Name: _____

Email address (UWNetID): _____

CSE 332 Autumn 2016 Final Exam

(closed book, closed notes, no calculators)

Instructions: Read the directions for each question carefully before answering. We may give partial credit based on the work you **write down**, so show your work! Use only the data structures and algorithms we have discussed in class so far. Writing after time has been called will result in a loss of points on your exam.

Note: For questions where you are drawing pictures, please circle your final answer.

You have 1 hour and 50 minutes, work quickly and good luck!

Total: Time: 1 hr and 50 minutes.

Question	Max Points	Score
1	12	
2	12	
3	10	
4	10	
5	14	
6	10	
7	10	
8	11	
9	11	
Total	100	

1) [12 points total] Hash Tables

For a) and b) below, insert the following elements in this order: 50, 21, 29, 10, 39, 19. For each table, TableSize = 10, and you should use the primary hash function h(k) = k%10. If an item cannot be inserted into the table, please indicate this and continue inserting the remaining values.

a) Quadratic probing hash table

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

b) Separate chaining hash table – use a linked list for each bucket where the values are ordered by **increasing value**

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

c) What is the load factor in Table b)?

d) In a sentence or two, describe **double hashing**.

e) What is one advantage of **double hashing** over **quadratic probing**, **be specific**.

f) What is the big-O worst case runtime of a *find operation on a table like table b?*

g) What is the big-O worst case runtime of an *Insert in a **separate chaining hash table** containing N elements where each bucket points to an AVL tree?*

2) [12 points total] Graphs!

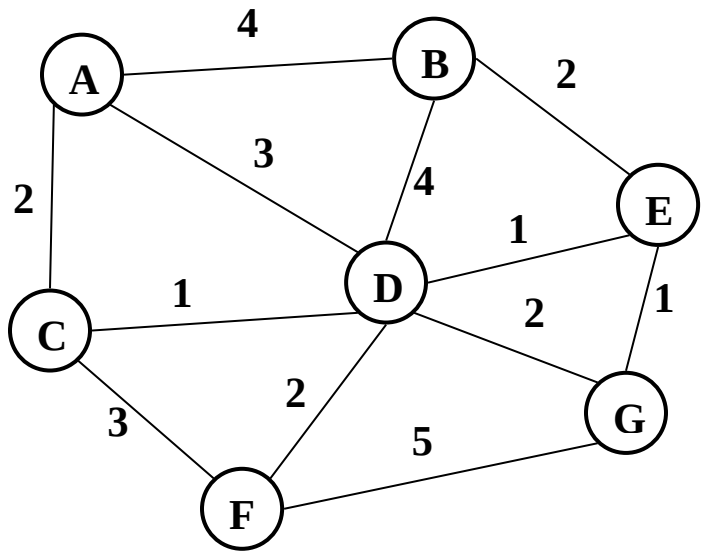
a) **[2 points]** What is the big-O running time of Dijkstra's algorithm (assuming an adjacency list representation) if:

(i) A priority queue is used?

(ii) An unsorted list is used?

b) **[2 points]** Which implementation of Dijkstra's (priority queue vs. unsorted list) is likely to be faster if the graph is known to be dense? **Explain your answer in ~one sentence for any credit**.

c) **[2 points]** Give a Minimum Spanning Tree (MST) of the graph below by highlighting the edges that would be part of the MST.



d) **[6 points]** For (ii) and (iv) below, ***you are given a perfect binary tree of height h containing n nodes***. Your answer should be an **exact formula**, such as 3/2 log h, or 5×3^n, not big-O notation.

(i) Depth First Search: What is the name of the data structure used in DFS?

(ii) What is the maximum size of that data structure during a DFS ?

(iii)Breadth First Search: What is the name of the data structure used in BFS?

(iv)What is the maximum size of that data structure during a BFS ?

3) [10 points total] More Graphs!

a) **[4 points] <u>Draw a picture</u>** of a connected directed graph with 5 nodes that has the largest possible number of topological sorts. How many different topological sorts does it have?

