CSE 332 Winter 2023 Final Exam

Name: _____**Sample Solution**_____

Email address (UWNetID): _____

Instructions:

- The allotted time is 1 hour and 50 minutes.
- Please do not turn the page until the staff says to do so.
- This is a closed-book and closed-notes exam. You are NOT permitted to access electronic devices including calculators.
- Read the directions carefully, especially for problems that require you to show work or provide an explanation.
- When provided, write your answers in the box or on the line provided.
- Unless otherwise noted, every time we ask for an O, Ω, or Θ bound, it must be simplified and tight.
- For answers that involve bubbling in a ◯ or ☐, make sure to fill in the shape completely.
- If you run out of room on a page, indicate where the answer continues. Try to avoid writing on the very edges of the pages: we scan your exams and edges often get cropped off.
- A formula sheet has been included at the end of the exam.

Advice

- If you feel like you're stuck on a problem, you may want to skip it and come back at the end if you have time.
- Look at the question titles on the cover page to see if you want to start somewhere other than problem 1.
- **Relax and take a few deep breaths. You've got this! :-).**

Total: 96 points

Q1: Short-answer questions (10 pts)

- For questions asking you about runtime, give a simplified, tight Big-O bound. This means that, for example, $O(5n^2 + 7n + 3)$ (not simplified) or $O(2^{n!})$ (not tight enough) are unlikely to get points. Unless otherwise specified, all logs are base 2.
- For questions with a mathematical answer, you may leave you answer as an unsimplified formula (e.g. $7 \cdot 10^3$).

We will only grade what is in the provided answer box.

a. Give a **best-case** runtime to find the second largest value in an AVL tree with **N** elements.

O(log N)

b. Given a B-tree of height **3**, with M = **5** and L = **4**, what is the **MINIMUM** possible total number of key-value pairs in the entire tree?

36

c. Give a simplified, tight big-O bound for: $f(N) = N(\log(N^2)) + N^2(\log N)$

$O(N^2(\log N))$

d. Give the **worst-case** runtime of a decreaseKey operation on a Binary Min Heap containing N values.

O(log N)

e. Give the **worst-case** runtime of a pre-order traversal on a Binary Search Tree containing N values

O(N)

Q2: More Short-answer questions (10 pts)

(Same instructions as for Q1)

a. **True** or **False**: Dijkstra's algorithm can find the shortest path on a graph where all edges have positive weights, but the graph contains cycles.

● True ○ False

b.Give the **worst-case** runtime of a breadth-first search on a dense graph containing V vertices. Assume an adjacency list graph representation is used. **Give your answer in terms of V.**

$O(V^2)$

c. Give the **maximum number of edges** in a Minimum Spanning tree of a connected graph containing V vertices and E edges.

V - 1

d. Give the **best-case** runtime of the sequential Merge Sort algorithm on an array of **N** elements.

O(Nlog N)

e. What fraction of a program must be parallelizable in order to get **4x** speedup on **6** processors?

9/10

Q3: Hashing (9 pts)

For questions a) & b), insert **91, 3, 66, 12, 31, 23, 11** into the tables below. For each table, TableSize = **10**, and you should use the primary hash function h(k) = k%10. If an item cannot be inserted into the table, please indicate this and continue inserting the remaining values.

a) [3 pts] **Quadratic Probing Hashtable.**
If any values cannot be inserted, write them here:_____

0	**11**
1	**91**
2	**12**
3	**3**
4	**23**
5	**31**
6	**66**
7	
8	
9	

b) [3 pts] **Double Hashing Hashtable**. You should use the primary hash function: **h(k) = k % 10** and a secondary hash function: **g(k) = 1 + (k % 4)**.
If any values cannot be inserted, write them here:_____

0	
1	**91**
2	**12**
3	**3**
4	
5	**31**
6	**66**
7	**23**
8	
9	**11**

c) [1 pt] What is the load factor for the table in part a)? _____**7 / 10**_____

d) [2 pts] What is one advantage and one disadvantage of **quadratic probing** as a collision resolution strategy **compared to linear probing**? Explain each briefly in 1-2 sentences.

