



# CSE 332: Data Structures & Parallelism

## Lecture 7: Dictionaries; Binary Search Trees

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Winter 2022

# *Today*

- Dictionaries
- Trees

# *Where we are*

Studying the absolutely essential ADTs of computer science and classic data structures for implementing them

ADTs so far:

1. Stack: `push, pop, isEmpty, ...`
2. Queue: `enqueue, dequeue, isEmpty, ...`
3. Priority queue: `insert, deleteMin, ...`

Next:

4. Dictionary (a.k.a. Map): associate keys with values
  - probably the most common, way more than priority queue

# The Dictionary (a.k.a. Map) ADT

Data:

- set of (key, value) *pairs*
- keys must be *comparable*

Operations:

- **insert(key, val)** :
  - places (key, val) in map  
(If key already used, overwrites existing entry)
- **find(key)** :
  - returns val associated with key
- **delete(key)**

**insert(rea, Ruth Anderson)**



**find(jhsia)**



**Justin Hsia,...**

• **rea**  
**Ruth**  
**Anderson**  
...

• **jhsia**  
**Justin**  
**Hsia**  
...

– ...

*We will tend to emphasize the keys, but don't forget about the stored values!*

# *Comparison: Set ADT vs. Dictionary ADT*

The *Set* ADT is like a Dictionary without any values

- A key is *present* or not (no repeats)

For **find**, **insert**, **delete**, there is little difference

- In dictionary, values are “just along for the ride”
- So *same data-structure ideas* work for dictionaries and sets
  - Java HashSet implemented using a HashMap, for instance

Set ADT may have other important operations

- **union**, **intersection**, **is\_subset**, etc.
- Notice these are binary operators on sets
- We will want different data structures to implement these operators

# *A Modest Few Uses for Dictionaries*

Any time you want to store information according to some key and be able to retrieve it efficiently – a **dictionary** is the ADT to use!

– Lots of programs do that!

- Networks: router tables
- Operating systems: page tables
- Compilers: symbol tables
- Databases: dictionaries with other nice properties
- Search: inverted indexes, phone directories, ...
- Biology: genome maps
- ...

# *Simple implementations*

For dictionary with  $n$  key/value pairs

**insert**      **find**      **delete**

- Unsorted linked-list
- Unsorted array
- Sorted linked list
- Sorted array

We'll see a Binary Search Tree (BST) probably does better, but not in the worst case unless we keep it balanced

# Lazy Deletion (e.g. in a sorted array)

10	12	24	30	41	42	44	45	50
✓	✗	✓	✓	✓	✓	✗	✓	✓

A general technique for making **delete** as fast as find:

- Instead of actually removing the item just mark it deleted
- No need to shift values, etc.

Plusses:

- Simpler
- Can do removals later in batches
- If re-added soon thereafter, just unmark the deletion

Minuses:

- Extra *space* for the “is-it-deleted” flag
- Data structure full of deleted nodes wastes *space*
- **find**  $O(\log m)$  *time* where  $m$  is data-structure size ( $m \geq n$ )
- May complicate other operations

# *Better Dictionary data structures*

Will spend the next several lectures looking at dictionaries with three different data structures

## 1. AVL trees

- Binary search trees with *guaranteed balancing*

## 2. B-Trees

- Also always balanced, but different and shallower
- B≠Binary; B-Trees generally have large branching factor

## 3. Hashtables

- Not tree-like at all

Skipping: Other balanced trees (red-black, splay)