How Many topo sorts?

b) [6 points total] Given a weighted, undirected graph with |V| nodes, answer the following. Assume all weights are non-negative.

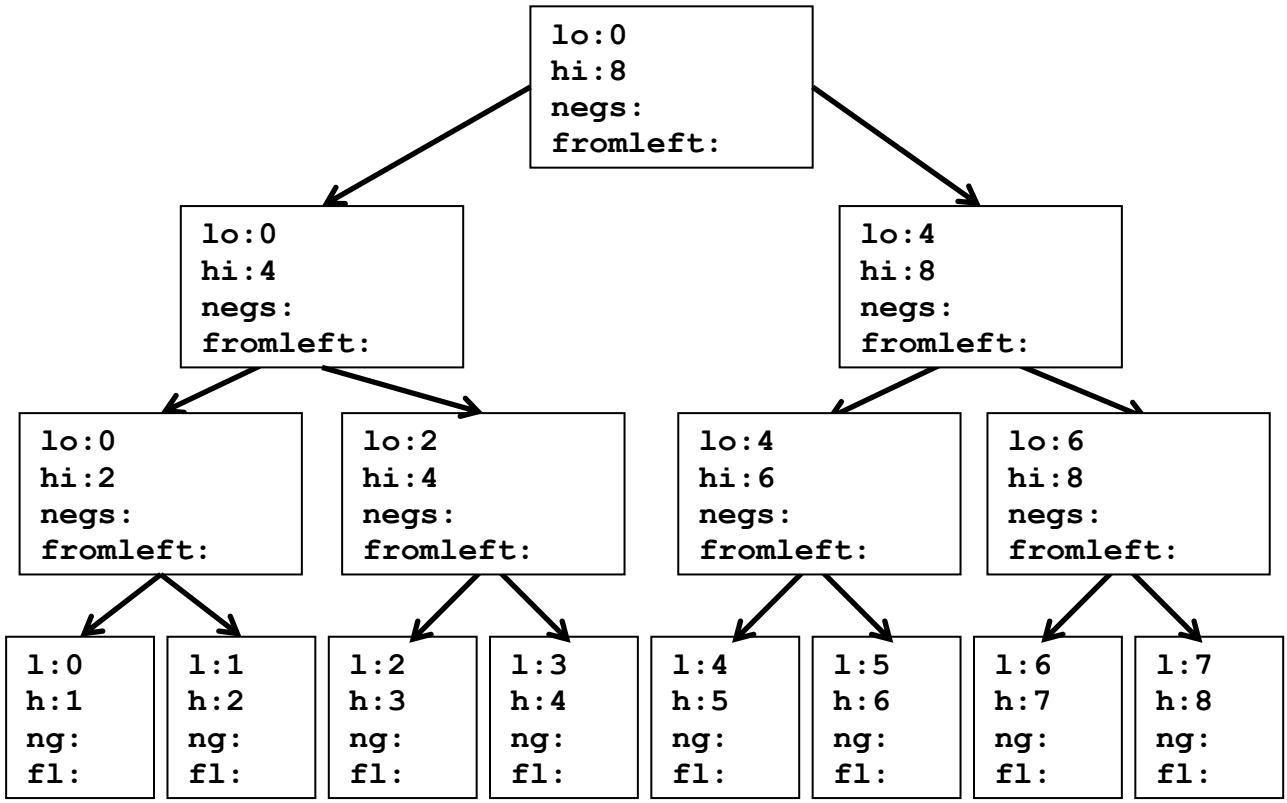
(i) [2 points] If each edge has weight ≤ *w*, what can you say about the cost of an MST? Your answer should give a lower bound, or an upper bound on the cost of the MST, e.g. "the cost of the MST is ≥ 2^{w+n}", or "the cost of the MST is ≤ log(w log n)".