One advantage compared to linear probing:

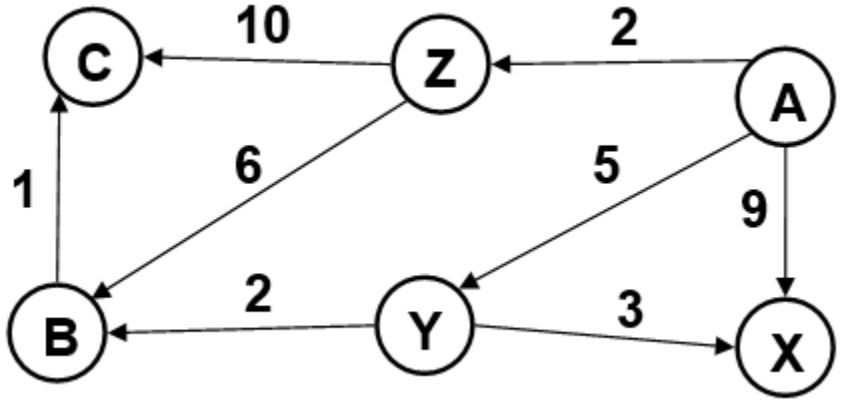
Quadratic probing avoids primary clustering.

One disadvantage compared to linear probing:

In quadratic probing, unless the table is prime-sized and less than half full ($\lambda < 0.5$) then you are not guaranteed to be able to find an empty spot. In linear probing, if there is an empty spot in the table we will always be able to find it.

Q4: Graphs (16 pts)

Use the following graph for the problems on this page:



a) [2 pts] List a valid **topological ordering** of the nodes in the graph above (if there are no valid orderings, state why not).

Some options: A, Y, X, Z, B, C A, Y, Z, X, B, C A, Y, Z, B, X, C
A, Y, Z, B, C, X A, Z, Y, X, B, C A, Z, Y, B, C, X A, Z, Y, B, X, C

b) [4 pts] Step through Dijkstra's Algorithm to calculate the **single source shortest path from A** to every other vertex. Break ties by choosing the lexicographically smallest letter first; ex. if B and C were tied, you would explore B first. *Note that the next question asks you to recall what order vertices were declared known*. Make sure the final distance and predecessor are clear in the table below.

Vertex	Known	Distance	Predecessor
A	T	0	------
B	T	~~∞~~, ~~8~~, 7	~~Z~~, Y
C	T	~~∞~~, ~~12~~, 8	~~Z~~, B
X	T	~~∞~~, ~~9~~, 8	~~A~~, Y
Y	T	~~∞~~, 5	A
Z	T	~~∞~~, 2	A

c) [1 pt] In what order would Dijkstra's algorithm mark each node as *known*?

A, Z, Y, B, C, X

d) [1 pt] List the **shortest path** from A to C. (Give the actual path **NOT** the cost.)

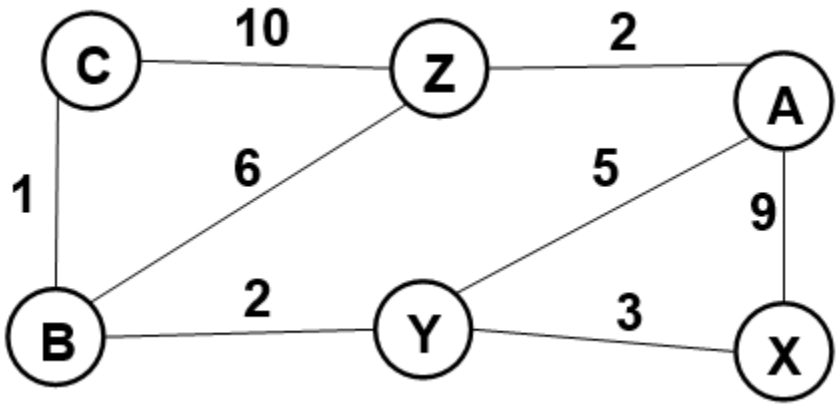
A, Y, B, C

e) [2 pts] Is this graph **strongly connected**? **Explain your answer in 1-2 sentences for any credit.**