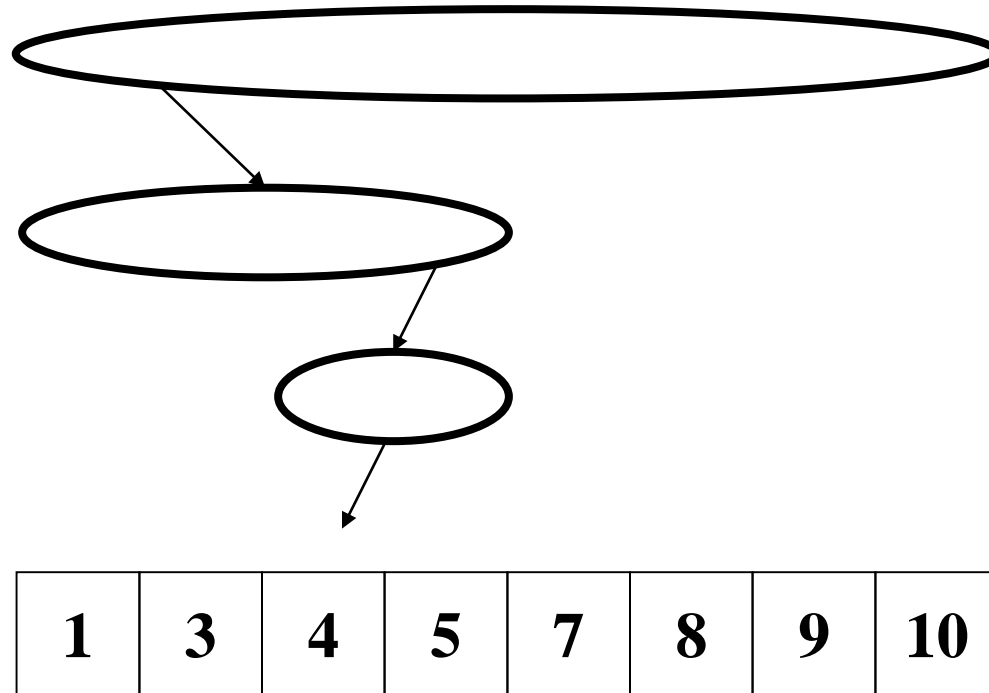
# *Why Trees?*

Trees offer speed ups because of their branching factors

- Binary Search Trees are structured forms of *binary search*

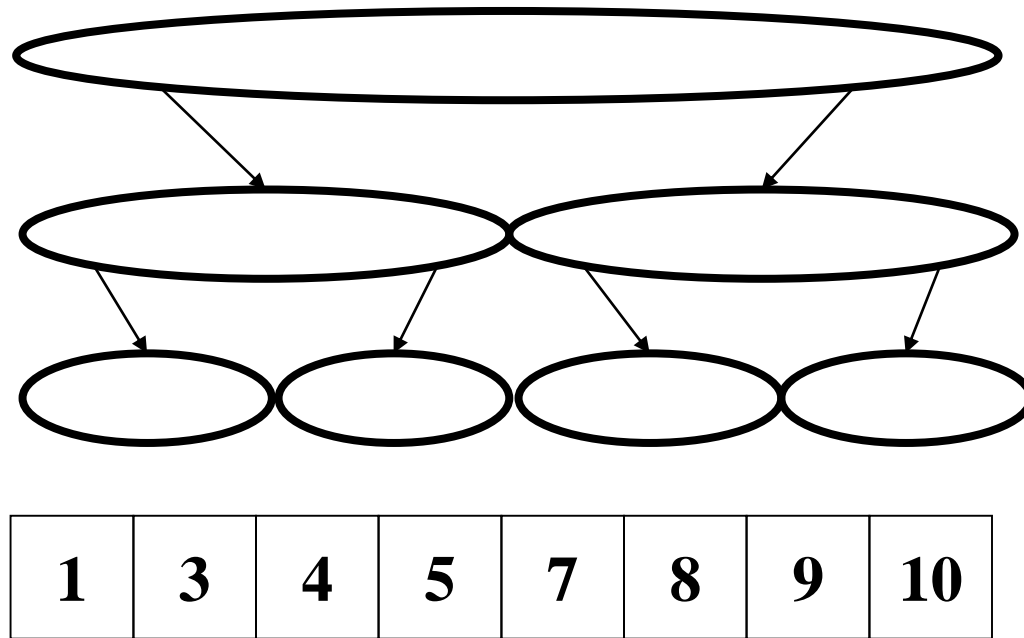
# *Binary Search*

**find(4)**



# *Binary Search Tree*

Our goal is the performance of binary search in a tree representation



# Why Trees?

Trees offer speed ups because of their branching factors

- Binary Search Trees are structured forms of *binary search*

Even a basic BST is fairly good

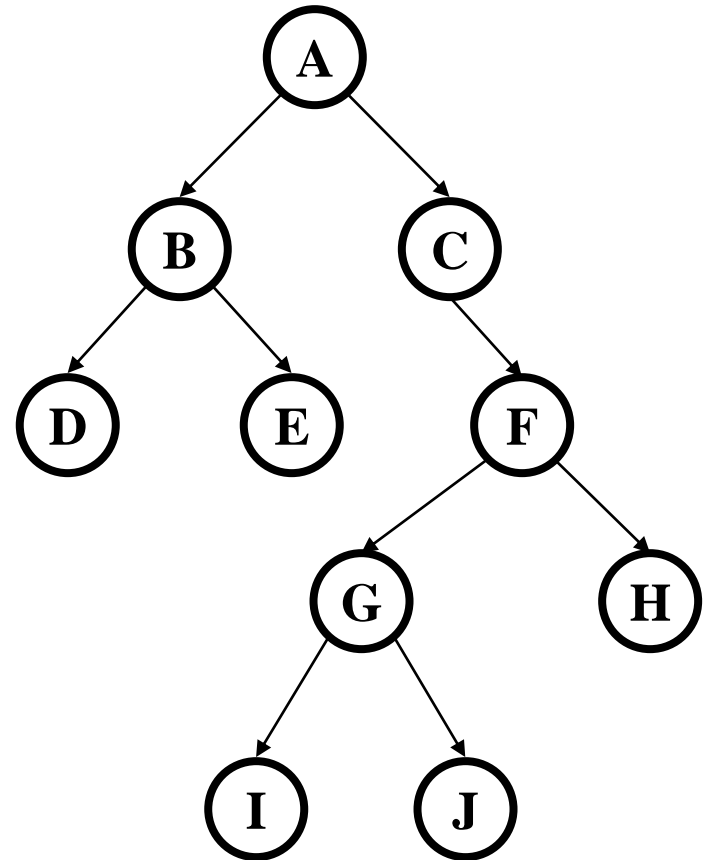
	Insert	Find	Delete
Worse-Case	$O(n)$	$O(n)$	$O(n)$
Average-Case	$O(\log n)$	$O(\log n)$	$O(\log n)$

# Binary Trees

- Binary tree is empty or
  - a root (*with data*)
  - a left subtree (*maybe empty*)
  - a right subtree (*maybe empty*)
- Representation:

<b>Data</b>	
left pointer	right pointer

- For a dictionary, data will include a key and a value



# *Binary Tree: Some Numbers*

Recall: height of a tree = longest path from root to leaf (count # of edges)

For binary tree of height  $h$ :

- max # of leaves:
- max # of nodes:
- min # of leaves:
- min # of nodes:

