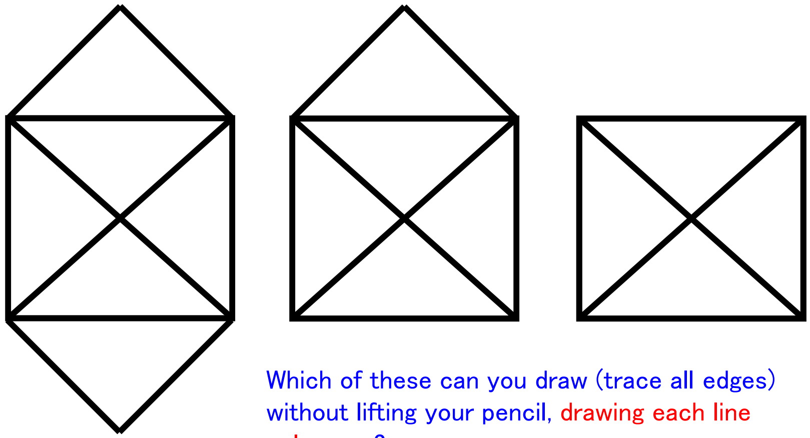
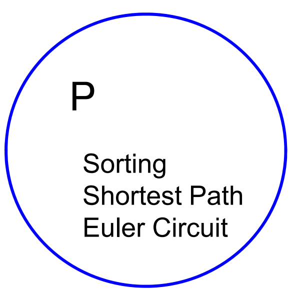
Try it!



Which of these can you draw (trace all edges) without lifting your pencil, drawing each line only once?
Can you start and end at the same point?



Hamiltonian Circuit
Satisfiability (SAT)
Vertex Cover
Travelling Salesman

Euler Circuits

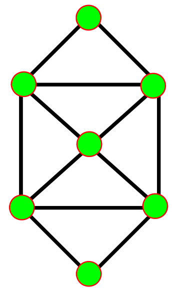
- Euler circuit: a path through a graph that *visits each **edge** exactly once* and *starts and ends at the same vertex*
- Named after Leonhard Euler (1707-1783), who cracked this problem and founded graph theory in 1736
- An Euler circuit exists *iff*
  - the graph is connected and
  - each vertex has even degree (= # of edges on the vertex)

The Road Inspector: Finding Euler Circuits

Given a connected, undirected graph G = (V,E), find an Euler circuit in G

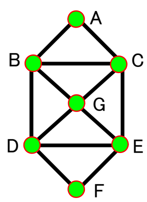
Can check if one exists:
- Check if all vertices have even degree

Basic Euler Circuit Algorithm:
1. Do an edge walk from a start vertex until you are back to the start vertex.
   - You never get stuck because of the even degree property.
2. "Remove" the walk, leaving several components each with the even degree property.
   - Recursively find Euler circuits for these.
3. Splice all these circuits into an Euler circuit



Running time?

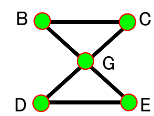
Euler Circuit Example



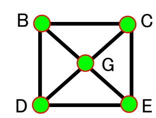
Euler(A) :

Hamiltonian Circuits

- **Euler circuit**: A cycle that goes through each *edge* exactly once
- **Hamiltonian circuit**: A cycle that goes through each *vertex* exactly once
- Does graph **I** have:
  - An Euler circuit?
  - A Hamiltonian circuit?
- Does graph **II** have:
  - An Euler circuit?
  - A Hamiltonian circuit?
- Which problem sounds harder?



I



II

Polynomial vs. Exponential Time

- All of the algorithms we have discussed in this class have been **polynomial time** algorithms:
  - Examples: O(log N), O(N), O(N log N), $O(N^2)$
  - Algorithms whose running time is $O(N^k)$ for some k > 0
- **Exponential time** b^N is asymptotically worse than any polynomial function N^k for any k

The Complexity Class P

- P is the set of all problems that can be solved in *polynomial worst case time*
  - All *problems* that have some *algorithm* whose running time is $O(N^k)$ for some *k*
- Examples of problems in P: sorting, shortest path, Euler circuit, *etc.*