## CSE 332: Data Structures and Parallelism

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## Section 2: Heaps and Asymptotics

Definition of Big-Oh:

Suppose $f: \mathbb{N} \to \mathbb{R}$, $g: \mathbb{N} \to \mathbb{R}$ are two functions,

$$f(n) \in \mathcal{O}(g(n)) \equiv \exists_{c \in \mathbb{R}_{>0},\, n_0 \in \mathbb{N}} \forall_{n \in \mathbb{N} \geq n_0} f(n) \leq c \cdot g(n)$$

Definition of Big-Omega:

Suppose $f: \mathbb{N} \to \mathbb{R}$, $g: \mathbb{N} \to \mathbb{R}$ are two functions,

$$f(n) \in \Omega(g(n)) \equiv \exists_{c \in \mathbb{R}_{>0},\, n_0 \in \mathbb{N}} \forall_{n \in \mathbb{N} \geq n_0} f(n) \geq c \cdot g(n)$$

Definition of Big-Theta:

Suppose $f: \mathbb{N} \to \mathbb{R}$, $g: \mathbb{N} \to \mathbb{R}$ are two functions,

$$f(n) \in \Theta(g(n))$$
$$\equiv f(n) \in \mathcal{O}(g(n)) \wedge f(n) \in \Omega(g(n))$$
$$\equiv \exists_{c_0 \in \mathbb{R}_{>0},\, n_0 \in \mathbb{N}} \forall_{n \in \mathbb{N} \geq n_0} f(n) \leq c_0 \cdot g(n) \wedge \exists_{c_1 \in \mathbb{R}_{>0},\, n_1 \in \mathbb{N}} \forall_{n \in \mathbb{N} \geq n_1} f(n) \geq c_1 \cdot g(n)$$

# 0. Big-Oh Proofs

For each of the following, prove that $f(n) \in \mathcal{O}(g)$:

a) $f(n) = 7n$ $\qquad\qquad$ $g(n) = \frac{n}{10}$

b) $f(n) = 1000$ $g(n) = 3n^3$

c) $f(n) = 7n^2 + 3n$ $g(n) = n^4$

d) $f(n) = n + 2n \lg n$ $g(n) = n \lg n$

# 1. Is Your Program Running? Better Catch It!

For each of the following, determine the tight $\Theta(\cdot)$ bound for the worst-case runtime in terms of the free variables of the code snippets.

a)

```
1 int x = 0
2 for (int i = n; i >= 0; i--) {
3     if ((i % 3) == 0) {
4         break
5     }
6     else {
7         x += n
8     }
9 }
```

b)

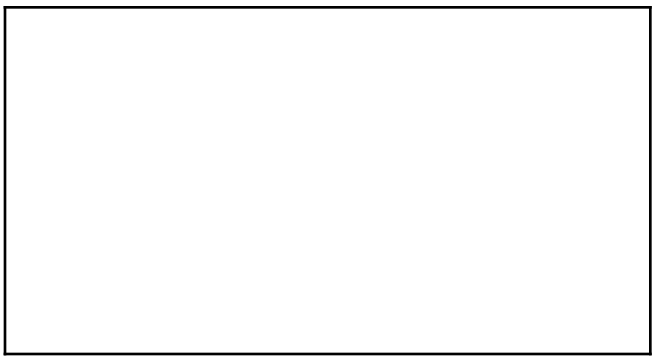
```
1 int x = 0
2 for (int i = 0; i < n; i++) {
3     for (int j = 0; j < (n * n / 3); j++) {
4         x += j
5     }
6 }
```

c)

```
1 int x = 0
2 for (int i = 0; i < n; i++) {
3     for (int j = 0; j < i; j++) {
4     x += j
5     }
6 }
```

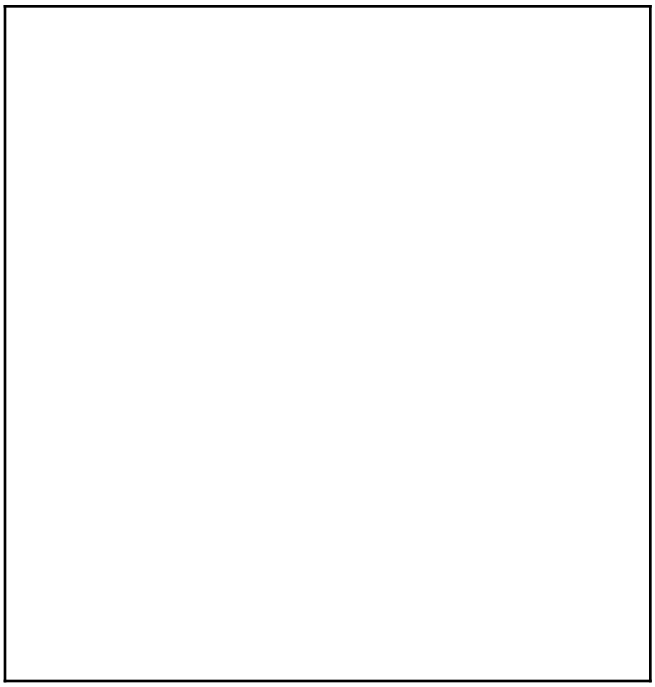
d)

```
1  int x = 0
2  for (int i = 0; i < n; i++) {
3      if (n < 100000) {
4          for (int j = 0; j < i * i * n; j++) {
5              x += 1
6          }
7      } else {
8          x += 1
9      }
10 }
```