# *Calculating height*

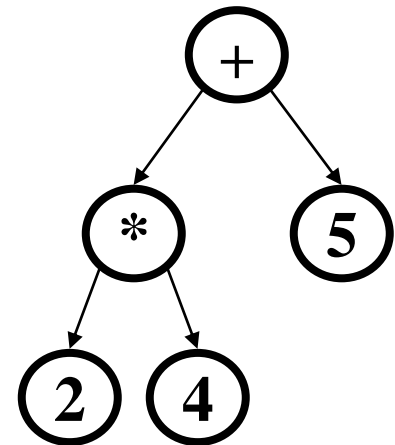
What is the height of a tree with root `root`?

```
int treeHeight(Node root) {  
    ???  
}
```

# Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree

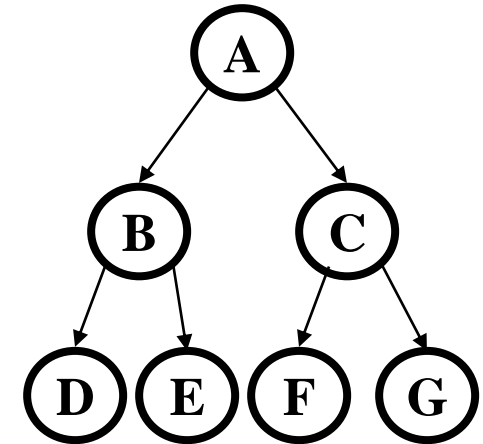
- *Pre-order*: root, left subtree, right subtree
- *In-order*: left subtree, root, right subtree
- *Post-order*: left subtree, right subtree, root



**(an expression tree)**

# More on traversals

```
void inOrdertraversal(Node t) {  
    if(t != null) {  
        traverse(t.left);  
        process(t.element);  
        traverse(t.right);  
    }  
}
```



Sometimes order doesn't matter

- Example: sum all elements

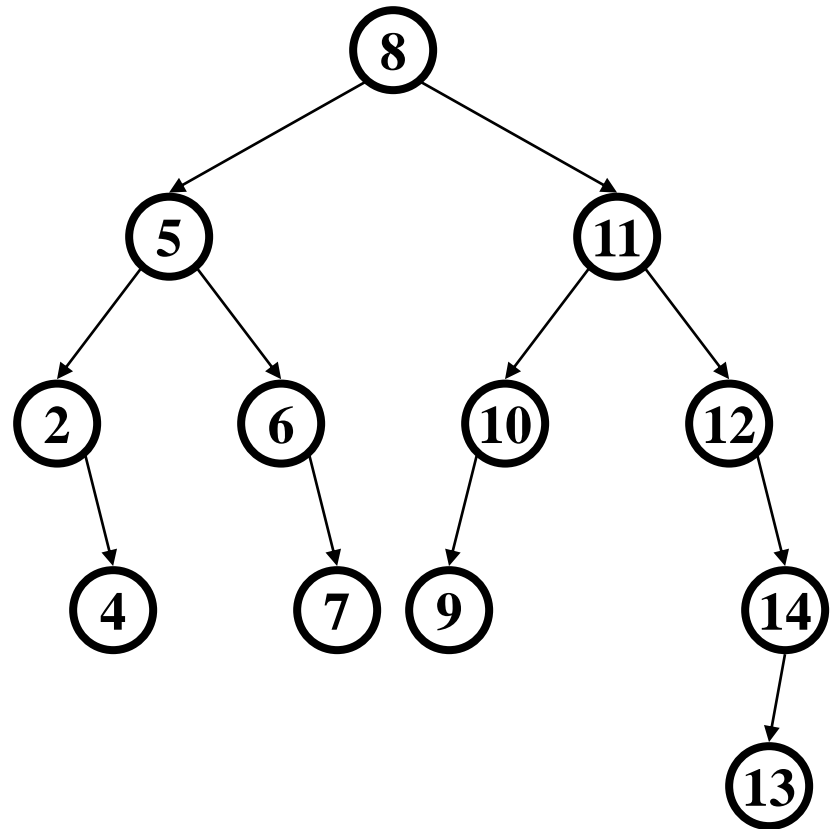
Sometimes order matters

- Example: print tree with parent above indented children (pre-order)
- Example: evaluate an expression tree (post-order)