(ii) [2 points] If the cost of an MST is *c*, what can you say about the shortest distances returned by Dijkstra's algorithm when run with an arbitrary vertex *s* as the source? You should give a lower bound or an upper bound for the distance between arbitrary vertices u and v.

(iii)[2 points] If there exists a Hamiltonian circuit of cost c, then what can you say about the cost of the minimum spanning tree ?

4) [10 points] Parallel Prefix <u>CountNegatives</u>:

a) Given the following array as input, perform the parallel prefix algorithm to fill the **output** array with the **<u>number of negative values</u> contained in all of the cells to the left** (including the value contained in that cell) in the input array. Fill in the values for: **negs,** and **fromLeft** in the tree below. Do not use a sequential cutoff. **Note**: This is NOT sum!



Index	0	1	2	3	4	5	6	7
Input	-3	5	-4	-7	2	-8	9	-5
Output								

b) How is the **fromLeft** value computed for a node in the tree? Specifically, if you have a node with **negs** & **fromLeft** computed, how do you compute **fromLeft** for its left & right children (both of which have **negs** already computed).

Left child's fromLeft:

Right child's fromLeft:

5) **[14 points]** In Java using the ForkJoin Framework, write code to solve the following problem:

• **Input**: An int k, and an array of ints of size n containing values in the range 0…k-1
• **Output**: the value (between 0 and k-1) occurring most often in the Input array. If there is a tie for the most frequently occurring value, return the *smallest* number.

For example, if k=7 and input array of size 9 is {0, 2, 5, 2, 6, 5, 4, 5, 2}, the output would be 2.

- Do **not** employ a sequential cut-off: **the base case should process one element**. (You can assume the input array will contain at least one element.)
- Assume k is small (e.g. less than 50)
- Fill in the function `findMostCommon` **below.**
- Give a class definition, `FindComTask`, **on the next page.**

You may not use any global data structures or synchronization primitives (locks).

```
import java.util.concurrent.ForkJoinPool;
import java.util.concurrent.RecursiveTask;

class Main{
      static final ForkJoinPool fjPool = new ForkJoinPool();

      // Returns the most common value in the array input.
      // Values in input are in the range 0 to k.
      int findMostCommon (int k, int[] input) {
```

Please fill in the function above and write your class on the next page.

5) (Continued) Write your class on this page.

6) **[10 points] Concurrency:** Once again, we are helping out with the Aviation Management System. This time they have a data structure that keeps track of whether there is a flight from one airport to another. If there is a flight from airport x to airport y, then we should be able to assume that there is also a flight from airport y to airport x. This information is being kept in an array (indexed by unique airport number), where each location in the array points to a list of the airport numbers that can be reached from this airport by a single flight (an **`AirportList`**). We would like to allow as much concurrent access to this data structure as possible, while assuring that each thread always sees a consistent state of the data structure. We attempt this by having a different lock on each of the **`AirportLists`**.

a) Our first attempt at the **`removeRoute`** method is below. Assume **`hasFlightTo`** and **`removeFlight`** are methods on an **`AirportList`**. **`removeFlight`** will throw an exception if the specified flight is not present.

```
void removeRoute1(int x, int y, AirportList[] airports) {
    synchronized(airports[x]) {
        synchronized(airports[y]) {
            if(airports[x].hasFlightTo(y)) {
                airports[x].removeFlight(y);
                airports[y].removeFlight(x);
            }
        }
    }
}
```

i. Does the code above have (circle all that apply):

potential for deadlock, a data race, a race condition, none of these

ii. If possible, show (as done in class) an interleaving of two or more threads calling **`removeRoute1`** that demonstrates a concurrency error. If not possible, explain why not.

