CSE 332 Autumn 2023
Lecture 13: Hashing

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## Dictionary Data Structures

<table>
<thead>
<tr>
<th>Data Structure</th>
<th>Time to insert</th>
<th>Time to find</th>
<th>Time to delete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsorted Array</td>
<td>$\Theta(n)$</td>
<td>$\Theta(n)$</td>
<td>$\Theta(n)$</td>
</tr>
<tr>
<td>Unsorted Linked List</td>
<td>$\Theta(n)$</td>
<td>$\Theta(n)$</td>
<td>$\Theta(n)$</td>
</tr>
<tr>
<td>Sorted Array</td>
<td>$\Theta(n)$</td>
<td>$\Theta(\log n)$</td>
<td>$\Theta(n)$</td>
</tr>
<tr>
<td>Sorted Linked List</td>
<td>$\Theta(n)$</td>
<td>$\Theta(n)$</td>
<td>$\Theta(n)$</td>
</tr>
<tr>
<td>Binary Search Tree</td>
<td>$\Theta(n)$</td>
<td>$\Theta(n)$</td>
<td>$\Theta(n)$</td>
</tr>
<tr>
<td>AVL Tree</td>
<td>$\Theta(\log n)$</td>
<td>$\Theta(\log n)$</td>
<td>$\Theta(\log n)$</td>
</tr>
<tr>
<td>Hash Table (Worst case)</td>
<td>$\Theta(n)$</td>
<td>$\Theta(n)$</td>
<td>$\Theta(n)$</td>
</tr>
<tr>
<td>Hash Table (Average)</td>
<td>$\Theta(1)$</td>
<td>$\Theta(1)$</td>
<td>$\Theta(1)$</td>
</tr>
</tbody>
</table>
Hash Tables

- Idea:
  - Have a small array to store information
  - Use a hash function to convert the key into an index
    - Hash function should “scatter” the keys, behave as if it randomly assigned keys to indices
  - Store key at the index given by the hash function
  - Do something if two keys map to the same place (should be very rare)
    - Collision resolution

| Key Object | $h(k)$ | Index between 0 and size-1 | Insert / find / delete | & value |
What Influences Running time?

• How “spread out” our input keys are
  • How much do keys repeat
• Hash the function itself will take time
• Size of the table relative to the number things inserted
• How well our hash function scatters the keys
• What do we do when two things hash to the same spot
Properties of a “Good” Hash

• Definition: A hash function maps objects to integers

• Should be very efficient
  • Calculating the hash should be negligible

• Should randomly scatter objects
  • Objects that are similar to each other should be likely to end up far away

• Should use the entire table
  • There should not be any indices in the table that nothing can hash to
  • Picking a table size that is prime helps with this

• Should use things needed to “identify” the object
  • Use only fields you would check for a .equals method be included in calculating the hash
  • More fields typically leads to fewer collisions, but less efficient calculation
Collision Resolution

• A Collision occurs when we want to insert something into an already-occupied position in the hash table

• 2 main strategies:
  • Separate Chaining
    • Use a secondary data structure to contain the items
      • E.g. each index in the hash table is itself a linked list
  • Open Addressing
    • Use a different spot in the table instead
      • Linear Probing
      • Quadratic Probing
      • Double Hashing
Separate Chaining Insert

• To insert $k, v$:
  • Compute the index using $i = h(k) \% \text{size}$
  • Add the key-value pair to the data structure at $table[i]$
Separate Chaining Find

• To find $k$:
  • Compute the index using $i = h(k) \% \text{size}$
  • Call find with the key on the data structure at $table[i]$
Separate Chaining Delete

• To delete $k$:
  • Compute the index using $i = h(k) \% \text{size}$
  • Call delete with the key on the data structure at $\text{table}[i]$
Formal Running Time Analysis

• The load factor of a hash table represents the average number of items per “bucket”
  • \( \lambda = \frac{n}{\text{size}} \)
• Assume we have a has table that uses a linked-list for separate chaining
  • What is the expected number of comparisons needed in an unsuccessful find?
  • What is the expected number of comparisons needed in a successful find?
• How can we make the expected running time \( \Theta(1) \)?
Load Factor?

\[ k, v \]

\[ k, v \]

0 1 2 3 4 5 6 7 8 9
Load Factor?
Load Factor?
Collision Resolution: Linear Probing

• When there’s a collision, use the next open space in the table
Linear Probing: Insert Procedure

• To insert $k, v$
  • Calculate $i = h(k) \% \text{size}$
  • If $table[i]$ is occupied then try $(i + 1)\% \text{size}$
  • If that is occupied try $(i + 2)\% \text{size}$
  • If that is occupied try $(i + 3)\% \text{size}$
  • ...
Linear Probing: Find

- Let’s do this together!
Linear Probing: Find

• To find key $k$
  • Calculate $i = h(k) \% \text{size}$
  • If $\text{table}[i]$ is occupied and does not contain $k$ then look at $(i + 1) \% \text{size}$
  • If that is occupied and does not contain $k$ then look at $(i + 2) \% \text{size}$
  • If that is occupied and does not contain $k$ then look at $(i + 3) \% \text{size}$
  • Repeat until you either find $k$ or else you reach an empty cell in the table
Linear Probing: Delete

• Let’s do this together!
Linear Probing: Delete

- Option 1: Find the last thing with a matching hash, move that into the spot you deleted from

- Option 2: Called “tombstone” deletion. Leave a special object that indicates an object was deleted from there
  - The tombstone does not act as an open space when finding (so keep looking after its reached)
  - When inserting you can replace a tombstone with a new item

\[
\begin{array}{ccccccccc}
  k, v & k, v & k, v & \text{grave} & k, v & \\
  0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9
\end{array}
\]
Downsides of Linear Probing

• What happens when $\lambda$ approaches 1?
• What happens when $\lambda$ exceeds 1?
Quadratic Probing: Insert Procedure

• To insert $k, v$
  • Calculate $i = h(k) \% \text{size}$
  • If $table[i]$ is occupied then try $(i + 1^2)\% \text{size}$
  • If that is occupied try $(i + 2^2)\% \text{size}$
  • If that is occupied try $(i + 3^2)\% \text{size}$
  • If that is occupied try $(i + 4^2)\% \text{size}$
  • ...

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
Quadratic Probing: Example

- Insert:
  - 76
  - 40
  - 48
  - 5
  - 55
  - 47
Using Quadratic Probing

• If you probe $tablesizet$ times, you start repeating the same indices

• If $tablesizet$ is prime and $\lambda < \frac{1}{2}$ then you’re guaranteed to find an open spot in at most $tablesizet/2$ probes

• Helps with the clustering problem of linear probing, but does not help if many things hash to the same value
Double Hashing: Insert Procedure

• Given $h$ and $g$ are both good hash functions
• To insert $k, v$
  • Calculate $i = h(k) \% \text{size}$
  • If $\text{table}[i]$ is occupied then try $(i + g(k)) \% \text{size}$
  • If that is occupied try $(i + 2 \cdot g(k)) \% \text{size}$
  • If that is occupied try $(i + 3 \cdot g(k)) \% \text{size}$
  • If that is occupied try $(i + 4 \cdot g(k)) \% \text{size}$
  • ...
Rehashing

• If your load factor $\lambda$ gets too large, copy everything over to a larger hash table
  • To do this: make a new array with a new hash function
  • Re-insert all items into the new hash table with the new hash function
  • New hash table should be “roughly” double the size (but probably still want it to be prime)