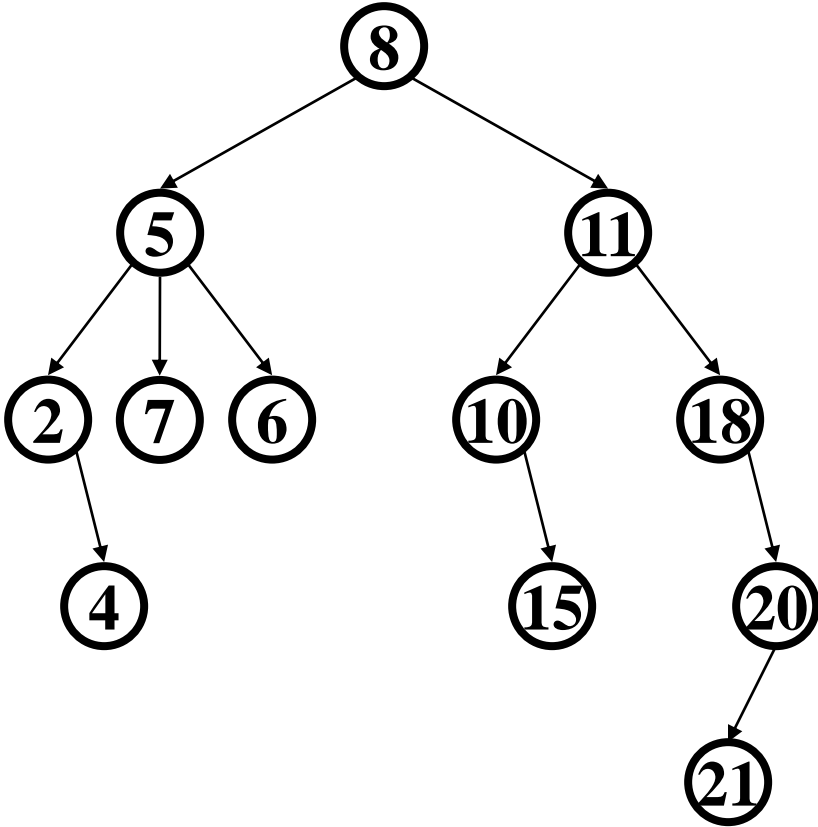
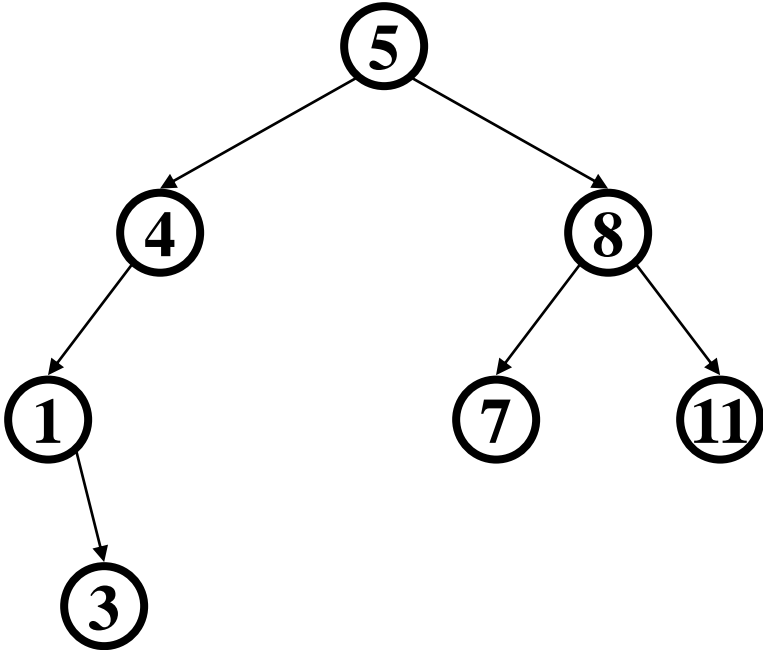
A  
  B  
    D  
    E  
  C  
    F  
    G

# Binary Search Tree

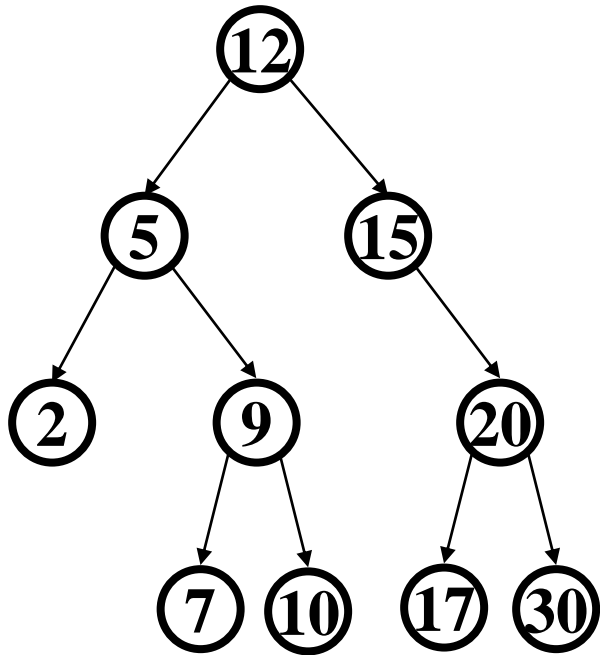
- Structural property (“binary”)
  - each node has  $\leq 2$  children
  - result: keeps operations simple
- Order property
  - all keys in left subtree smaller than node’s key
  - all keys in right subtree larger than node’s key
  - result: easy to find any given key



*Are these BSTs?*

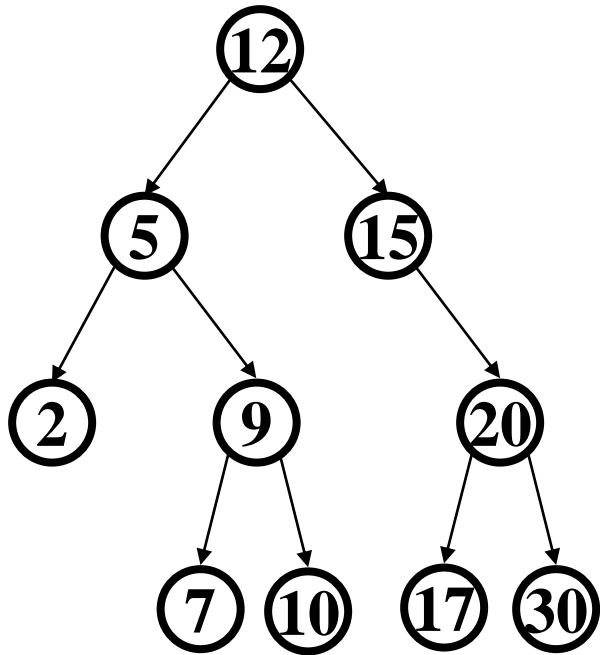


# Find in BST, Recursive



```
Data find(Key key, Node root) {  
    if(root == null)  
        return null;  
    if(key < root.key)  
        return find(key, root.left);  
    if(key > root.key)  
        return find(key, root.right);  
    return root.data;  
}
```