e)

```
1  int x = 0
2  for (int i = 0; i < n; i++) {
3      if (i % 5 == 0) {
4          for (int j = 0; j < n; j++) {
5              if (i == j) {
6                  for (int k = 0; k < n; k++) {
7                      x += i * j * k
8                  }
9              }
10         }
11     }
12 }
```



# 2. Asymptotics Analysis

Consider the following method which finds the number of unique Strings within a given array of length $n$.

```
1  int numUnique(String[] values) {
2      boolean[] visited = new boolean[values.length]
3      for (int i = 0; i < values.length; i++) {
4          visited[i] = false
5      }
6      int out = 0
7      for (int i = 0; i < values.length; i++) {
8          if (!visited[i]) {
9              out += 1
10             for (int j = i; j < values.length; j++) {
11                 if (values[i].equals(values[j])) {
12                     visited[j] = true
13                 }
14             }
15         }
16     }
17     return out;
18 }
```

Determine the tight $\mathcal{O}(\cdot)$, $\Omega(\cdot)$, and $\Theta(\cdot)$ bounds of each function below. If there is no $\Theta(\cdot)$ bound, explain why. Start by (1) constructing an equation that models each function then (2) simplifying and finding a closed form.

a) $f(n) =$ the worst-case runtime of `numUnique`

b) $g(n)$ = the best-case runtime of `numUnique`

c) $h(n)$ = the amount of memory used by `numUnique` (the space complexity)

# 3. Oh Snap!

For each question below, explain what's wrong with the provided answer. The problem might be the reasoning, the conclusion, or both!

a) Determine the tight $\Theta(\cdot)$ bound worst-case runtime of the following piece of code:

```
1  public static int waddup(int n) {
2      if (n > 10000) {
3          return n
4      } else {
5          for (int i = 0; i < n; i++) {
6              System.out.println("It's dat boi!")
7          }
8          return 0
9      }
10 }
```

**Bad answer**: The runtime of this function is $\mathcal{O}(n)$, because when searching for an upper bound, we always analyze the code branch with the highest runtime. We see the first branch is $\mathcal{O}(1)$, but the second branch is $\mathcal{O}(n)$.

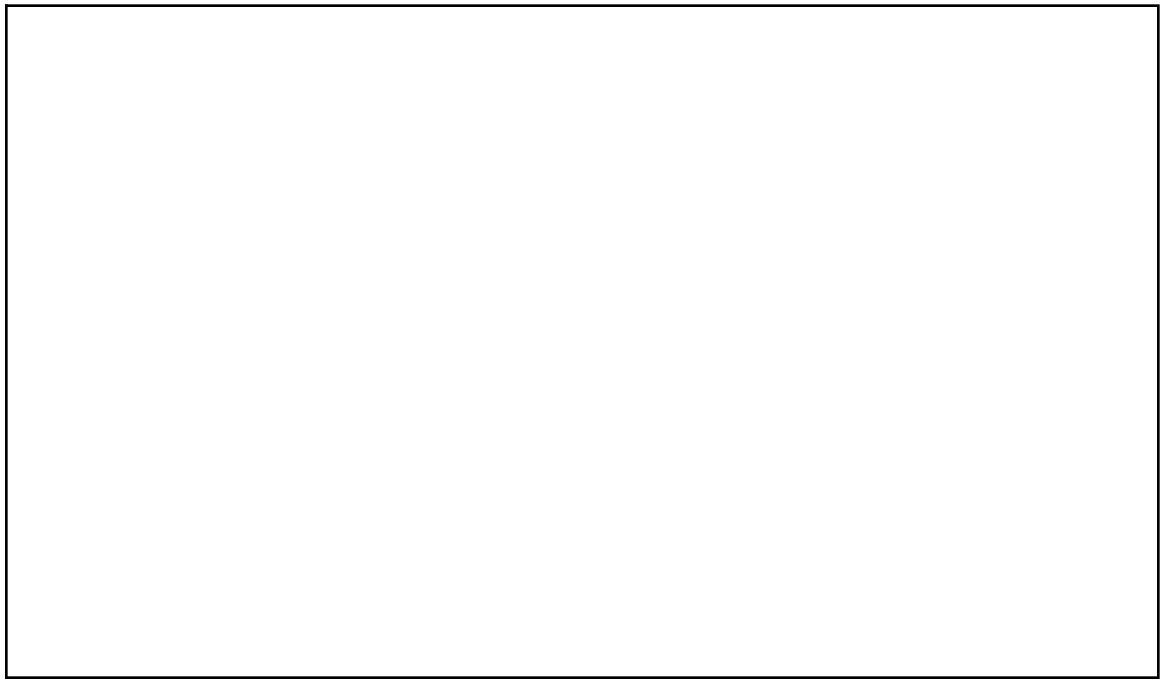
b) Determine the tight $\Theta(\cdot)$ bound worst-case runtime of the following piece of code:

```
1  public static void trick(int n) {
2      for (int i = 1; i < Math.pow(2, n); i *= 2) {
3          for (int j = 0; j < n; j++) {
4              System.out.println("(" + i + "," + j + ")")
5          }
6      }
7  }
```

