequeue`

But the whole point is to use priorities instead of FIFO

Applications

Like all good ADTs, the priority queue arises often

- Sometimes “directly”, sometimes less obvious

- Run multiple programs in the operating system
  - “critical” before “interactive” before “compute-intensive”
  - Maybe let users set priority level
- Treat hospital patients in order of severity (or triage)
- Select print jobs in order of decreasing length?
- Forward network packets in order of urgency
- Select most frequent symbols for data compression (cf. CSE143)
- Sort: **`insert`** all, then repeatedly **`deleteMin`**

More applications

- "Greedy" algorithms
  - Select the 'best-looking' choice at the moment
  - Will see an example when we study graphs in a few weeks
- Discrete event simulation (system modeling, virtual worlds, …)
  - Simulate how state changes when events fire
  - Each event *e* happens at some time t and generates new events *e1*, …, *en* at times *t+t1*, …, *t+tn*
  - Naïve approach: advance "clock" by 1 unit at a time and process any events that happen then
  - Better:
    - *Pending events* in a priority queue (priority = time happens)
    - Repeatedly: **`deleteMin`** and then **`insert`** new events
    - Effectively, "set clock ahead to next event"

<u>Preliminary</u> Implementations of Priority Queue ADT

	insert	deleteMin
Unsorted Array		
Unsorted Linked-List		
Sorted Circular Array		
Sorted Linked-List		
Binary Search Tree (BST)		

Notes: Worst case, Assume arrays have enough space

Aside: More on possibilities

- Note: If priorities are inserted in random order, binary search tree will likely do better than $O(n)$
  - $O($**`log`** $n)$ **`insert`** and $O($**`log`** $n)$ **`deleteMin`** on average
  - Could get same performance from a *balanced* binary search tree (e.g. AVL tree we will study later)

- One more idea: if priorities are 0, 1, …, *k* can use array of lists
  - **`insert`**: add to front of list at **`arr[priority]`**, $O(1)$
  - **`deleteMin`**: remove from lowest non-empty list $O(k)$

Our Data Structure: The Heap

The Heap:

- Worst case: O(log n) for insert
- Worst case: O(log n) for deleteMin
- If items arrive in random order, then the average-case of insert is O(1)
- Very good constant factors

Key idea: Only pay for functionality needed

- We need something better than scanning unsorted items
- But we do not need to maintain a full sorted list

- We will *visualize* our heap as a tree, so we need to review some tree terminology