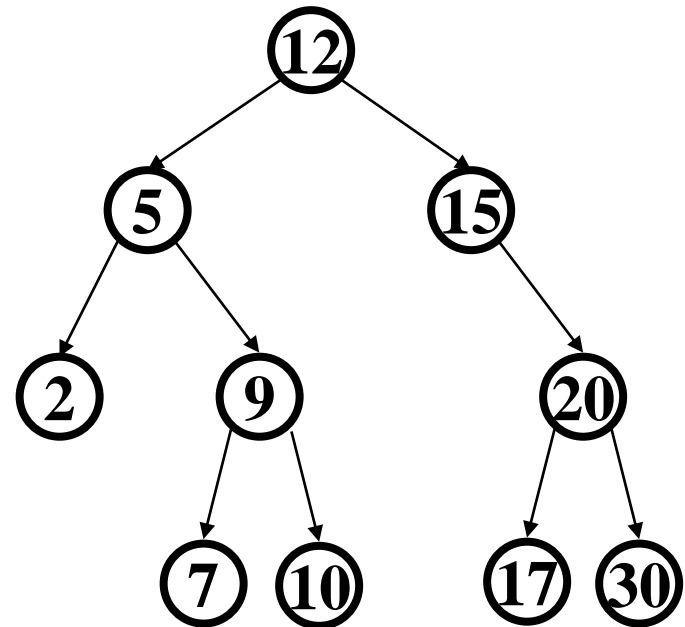
# Find in BST, Iterative



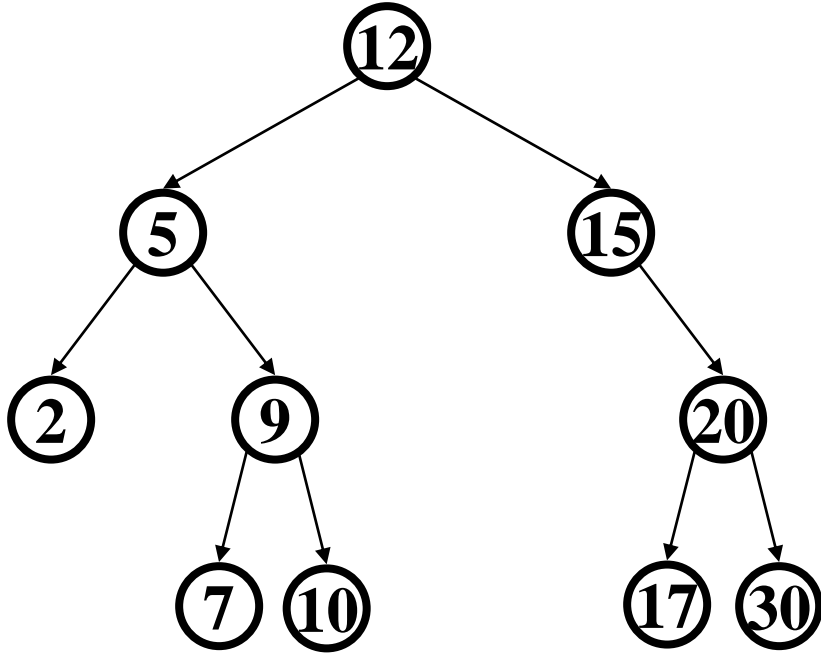
```
Data find(Key key, Node root) {  
    while(root != null  
        && root.key != key) {  
        if(key < root.key)  
            root = root.left;  
        else(key > root.key)  
            root = root.right;  
        }  
    if(root == null)  
        return null;  
    return root.data;  
}
```

# Other “finding operations”

- Find *minimum* node
- Find *maximum* node



# Insert in BST



`insert(13)`

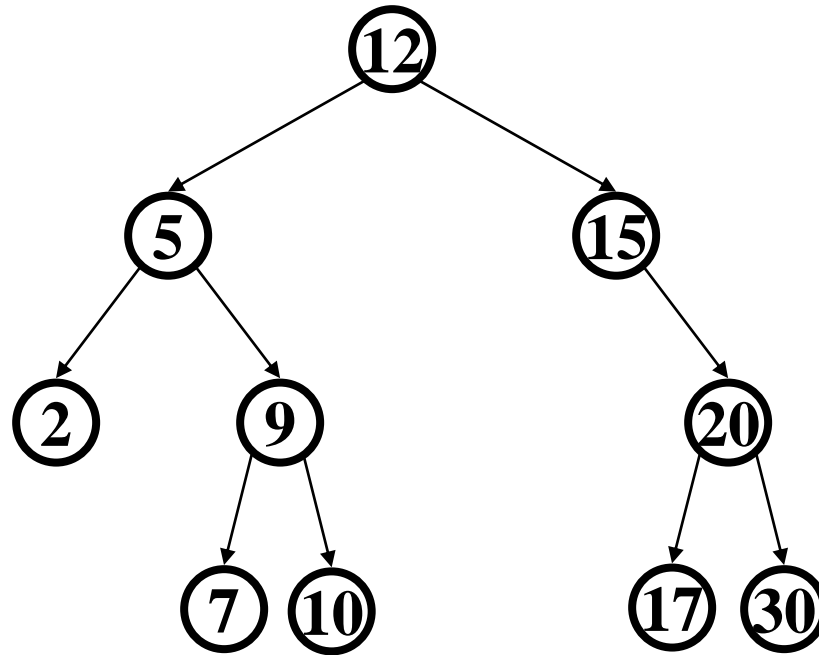
`insert(8)`

`insert(31)`

(New) insertions happen only at leaves – easy!

