

Graphs

- A graph is a formalism for representing relationships among items
 - Very general definition because very general concept

- A **graph** is a pair

$$G = (V, E)$$

- A set of **vertices**, also known as **nodes**

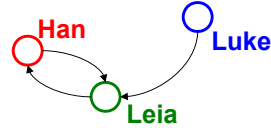
$$V = \{v_1, v_2, \dots, v_n\}$$

- A set of **edges**

$$E = \{e_1, e_2, \dots, e_m\}$$

- Each edge e_i is a pair of vertices (v_j, v_k)
- An edge “connects” the vertices

- Graphs can be **directed** or **undirected**



$$V = \{\text{Han}, \text{Leia}, \text{Luke}\}$$

$$E = \{(\text{Luke}, \text{Leia}), (\text{Han}, \text{Leia}), (\text{Leia}, \text{Han})\}$$

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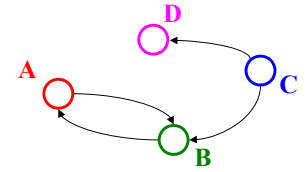
More notation

For a graph $G = (V, E)$:

- $|V|$ is the number of vertices
- $|E|$ is the number of edges
 - Minimum?
 - Maximum for undirected?
 - Maximum for directed?

- If $(u, v) \in E$

- Then v is a **neighbor** of u , i.e., v is **adjacent** to u
- Order matters for directed edges
 - u is not **adjacent** to v unless $(v, u) \in E$



$$V = \{A, B, C, D\}$$

$$E = \{(C, B), (A, B), (B, A), (C, D)\}$$

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Examples again

Which would use **directed edges**? Which would have **self-edges**?
Which could have **0-degree nodes**?

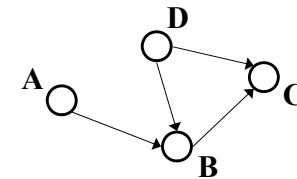
- Web pages with links
- Facebook friends
- “Input data” for the Kevin Bacon game
- Methods in a program that call each other
- Road maps (e.g., Google maps)
- Airline routes
- Family trees
- Course pre-requisites
- ...

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Paths/cycles in directed graphs

Example:



Is there a **path** from A to D?

Does the graph contain any **cycles**?

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Density / sparsity

- Recall: In an undirected graph, $0 \leq |E| < |V|^2$
- Recall: In a directed graph: $0 \leq |E| \leq |V|^2$
- So for any graph, $|E|$ is $O(|V|^2)$
- One more fact: If an undirected graph is *connected*, then $|E| \geq |V|-1$
- Because $|E|$ is often much smaller than its maximum size, we do not always approximate as $|E|$ as $O(|V|^2)$
 - This is a correct bound, it just is often not tight
 - If it is tight, i.e., $|E|$ is $\Theta(|V|^2)$ we say the graph is **dense**
 - More sloppily, dense means “lots of edges”
 - If $|E|$ is $O(|V|)$ we say the graph is **sparse**
 - More sloppily, sparse means “most (possible) edges missing”

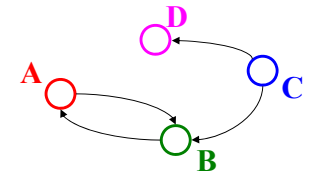
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Adjacency Matrix Properties

- Running time to:
 - Get a vertex's out-edges:
 - Get a vertex's in-edges:
 - Decide if some edge exists:
 - Insert an edge:
 - Delete an edge:
- Space requirements:
- Best for sparse or dense graphs?

	A	B	C	D
A	F	T	F	F
B	T	F	F	F
C	F	T	F	T
D	F	F	F	F



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Adjacency Matrix Properties

- How will the adjacency matrix vary for an *undirected graph*?
- How can we adapt the representation for *weighted graphs*?

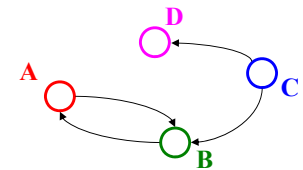
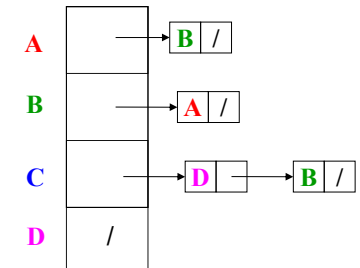
	A	B	C	D
A	F	T	F	F
B	T	F	F	F
C	F	T	F	T
D	F	F	F	F

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Adjacency List Properties

- Running time to:
 - Get all of a vertex's out-edges:
 - Get all of a vertex's in-edges:
 - Decide if some edge exists:
 - Insert an edge:
 - Delete an edge:
- Space requirements:
- Best for dense or sparse graphs?



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