Q: Reviewing Some Tree Terminology

***root*(T):**

***leaves*(T):**

***children*(B):**

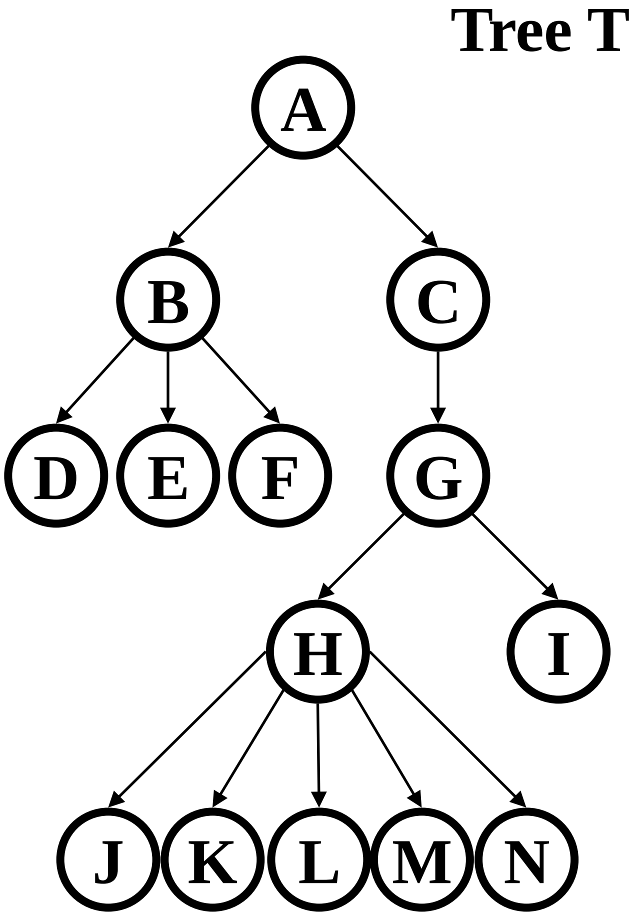
***parent*(H):**

***siblings*(E):**

***ancestors*(F):**

***descendents*(G):**

***subtree*(G):**



Q: Some More Tree Terminology

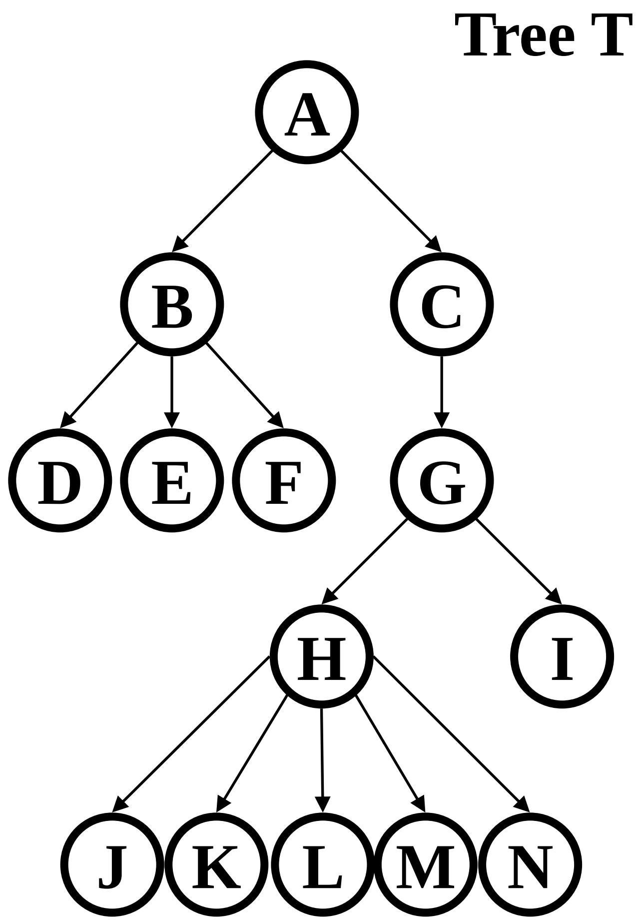
***depth*(B):**

***height*(G):**

***height*(T):**

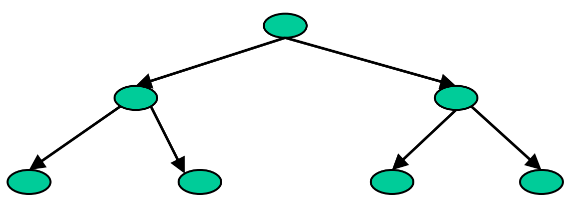
***degree*(B):**

***branching factor*(T):**

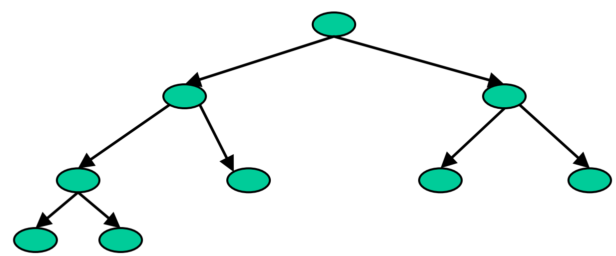


Types of Trees

Binary tree:	Every node has ≤2 children
n-ary tree:	Every node has ≤n children
Perfect tree:	Every row is completely full
Complete tree:	All rows except possibly the bottom are completely full, and it is filled from left to right



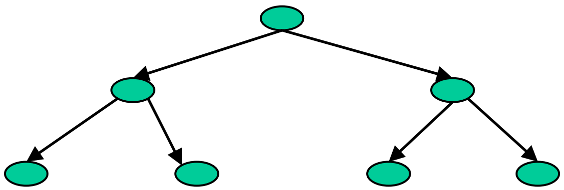
Perfect Tree



Complete Tree

More on Perfect Trees

Perfect tree: Every row is completely full



Perfect Tree

Some Basic Tree Properties

Nodes in a perfect binary tree of height h?