6) (Continued)

b) Our second attempt at the **`removeRoute`** method is below.

```
void removeRoute2(int x, int y, AirportList[] airports) {
    synchronized(airports[x]) {
        if(!(airports[x].hasFlightTo(y))) {
            return;
        }
    }
    synchronized(airports[x]) {
      airports[x].removeFlight(y);
    }
    synchronized(airports[y]) {
      airports[y].removeFlight(x);
    }
}
```

i. Does the code above have (circle all that apply):

potential for deadlock, a data race, a race condition, none of these

ii. If possible, show (as done in class) an interleaving of two or more threads calling **`removeRoute2`** that demonstrates a concurrency error. If not possible, explain why not.

c) Finally, the developers' consider scrapping the one-lock-per **`AirportList`** strategy in favor of a single lock, locking the entire array of **`AirportLists`**.

One benefit of a single-lock locking all airports:

One drawback of a single-lock locking all airports:

7) **[10 points] Sorting**

You are given a list of AVL trees. The keys in the AVL trees are ages of people. Each AVL tree represents the ages for people in a different community. Your task is to sort the AVL trees such that tree X comes before tree Y if and only if:

- The minimal age in tree X is less than the minimal age in tree Y, **or**
- The minimal ages are the same, but the maximal age in tree X is less than the maximal age in tree Y

Otherwise, ties are broken arbitrarily. You may assume that:

- There are *k* trees
- Each tree has *n* keys in it
- The range of ages is fixed (0-127)

a) [5 points] Describe in a few sentences or numbered steps how you could use Mergesort to sort these trees efficiently in the worst case. What is the running time in terms of k and n?

Running Time:

b) [5 points] Describe in a few sentences or numbered steps how you could use ideas from Radixsort to sort these trees efficiently in the worst case. What is the running time in terms of k and n?

Running Time:

8) [11 points] More Sorting

a) [3 points] Give the ***recurrence*** for Mergesort (parallel sort & sequential merge) – best case span: (Note: We are NOT asking for the closed form.)

b) [3 points] In the ____ spaces below, order these sorts from slowest to fastest in terms of big-O runtimes. For parallel sorts, use the span. **Draw a circle around any sorts whose big-O runtimes are the same.** You do not have to give the runtimes, just list the sort letters.

A) Mergesort (sequential) – worst case

B) Quicksort (parallel sort & parallel partition) – best case span

C) Quicksort (sequential) – best case

D) Quicksort (sequential) – worst case

E) Quicksort (parallel sort & parallel partition) – worst case span

_____ _____ _____ _____ _____

Slowest Fastest

c) [2 points] Suppose we choose the median of five items as the pivot in quicksort. If we have an N element array, then we find the median of the elements located at the following positions: left (= 0), right (= N – 1), center (the average of left and right, rounded down), leftOfCenter (the average of left and center, rounded down), and rightOfCenter (the average of right and center, rounded down). The median of these elements is the pivot.

What is the worst case running time of *this version* of quicksort?

d) [1 point] Any algorithm for sorting must take $\Omega(N \log N)$ time in the worst-case.

TRUE FALSE

e) [2 points] What does it mean for a sort to be stable?

9) **[11 points] P, NP, NP-Complete**

a) [2 points] "NP" stands for _____

b) [2 points] What does it mean for a problem to be in NP-complete?

c) [5 points] For the following problems, circle **ALL** the sets they belong to:

Determining if a chess move is the best move on an N x N board	NP	P	NP-complete	None of these
Finding the maximum value in an array	NP	P	NP-complete	None of these
Finding a cycle that visits each vertex in a graph exactly once	NP	P	NP-complete	None of these
Finding a cycle that visits each edge in a graph exactly once	NP	P	NP-complete	None of these
Determining if a program will ever stop running	NP	P	NP-complete	None of these

d) [1 point] If there exists a polynomial time algorithm to solve **Euler Circuit**, then there exists a polynomial time algorithm to solve **SAT**.

TRUE FALSE

e) [1 point] If there exists a polynomial time algorithm to solve **Hamiltonian Circuit** then any problem in **NP** can be solved by some polynomial time algorithm.

TRUE FALSE