CSE 332: Data Structures & Parallelism
Lecture 11: Hashing 2, Comparison Sorts

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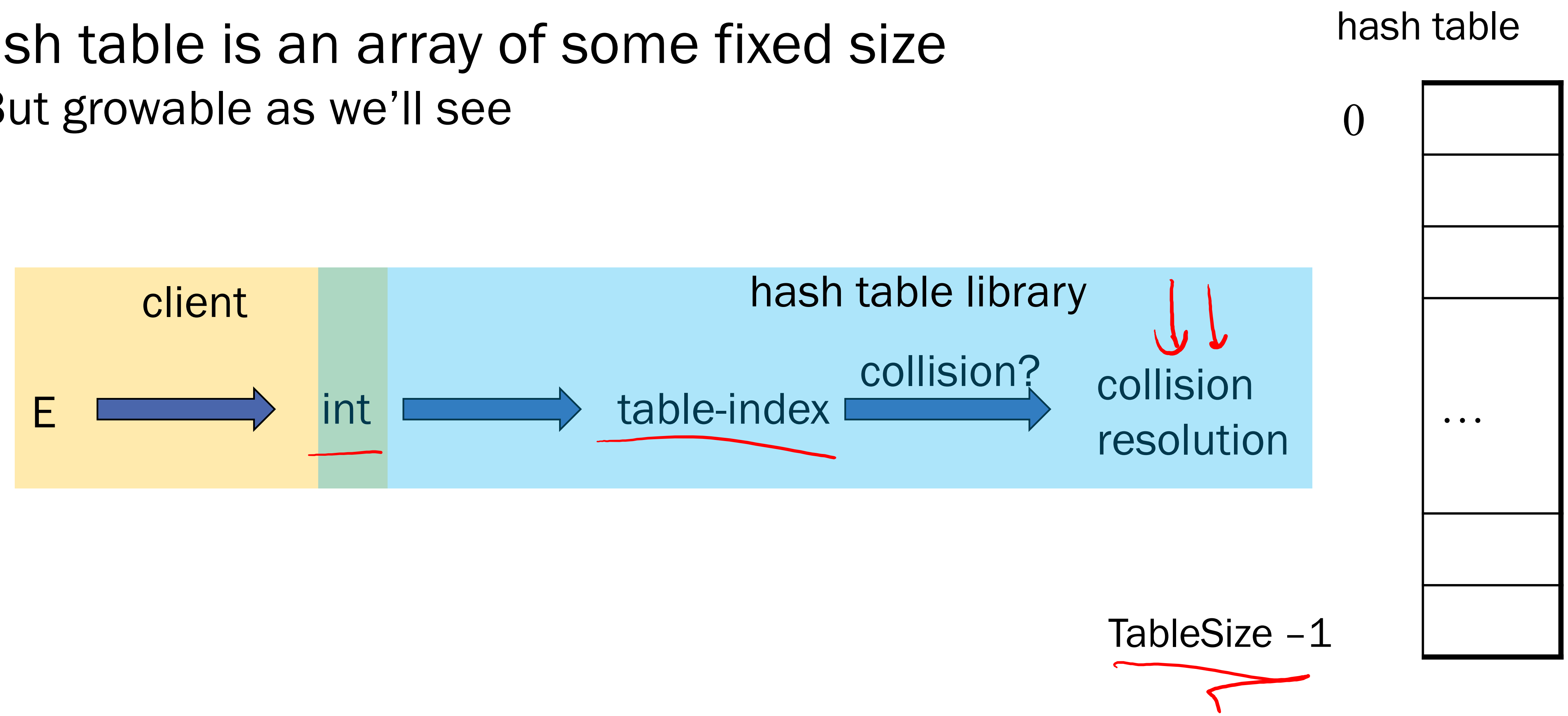
Outline

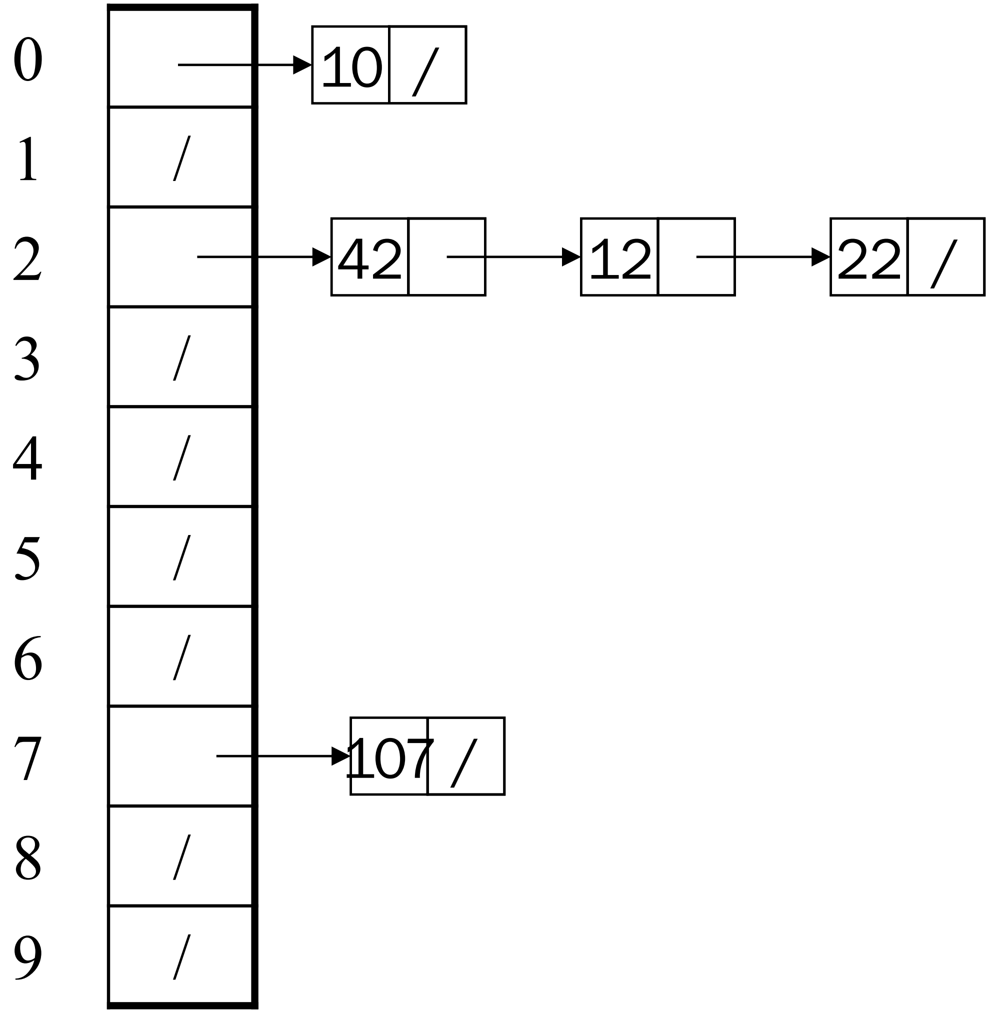
- Hashing
  - Open Addressing
  - Rehashing
  - Hashing in Practice
- Comparison Sorting

