Name: _____

Email address: _____

Quiz Section: _____

CSE 332 Winter 2013: Midterm Exam

(closed book, closed notes, no calculators)

Instructions: Read the directions for each question carefully before answering. We will give partial credit based on the work you **write down**, so show your work! Use only the data structures and algorithms we have discussed in class or that were mentioned in the book so far.

Note: For questions where you are drawing pictures, please circle your final answer for any credit.

Good Luck!

Total: 100 points. Time: 50 minutes.

Question	Max Points	Score
1	10	
2	12	
3	10	
4	6	
5	8	
6	8	
7	8	
8	14	
9	14	
10	10	
Total	100	

1. (10 pts) Big-Oh

(2 pts each) For each of the functions *f(N)* given below, indicate the tightest bound possible (in other words, giving $O(2^N)$ as the answer to every question is not likely to result in many points). Unless otherwise specified, all logs are base 2. **You MUST choose your answer from the following** (not given in any particular order), each of which could be re-used (could be the answer for more than one of a) – e)):

$O(N^2)$, $O(N^{½})$,$O(N^3 \log N)$, $O(N \log N)$, $O(N \log \log N)$, $O(N)$, $O(N^2 \log N)$, $O(N^5)$, $O(\log N)$,$O(1)$,$O(\log^2 N)$,$O(N^4)$,$O(N^N)$,$O(N^6)$,$O(N \log^2 N)$, $O(2^N)$, $O(\log^4 N)$, $O(N^3)$

You do not need to explain your answer.

a) *f(N)* = $50 \log N^2 + 100 N^2 \log N$ _____

b) *f(N)* = $(N^2)^3 + N^5$ _____

c) *f(N)* = $\log_4(N^4)$ _____

d) *f(N)* = $N \log^2 N + N \log \log N$ _____

e) *f(N)* = $N^{1/2} + \log N$ _____

2. **(12 pts) Big-Oh and Run Time Analysis:** Describe the worst case running time of the following pseudocode functions in Big-Oh notation in terms of the variable `n`. Give an answer in the most simplified form (similar to options to choose from in Problem 1). ***Showing your work is not required***

I.
```
void sunny (int n, int sum) {
     for (int i = 0; i < n * 1000; ++i)
        for (int j = 0; j < i; ++j)
            for (int k = n; k > 0; k--)
                 sum++;
 }
```
Runtime:

II.
```
int happy (int n, int sum) {
     if (n < 10)
        return n/3;
     else if (n < 100)
       return happy (n/2, sum);
     else {
       for (int i = 0; i < n * n; ++i)
         sum++;
       return happy (n - 2, sum);
     }
}
```

III.
```
void smiley (int n, int sum) {
     for (int k = 0; k < n; ++k) {
        if (n < 100) {
           for (int i = 0; i < k; ++i)
              sum++;
        }
        for (int j = n; j > 0; j--)
           sum++;
     }
 }
```

IV.
```
void funny (int n, int sum) {
     for (int k = n; k > 0; k = k/2) {
        for (int j = n; j > 0; j--)
           sum++;
        for (int i = 0; i < n; i++)
           sum++;
     }
}
```

3. (10 pts) Big-O, Big Ω, Big Θ

(2 pts each) For parts (a) – (c) circle whether the statement is true or false. Part (d) has its own instructions. You do not need to show your work for parts (a) – (c).

TRUE / FALSE a) $N^2 + N = \Theta(N^3)$

TRUE / FALSE b) $N^2 + N^3 = \Omega(N^3)$

TRUE / FALSE c) $100\ N^2 + 20N + 500 = \Theta(N^2)$

d) (4 points) Demonstrate that: $25n + 300$ is $O(n)$ by finding positive integers c and n_0 such that the definition of Big Oh is satisfied. You do not need to prove that these constants satisfy the definition, just specify the constants.

4. (6 pts) Recurrence Relationships -

Suppose that the running time of an algorithm satisfies the recurrence relationship

$$T(1) = 10.$$

and

$$T(N) = T(N/2) + 3 \quad \text{for integers } N > 1$$

Find the closed form for T(N) ***and show your work step by step***. In other words express T(N) as a function of N. Your answer should *not* be in Big-Oh notation – show the relevant *exact* constants in your answer (e.g. don't use "C" in your answer).