1. Find
2. Create a new node

# *Deletion in BST*



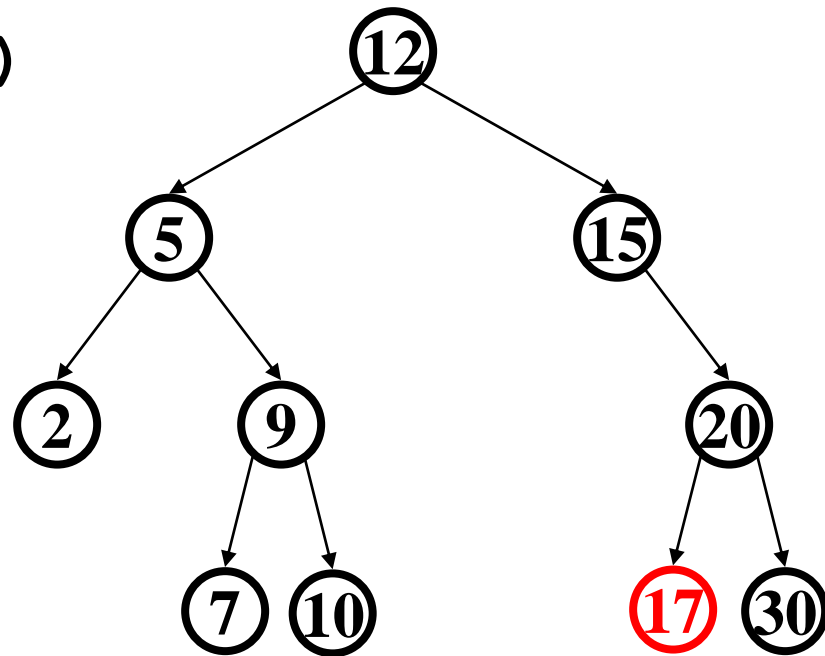
Why might deletion be harder than insertion?

# *Deletion*

- Removing an item disrupts the tree structure
- Basic idea:
  - **find** the node to be removed,
  - Remove it
  - “fix” the tree so that it is still a binary search tree
- Three cases:
  - node has no children (leaf)
  - node has one child
  - node has two children

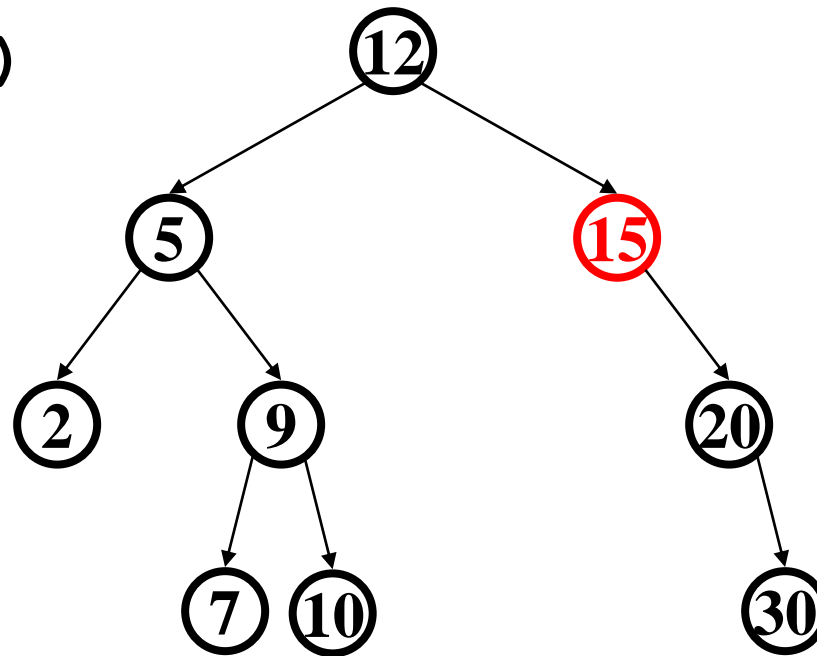
# Deletion – The Leaf Case

delete (17)