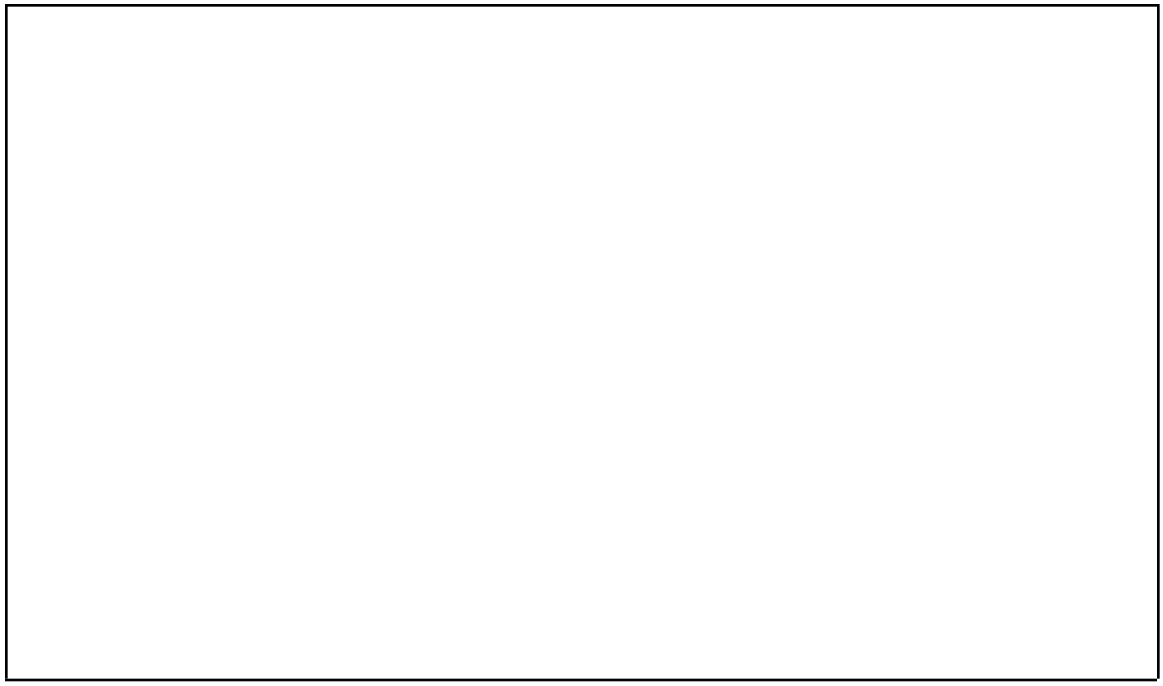
**Bad answer**: The runtime of this function is $\mathcal{O}(n^2)$, because the outer loop is conditioned on an expression with $n$ and so is the inner loop.