○ Yes ● No

The graph does not have a path from every vertex to every other vertex. (e.g. there is no path from X to B)

Use the following graph for the problems on this page:



f) [2 pts] What is the **total cost** of a minimum spanning tree in the graph above? **13**

g) [2 pts] ASSUMING the edges above are **unweighted**, give a valid **depth first search** of this graph, **starting at vertex A**, using the non-recursive algorithm described in lecture. **Break ties by choosing the lexicographically smallest letter first**; ex. if B and C were tied, you would add B to the data structure first. You only need to show the final depth first search.

___A___, ___Z___, ___C___, ___B___, ___Y___, ___X___

h) [2 pts] ASSUMING the edges above are **unweighted**, give a valid **breadth first search** of this graph, **starting at vertex A**, using the algorithm described in lecture. **Break ties by choosing the lexicographically smallest letter first**; ex. if B and C were tied, you would add B to the data structure first. You only need to show the final breadth first search.

___A___, ___X___, ___Y___, ___Z___, ___B___, ___C___

Note: For both g) and h) you were asked to break ties when adding to the data structure (stack for DFS and queue for BFS).

Q5: ForkJoin (14 pts)

In Java using the ForkJoin Framework, write code to solve the following problem:

- **Input**: An array of positive `int`s (does not contain duplicates).
  - You can assume the input array will contain at least two different `int`s.
- **Output**: Returns a **`Pair`** of a) the minimum value and b) its location in the array.

For example, Input array: {4, 6, 1, 2, 3} returns (1, 2), and Input array: {100, 4, 3, 25, 2} returns (2, 4)

- **Do **not** employ a sequential cut-off: **the base case should process one element**.**
  - i.e. you do not **need** to employ a sequential method and you can assume that the code will never process more than one element
- Give a class definition, **`MinLocTask`**, along with any other code or classes needed.
- Fill in the _____ in the function **`findMinLoc`** **below.**

*You may **NOT** use any **global data structures** or **synchronization primitives (locks).**
*Make sure your code has O(log n) span and O(n) work.

```
import java.util.concurrent.ForkJoinPool;
import java.util.concurrent.RecursiveTask;
import java.util.concurrent.RecursiveAction;

public class Pair { // You should use this class
      int min, loc;
      public Pair (int min, int loc) {
            this.min = min;
            this.loc = loc;
      }
}

public class Main {
    public static final ForkJoinPool fjPool = new ForkJoinPool();

    // Returns the minimum value and its location.
    public static Pair findMinLoc (int[] input) {
       return fjPool.invoke(new MinLocTask(__input, 0, input.length_____)
    }
}
```

Please fill in the function above and write your class on the next page.

Write your class here:

```
public class MinLocTask extends  __RecursiveTask<Pair>___  {
    // Fields go here
    private int[] array;
    private int lo;
    private int hi;


    public MinLocTask(__int[] array, int lo, int hi_________) {
        this.lo = lo;
        this.hi = hi;
        this.array = array;

    }

    public __Pair____compute() {
        if (hi - lo < 2) {
            return new Pair(array[lo], lo);
        } else {
            int mid = lo + (hi - lo) / 2;
            MinLocTask left = new MinLocTask(array, lo, mid);
            MinLocTask right = new MinLocTask(array, mid, hi);

            left.fork();
            Pair rightResult = right.compute();
            Pair leftResult = left.join();

            if (leftResult.min < rightResult.min)
                return leftResult;
            else
                return rightResult;
        }
    }
}
```

Q6: Concurrency (12 pts)