Leaf nodes in a perfect binary tree of height h?

Height of a perfect binary tree with n nodes?

Height of a complete binary tree with n nodes?

Properties of a Binary Min-Heap

More commonly known as a binary heap or simply a heap

- Structure Property:
  A complete [binary] tree
- Heap Property:
  Every non-root node has a priority value larger than (or possibly equal to) the priority of its parent

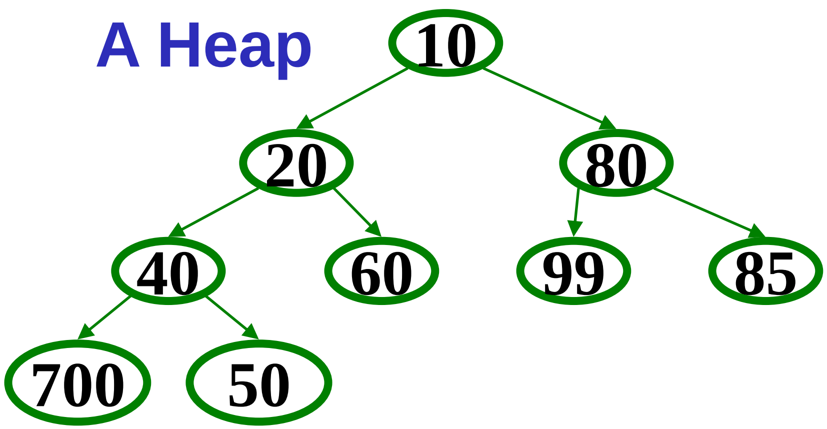
How is this different from a binary search tree?

Properties of a Binary Min-Heap

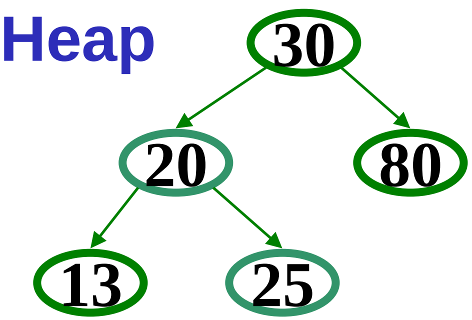
More commonly known as a binary heap or simply a heap

- Structure Property:
  A complete [binary] tree
- Heap Order Property:
  Every non-root node has a priority value larger than (or possibly equal to) the priority of its parent

A Heap



Not a Heap



Properties of a Binary Min-Heap

- Where is the minimum priority item?

- What is the height of a heap with n items?

Properties of a Binary Min-Heap

- Where is the minimum priority item?