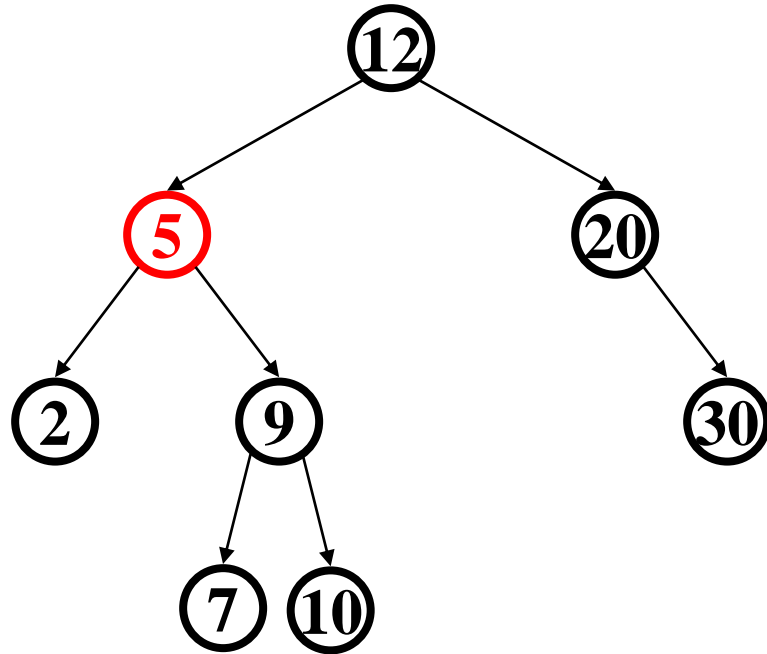
# Deletion – The One Child Case

delete (15)



# Deletion – The Two Child Case

delete (5)



What can we replace **5** with?

# *Deletion – The Two Child Case*

Idea: Replace the deleted node with a value guaranteed to be between the two child subtrees

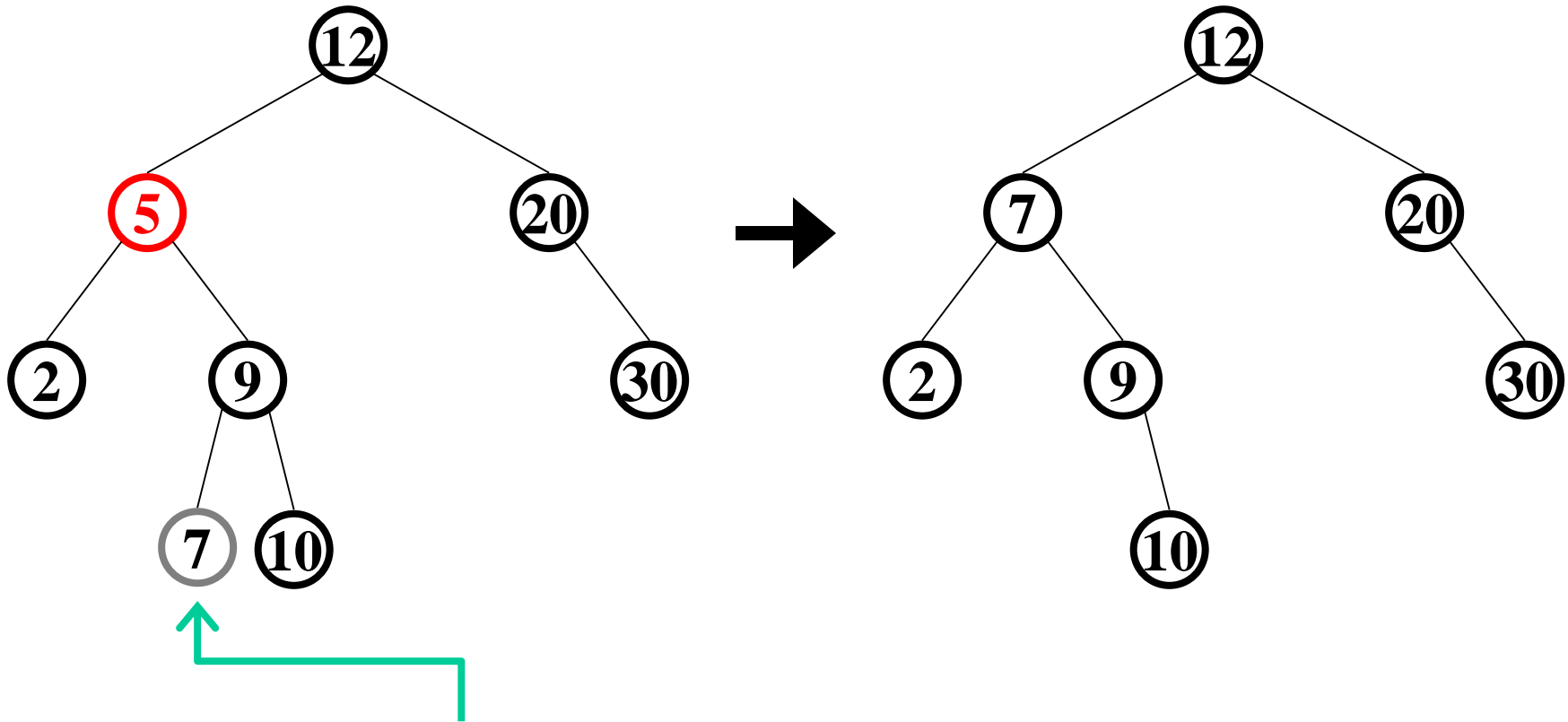
Options:

- *successor* from right subtree: `findMin(node.right)`
- *predecessor* from left subtree: `findMax(node.left)`
  - These are the easy cases of predecessor/successor

Now delete the original node containing *successor* or *predecessor*

- Leaf or one child case – easy cases of delete!