The following class manages moving charge between your home (solar) battery and your electric car.

```
public class ChargeManager {
    private int solarBatteryCharge = 0;
    private int carBatteryCharge = 0;
    private int chargePerMile = 20;
    private Object solarLock = new Object();
    private Object carLock = new Object();

    public void sunshineAdd(int sun){
        synchronized(solarLock) {
            solarBatteryCharge += sun;
        }
    }

    public void driveCar(int miles){
        synchronized(carLock) {
            if (miles * chargePerMile  > carBatteryCharge)
                throw new CarStrandedException();
            carBatteryCharge -= miles * chargePerMile;
        }
    }

    public void chargeCar(int charge) {
        synchronized(solarLock) {
            synchronized(carLock) {
                if (charge > solarBatteryCharge)
                    throw new NotEnoughSolarChargeException();
                solarBatteryCharge -= charge;
                carBatteryCharge += charge;
            }
        }
    }
}
```

a) [3 pts]Does the **`ChargeManager`** class above have (bubble in all that apply):

☐ a race condition ☐ potential for deadlock ☐ a data race ■ none of these

If there are any problems, describe each problem selected in 1-2 sentences.

b) [3 pts] Does the code above provide any more concurrency than having one lock on the entire `ChargeManager` object? **In 1-2 sentences explain why or why not.**

● Yes ○ No

Yes, one thread could be calling `sunshineAdd` at the same time as another thread is calling `driveCar`.

c) [3 pts] You decide to add one more method to the `ChargeManager` class:

```
public void chargeSolarFromCar(int charge) {
    synchronized(solarLock) {
        synchronized(carLock) {
            if (charge > carBatteryCharge)
                System.out.println("Warning: Charge values not accurate");
            carBatteryCharge -= charge;
            solarBatteryCharge += charge;
        }
    }
    System.out.println("You lose" + 1.0/chargePerMile + "miles per charge");
}
```

Does adding this method to the `ChargeManager` class **cause any new** (bubble in all that apply):

☐ a race condition ☐ potential for deadlock ☐ a data race ■ none of these

If there are any new problems, describe each problem selected in 1-2 sentences.

d) [3 pts] **Instead of** adding in the method above in part c), you add this to the `ChargeManager` class:

```
public int reportOverallCharge() {
    return solarBatteryCharge + carBatteryCharge;
}
```

Does adding this method to the `ChargeManager` class **cause any new** (bubble in all that apply):

■ a race condition ☐ potential for deadlock ■ a data race ☐ none of these

If there are any new problems, describe each problem selected in 1-2 sentences.

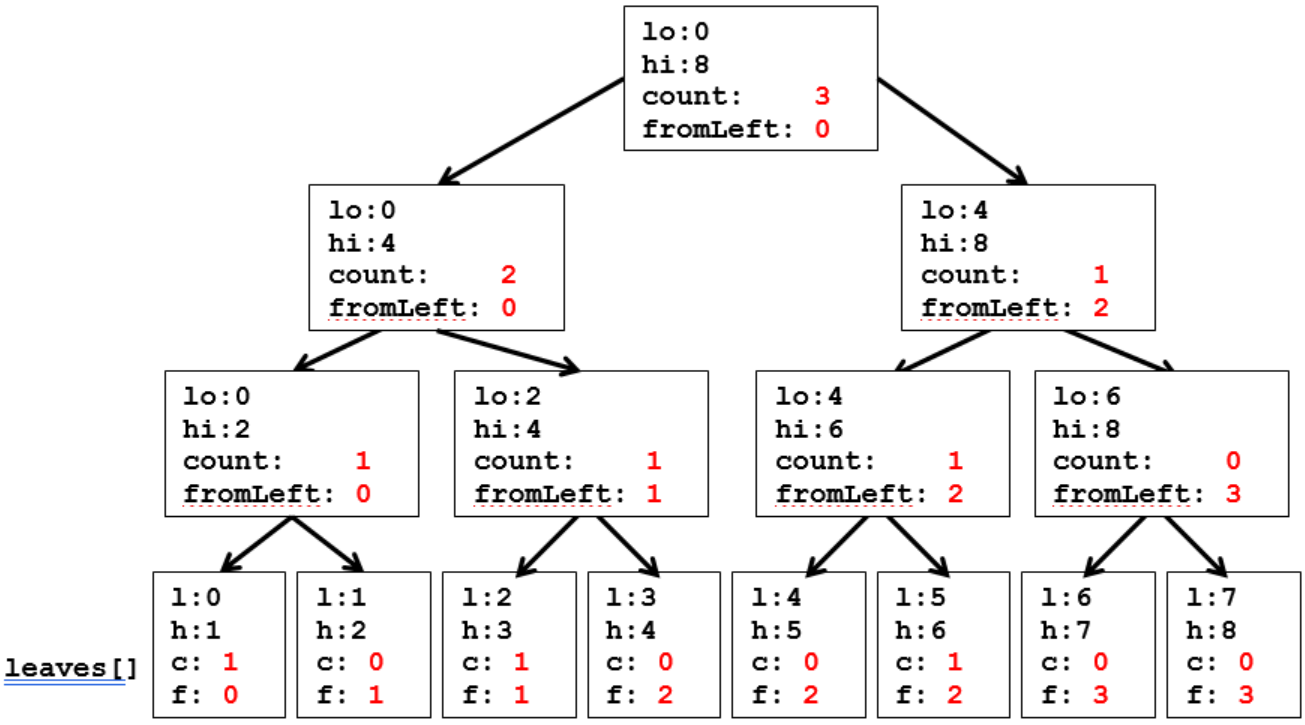
There is a data race on `solarBatteryCharge` and `carBatteryCharge` because one thread could be writing one or both of these values in any of `sunshineAdd`, `driveCar` or `chargeCar` at the same time as another thread calls `reportOverallCharge`. This would cause a write and a read at the same time: a data race, and a data race is a race condition.