# 4. Look Before You Heap

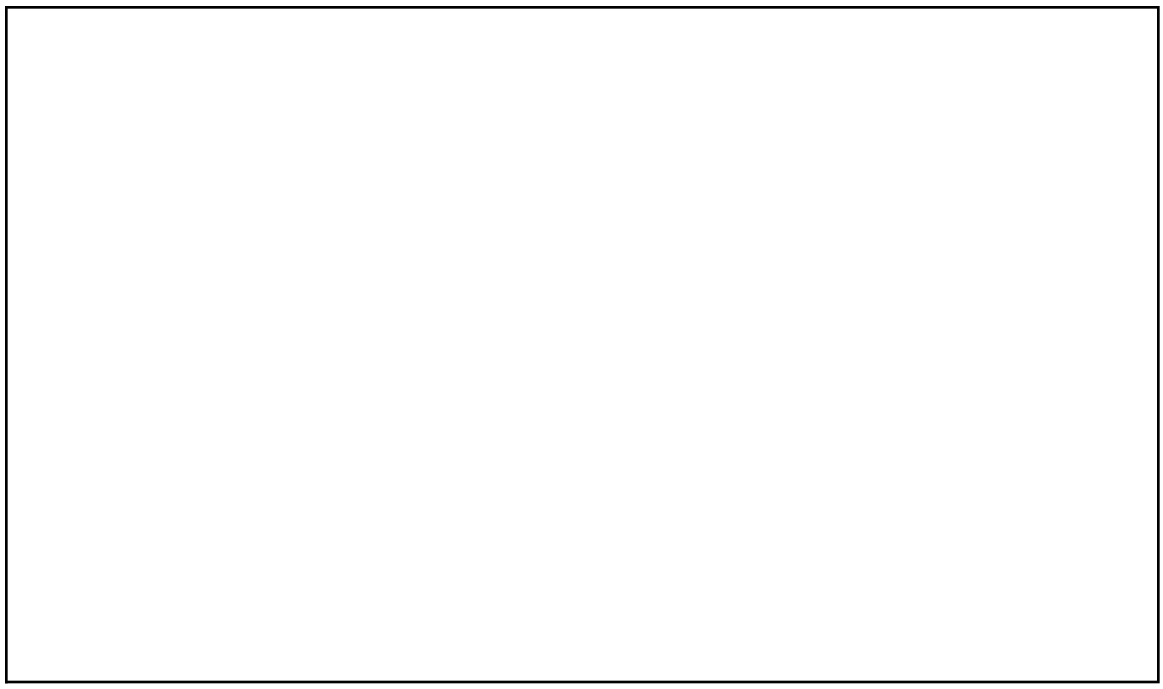
a) Insert 10, 7, 15, 17, 12, 20, 6, 32 into a *min* heap.



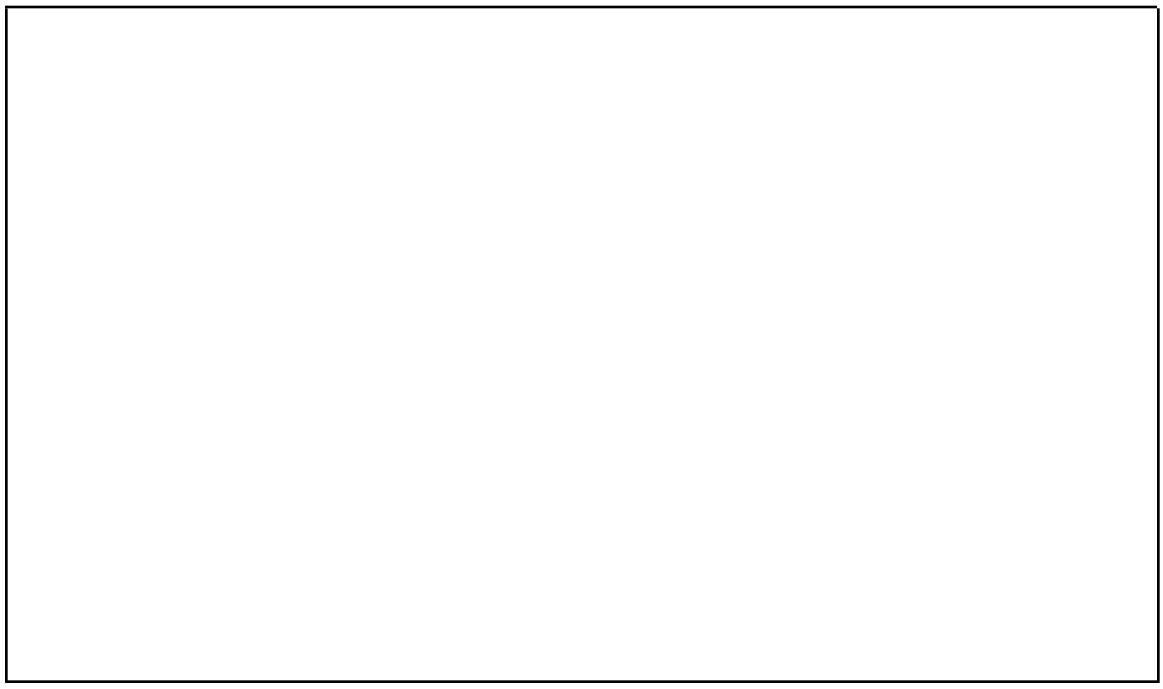
b) Now, insert the same values into a *max* heap.



c) Now, insert 10, 7, 15, 17, 12, 20, 6, 32 into a *min* heap, but use Floyd's `buildHeap` algorithm.



d) Insert 1, 0, 1, 1, 0 into a *min* heap.



## 5. $\mathcal{O}$ My God!

Prove that $4n^2 + n^5 \in \Omega(n)$. Use the definition of Big-Omega above.