5. (8 pts) Draw the AVL tree that results from inserting the keys 1, 4, 6, 8, 9, 5, in that order into an initially empty AVL tree. You are only required to show the final tree, although if you draw intermediate trees, ***please circle your final result for ANY credit.***

6. (8 pts) Trees

a) (4 pts) What is the minimum and maximum number of nodes in an **Binary min heap of height 6?** (Hint: the height of a tree consisting of a single node is 0) ***Give an exact number*** for both of your answers – not a formula.

Minimum =

Maximum =

b) (4 pts) What is the minimum and maximum number of nodes in an **AVL tree of height 5?** (Hint: the height of a tree consisting of a single node is 0) ***Give an exact number*** for both of your answers – not a formula.

Minimum =

Maximum =

7. (8 pts) B-trees

a) (4 pts) Given a B-tree (as defined in lecture and in Weiss) with M = 3 and L = 20, if you have inserted 100 items into the tree, what is **the minimum and maximum height of the tree**? Give an exact number, not a formula for your answer.

Minimum height =

Maximum height =

b) (4 pts) Given the following parameters for a B-tree with M= 37 and L = 5:

Key Size = 6 bytes

Pointer Size = 8 bytes

Data Size = 100 bytes per record (***includes*** the key)

Assuming that M and L were chosen appropriately, **what is the likely size of a disk block** on the machine where this implementation will be deployed? Give a numeric answer and a short justification based on one or more equations using the parameter values above.

8. (14 pts) Hash Tables

For a) and b) below, insert the following elements in this order: 80, 9, 8, 18, 29, 49. For each table, TableSize = 10, and you should use the primary hash function h(k) = h%10. If an item cannot be inserted into the table, please indicate this and continue inserting the remaining values.

a) Linear probing hash table:

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

b) Quadratic probing hash table

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

The following two questions are about hashing generally, NOT about the two tables above.

c) What is the big-O worst case running time for an **insert** operation in a **linear probing hash table** containing N elements? (Assume there is empty space in the table.)

d) What is the big-O worst case running time for a **find** operation in a **separate chaining hash table** containing N elements **where each bucket points to a sorted linked list**?

9. (14 pts) Binary Min Heaps

a) (8 pts) Draw the binary min heap that results from inserting 9, 4, 2, 6, 3, 7, 8, 5, 1 in that order into an **initially empty binary heap**. You do not need to show the array representation of the heap. You are only required to show the final tree, although if you draw intermediate trees, ***please circle your final result for ANY credit.***

9. Binary Min Heaps (continued)

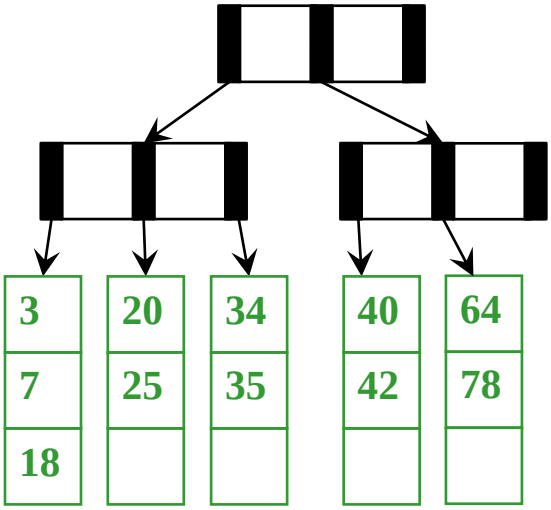
b) (4 pts) Draw the result of doing 1 deletemin on the heap you created in part a. You are only required to show the final tree, although if you draw intermediate trees, ***please circle your final result for ANY credit.***

c) (2 pts) What is the big-O worst case running time for a **findmin** operation in binary min heap containing N elements?

10. (10 pts) B-tree Insertion and Deletion

a) (2pts) In the B-Tree shown below, please write in the appropriate values for the interior nodes.

b) (4 pts) Starting with the B-tree shown below, insert **9**. Draw and circle the resulting tree (*including values for interior nodes*) below. Use the method for insertion described in lecture.



c) (4 pts) Starting with the ***original*** B-tree shown above on the left (***before* inserting 9**), delete **78**. Draw and circle the resulting tree (*including values for interior nodes)* below. Use the method for deletion described in lecture.