Q7: Parallel Prefix (9 pts)

Given the following array and character as inputs, perform the parallel prefix algorithm to fill the output array with the **sum of times the given character (stored in the variable `find`) appears in all of the cells to the left** (including the value contained in that cell) in the **`input`** array. Do not use a sequential cutoff. For example, for
`input = {'a', 'a', 'a', 'b', 'b', 'a', 'a', 'b'}` and **`find = 'b'`**,
`output` should be: **`{0, 0, 0, 1, 2, 2, 2, 3}`**.

a) [5 pts] Fill in the values for **`count`**, **`fromLeft`**, and the **`output`** array in the picture below given the following values for **`input`** and **`n`**. The **`input`** array has already been filled out for you.

```
char[] input = {'n', 'o', 'n', 's', 'e', 'n', 's', 'e'}
char find = 'n'
```



Index	0	1	2	3	4	5	6	7
input	'n'	'o'	'n'	's'	'e'	'n'	's'	'e'
output	1	1	2	2	2	3	3	3

Give formulas for the following values where **p** is a reference to a non-leaf tree node and **leaves[i]** refers to the leaf node in the tree visible just above the corresponding location in the **input** and **output** arrays in the picture on the previous page.

b) [1 pt] Give pseudocode for how you assigned a value to **leaves[i].count**

```
if (input[i] == find)
    leaves[i].count = 1
else
    leaves[i].count = 0
```

c) [1 pt] Give code for assigning **p.left.fromLeft.**

```
p.left.fromLeft = p.fromLeft
```

d) [1 pt] Give code for assigning **p.right.fromLeft.**

```
p.right.fromLeft = p.fromLeft + p.left.count
```

e) [1 pt] How is **output[i]** computed? Give exact code assuming **leaves[i]** refers to the leaf node in the tree visible just above the corresponding location in the **input** and **output** arrays in the picture above.

```
output[i] = leaves[i].count + leaves[i].fromLeft
```

Q8: Sorting (10 points)

a) [2 pts] Which of the sorts we discussed in our class would be best to use for this scenario:

Your entire candybar collection expires this year. You need to sort the candy bars by which month they expire in (January-December).

Name of Sort: __**Bucket/Bin Sort**_____
Justify why your choice is best in 1-2 sentences:

We know the range of possible values is 1-12, so we can use 12 bins. Total running time will be O(N + 12) which will be a better runtime than any of the other sorts we have discussed.

b) [1 pt] True or False: Insertion sort is an **in-place** sort.

● True ○ False

Explain why or why not in 1-2 sentences.