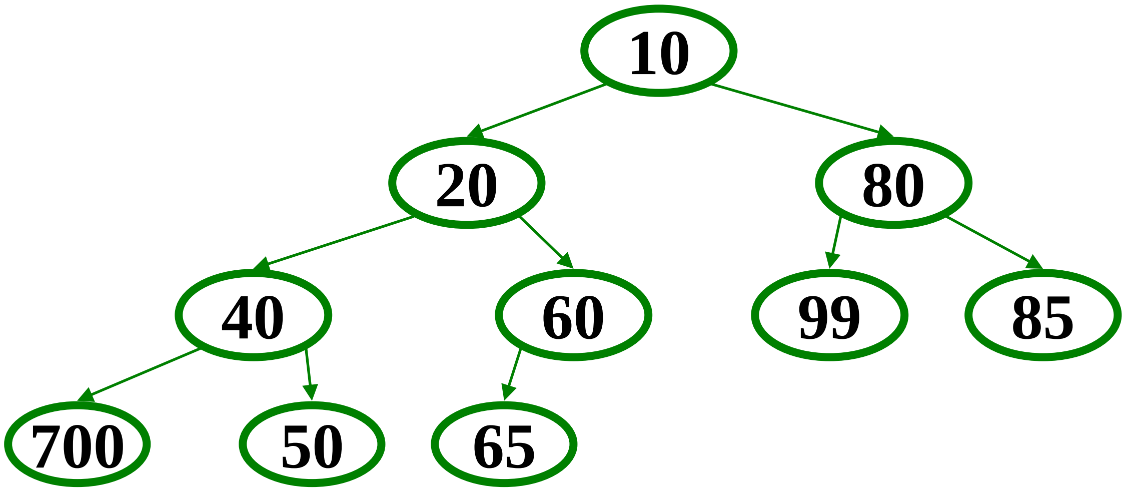
  At the root

- What is the height of a heap with n items?

  $\lfloor \log_2 n \rfloor$

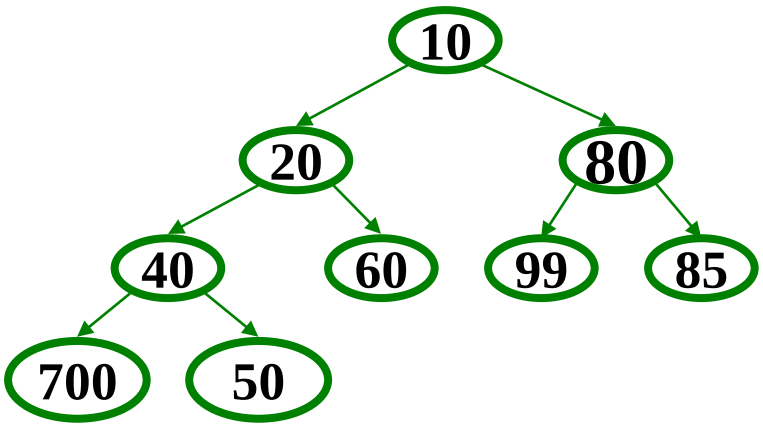
Heap Operations

- findMin:
- deleteMin: percolate down.
- insert(val): percolate up.



Operations: basic idea

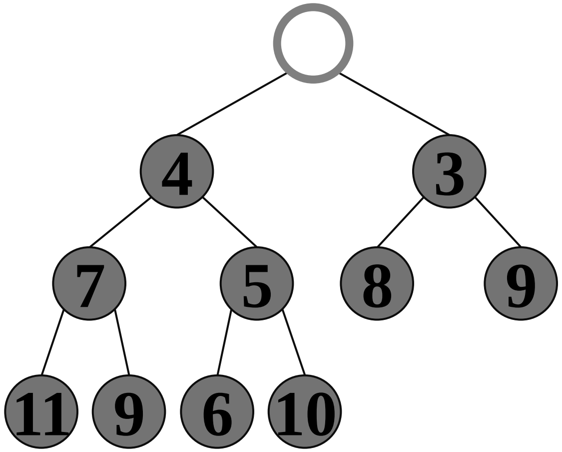
- **`findMin`**:
  return **`root.data`**
- **`deleteMin`**:
  1. **`answer = root.data`**
  2. Move right-most node in last row to root to restore structure property
  3. “Percolate down” to restore heap order property
- **`insert:`**
  1. Put new node in next position on bottom row to restore structure property
  2. “Percolate up” to restore heap order property



Overall strategy:

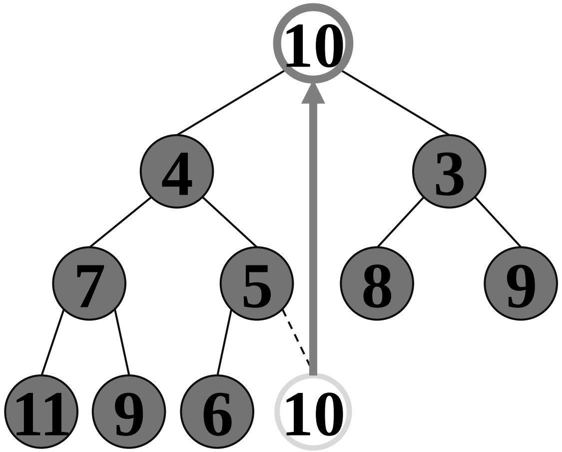
- *Preserve complete tree structure property*
- *This may break heap order property*
- *Percolate to restore heap order property*

DeleteMin Implementation

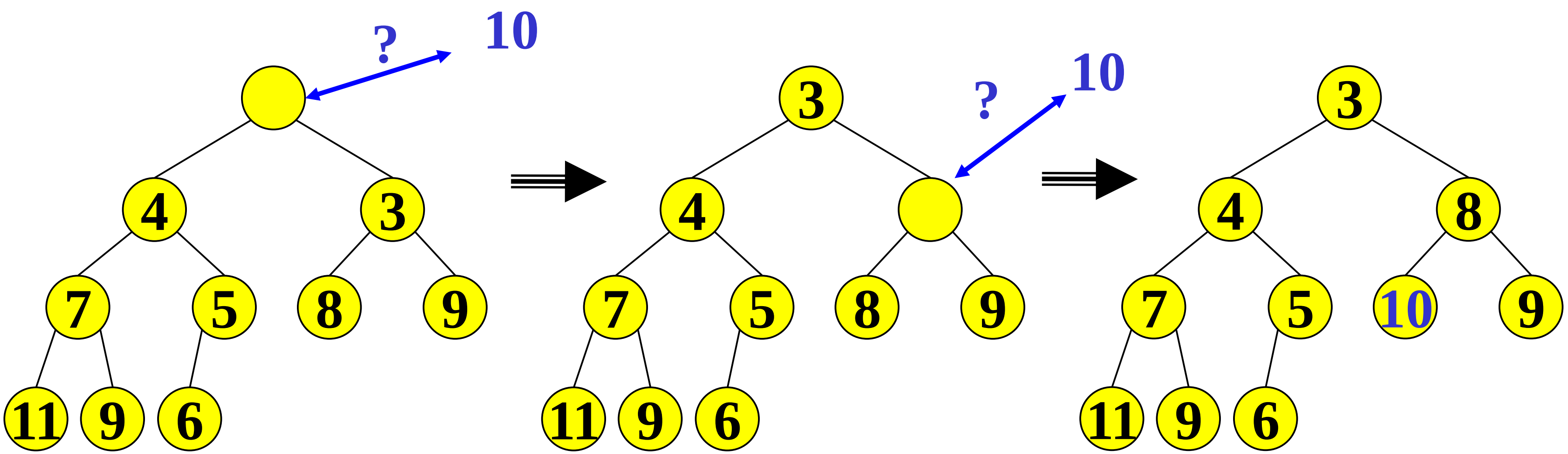


1. Delete value at root node (and store it for later return)
2. There is now a "hole" at the root. We must "fill" the hole with another value, must have a tree with one less node, and it must still be a complete tree
3. The "last" node is the is obvious choice, but now the heap order property is violated
4. We percolate down to fix the heap order:

```
While greater than either child
   Swap with smaller child
```



Percolate Down



Percolate down:

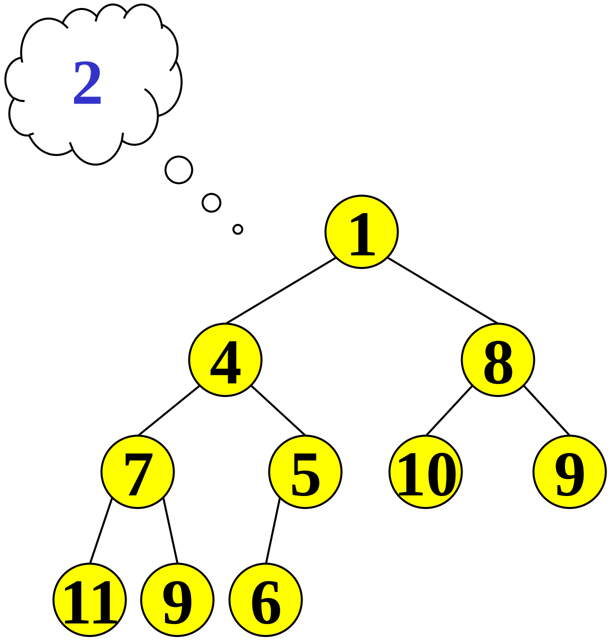
- Keep comparing with both children
- Move smaller child up and go down one level
- Done if both children are ≥ item or reached a leaf node
- Why does this work? What is the run time?

DeleteMin: Run Time Analysis

- Run time is *O*(height of heap)
- A heap is a complete binary tree
- Height of a complete binary tree of *n* nodes?
  - height = $\lfloor \log_2(n) \rfloor$
- Run time of **`deleteMin`** is $O(\log n)$

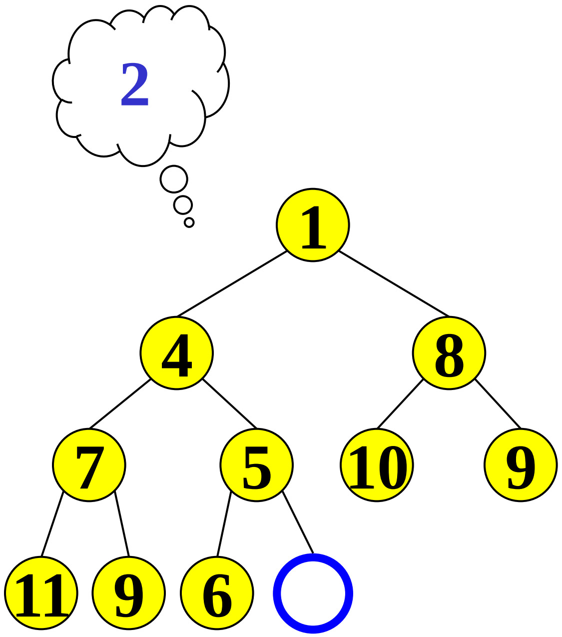
Insert

- Add a value to the tree
- Structure and heap order properties must still be correct afterwards