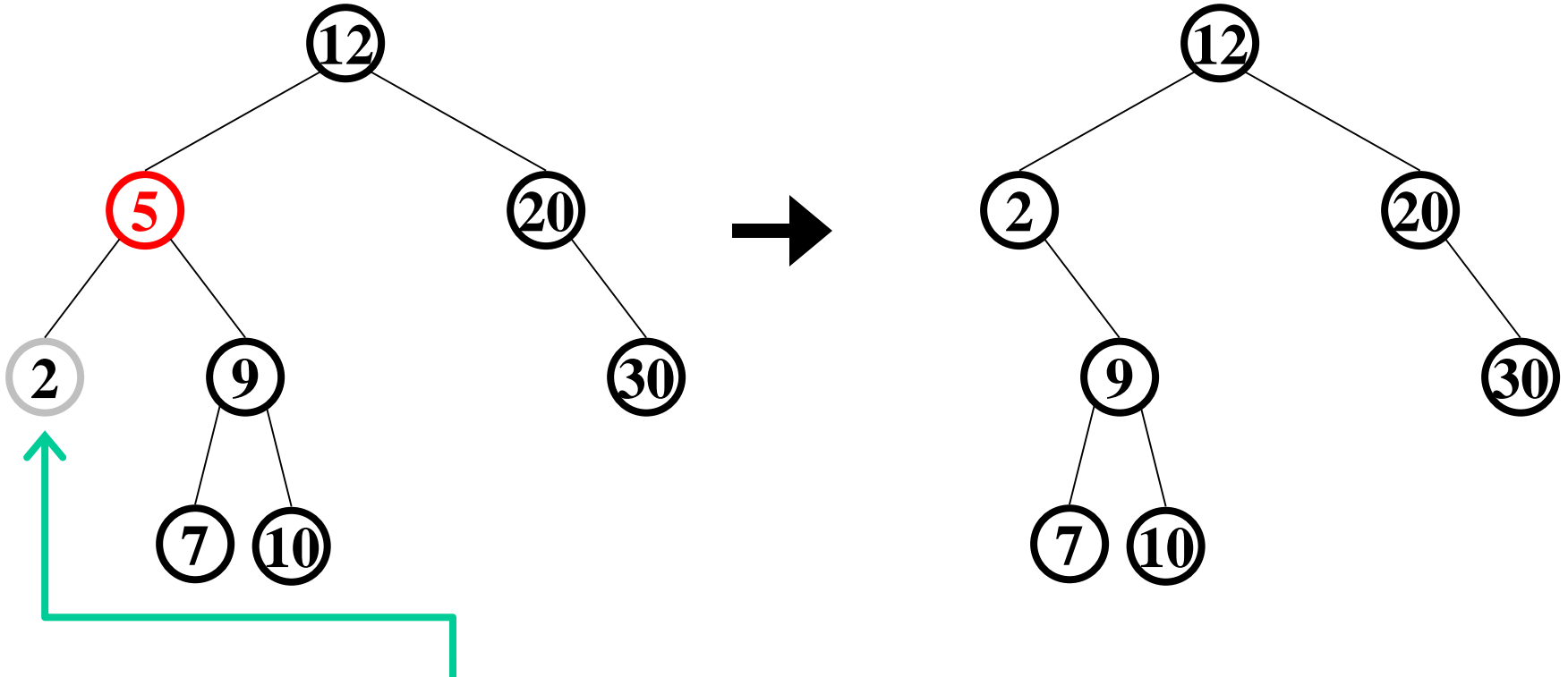
# Delete Using Successor



**findMin(right sub tree)  $\rightarrow$  7**

**delete (5)**

# Delete Using Predecessor

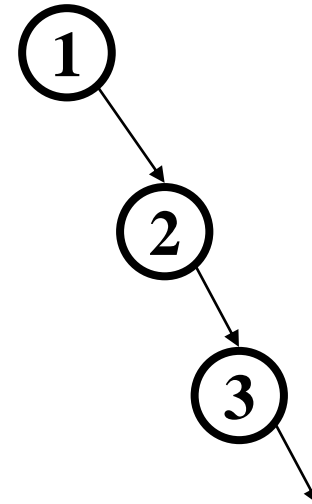


**findMax(left sub tree) → 2**

**delete (5)**

# *BuildTree for BST*

- We had `buildHeap`, so let's consider `buildTree`
- Insert keys 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST
  - If inserted in given order, what is the tree?
  - What big-O runtime for this kind of sorted input?
  - Is inserting in the reverse order any better?



# Balanced BST

## Observation

- BST: the shallower the better!
- For a BST with  $n$  nodes inserted in arbitrary order
  - Average height is  $O(\log n)$  – see text for proof
  - Worst case height is  $O(n)$
- Simple cases such as inserting in key order lead to the worst-case scenario

*Solution:* Require a **Balance Condition** that

1. ensures depth is always  $O(\log n)$  – strong enough!
2. is easy to maintain – not too strong!

# *Potential Balance Conditions*

1. Left and right subtrees of the *root* have equal number of nodes
2. Left and right subtrees of the *root* have equal *height*

# *Potential Balance Conditions*

3. Left and right subtrees of every node have equal number of nodes
4. Left and right subtrees of every node have equal *height*

# *The AVL Balance Condition*

Left and right subtrees of *every node*  
have *heights differing by at most 1*

*Definition:* **balance**(*node*) = height(*node*.left) – height(*node*.right)

*AVL property:* **for every node  $x$ ,  $-1 \leq \text{balance}(x) \leq 1$**

- Ensures small depth
  - Will prove this by showing that an AVL tree of height  $h$  must have a number of nodes *exponential* in  $h$
- Easy (well, efficient) to maintain
  - Using single and double rotations