Insertion sort takes the next value and inserts it into the correct place amongst the already sorted values in the original array. No extra memory is needed other than a few constant size temporary variables (e.g. no extra array is needed).

c) [2 pts] Give the *recurrence* for SEQUENTIAL Merge Sort - **best case**

$T(n) = 2 * T(n/2) + O(n)$ (or $+ c_1*n + c_2$)

d) [2 pts] Give the *recurrence* for SEQUENTIAL Quicksort - **worst case**.

$T(n) = T(n - 1) + O(n)$ (or $+ c_1*n + c_2$)

e) [2 pts] What **parallel algorithm pattern** did we discuss using to parallelize the partition step in Quicksort? **Describe in 1-2 sentences how it was used to do so**.

Name of Pattern: _**Pack or Filter**__

We can use parallel pack to place elements less than the pivot to the left side of the aux array and parallel pack to place elements greater than the pivot to the right side of the aux array. Put the pivot between them and you have now partitioned the array.

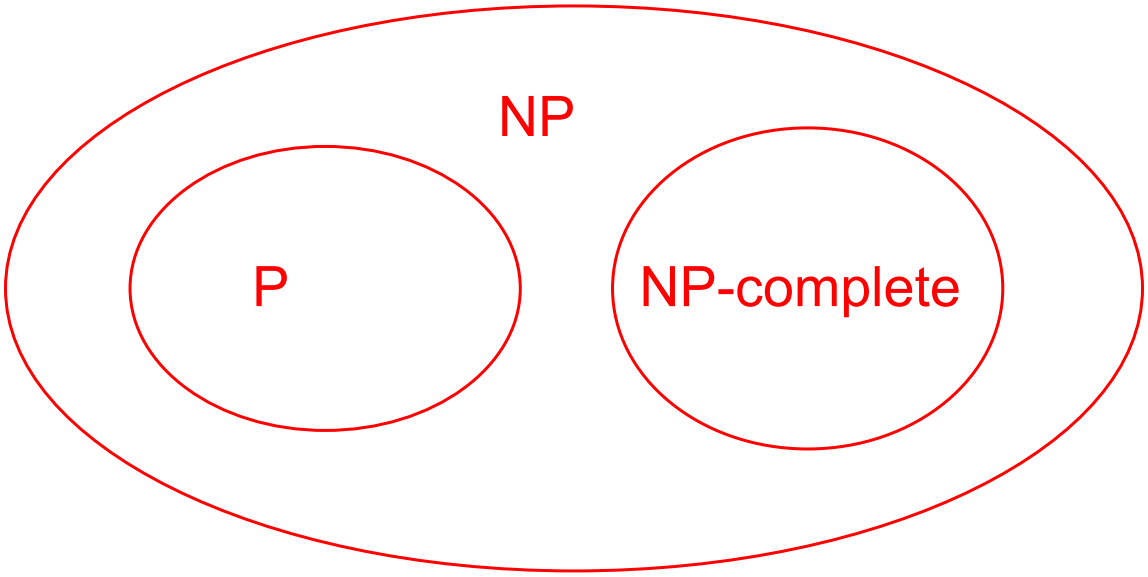
f) [1 pt] What was the new span for just the partitioning step after integrating the technique described in part e).

O(log N)

Q9: P/NP (6 pts)

a) [1 pt] "NP" stands for ___**Nondeterministic Polynomial**_____

b) [2 pt] Draw a diagram demonstrating how (we are pretty sure) the sets P, NP, and NP-Complete, overlap / don't overlap with each other.



For the following problems, bubble in **ALL** the sets each problem belongs to:

c) [1 pt] Given a separate-chaining hash table where each bucket is an AVL tree, determine whether the hash table contains the key *k* .

■ P ■ NP □ NP-complete □ none of these

d) [1 pt] Given a connected, undirected graph, determine if it has a path that starts and ends at the same vertex and goes through each **vertex** exactly once.

□ P ■ NP ■ NP-complete □ none of these

e) [1 pt] Given a connected, undirected graph, determine if it has a cycle that goes through each **edge** exactly once.

■ P ■ NP □ NP-complete □ none of these

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