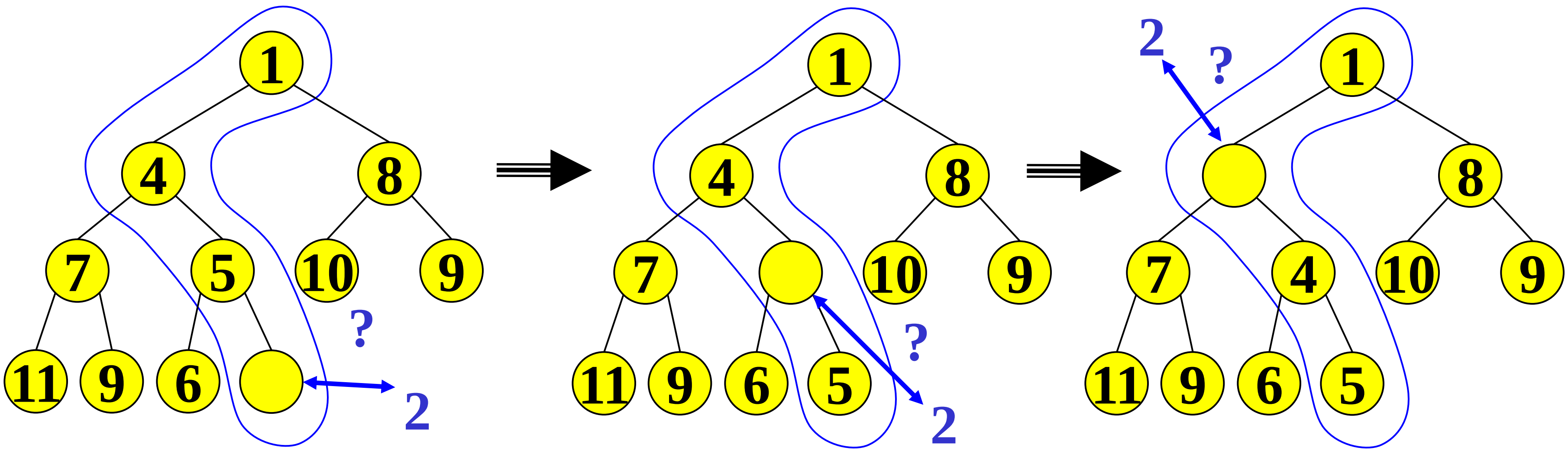


Insert: Maintain the Structure Property

- There is only **one** valid tree shape after we add one more node!
- So put our new data there and then focus on restoring the heap order property



Maintain the heap order property



Percolate up:

- Put new data in new location
- If parent larger, swap with parent, and continue
- Done if parent ≤ item or reached root
- Why does this work? What is the run time?

A Clever Trick for Storing the Heap…

Clearly, insert and deleteMin are worst-case O(log n)

- But we promised average-case O(1) insert (how??)

Insert requires access to the “next to use” position in the tree

- Walking the tree from root to leaf requires O(log n) steps
- Insert and Deletemin would have to update the “next to use” reference each time: O(log n)

We should only pay for the functionality we need!!

- Why have we insisted the tree be complete? ☺

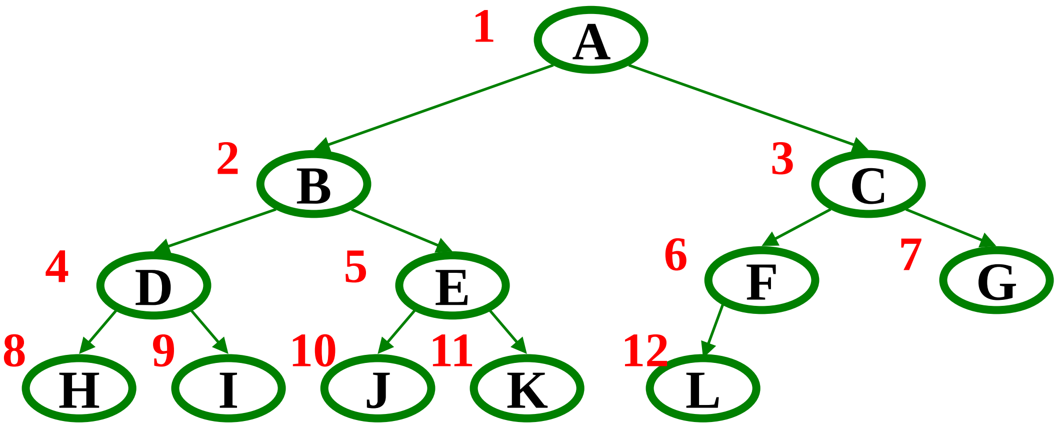
All complete trees of size n contain the same edges

- So why are we even representing the edges?

Here comes the really clever bit about implementing heaps!!!

Note: Exercises and P2 start counting from 0

Array Representation of a Binary Heap



From node i:

- left child:
- right child:
- parent:

	A	B	C	D	E	F	G	H	I	J	K	L	
0	1	2	3	4	5	6	7	8	9	10	11	12	13

- We skip index 0 to make the math simpler
- Actually, it can be a good place to store the current size of the heap