Hash Tables: Review

- Aim for constant-time (i.e., $O(1)$) **`find`**, **`insert`**, and **`delete`**
  - "On average" under some reasonable assumptions
- A hash table is an array of some fixed size
  - But growable as we'll see



Separate Chaining: Review



Chaining: All keys that map to the same table location are kept in a list (a.k.a. a "chain" or "bucket")

As easy as it sounds

Example: insert 10, 22, 107, 12, 42 with mod hashing and **`TableSize`** = 10

Worst case time for find?

Average case time for find?

More rigorous separate chaining analysis

Definition: The load factor, λ, of a hash table is

$$\lambda = \frac{\text{N}}{\text{TableSize}}$$

← number of elements

Under chaining, the average number of elements per bucket is $\underline{\lambda}$

So if some inserts are followed by *random* finds, then on average:

- **Each unsuccessful `find` compares against $\underline{\lambda}$ items**
- **Each successful `find` compares against $\underline{\lambda/2}$ items**

$O(\lambda)$

- **How big should TableSize be??**

$\lambda = 1$

Equal objects must hash the same

The Java library (and your project hash table) make a very important assumption that clients must satisfy…

- Object-oriented way of saying it:

  If `a.equals(b)`, then we must require `a.hashCode()==b.hashCode()`

- Function object way of saying it:

  If `c.compare(a,b) == 0`, then we must require

  `h.hash(a) == h.hash(b)`

- If you ever override equals
  - You need to override hashCode also in a consistent way
  - See CoreJava book, Chapter 5 for other "gotchas" with equals

Hashing Choices

1. Choose a Hash function
2. Choose TableSize
3. Choose a Collision Resolution Strategy from these:
    - Separate Chaining
    - Open Addressing
        - Linear Probing
        - Quadratic Probing
        - Double Hashing

Other issues to consider:

- Deletion?
- What to do when the hash table gets “too full”?

Open Addressing: Linear Probing

- Why not use up the empty space in the table?
- Store directly in the array cell (no linked list)
- How to deal with collisions?
- If **`h(key)`** is already full,
    - try **`(h(key) + 1) % TableSize`**. If full,
    - try **`(h(key) + 2) % TableSize`**. If full,
    - try **`(h(key) + 3) % TableSize`**. If full…

- Example: insert 38, 19, 8, 109, 10

0	8_2
1	109_2
2	10_2
3	
4	
5	
6	
7	
8	38
9	19

109_1 10_0

10_1

8_0

8_1 109_0

Open Addressing: Linear Probing

- Another simple idea: If **`h(key)`** is already full,
  - try **`(h(key) + 1) % TableSize`**. If full,
  - try **`(h(key) + 2) % TableSize`**. If full,
  - try **`(h(key) + 3) % TableSize`**. If full…

- Example: insert 38, 19, 8, 109, 10

$h(key) + \underbrace{i}$

0	8
1	109
2	10
3	/
4	/
5	/
6	/
7	/
8	38
9	19

Open addressing

Linear probing is *one example* of open addressing

In general, open addressing means resolving collisions by trying a sequence of other positions in the table.

Trying the *next* spot is called probing

- We just did linear probing:
  - `i`th probe: `(h(key) + i) % TableSize`
- In general have some probe function `f` and :
  - `i`th probe: `(h(key) + f(i)) % TableSize`

Open addressing does poorly with high load factor λ

- **So, want larger tables**
- **Too many probes means no more $O(1)$**

Aside: Terminology

We and the book use the terms

- “chaining” or “separate chaining”
- “open addressing”

Very confusingly,

- “open hashing” is a synonym for “chaining”
- “closed hashing” is a synonym for “open addressing”

Questions: Open Addressing: Linear Probing

How should `find` work? If value is in table? If not there?

Worst case scenario for `find`?

How should we implement `delete`?

How does open addressing with linear probing compare to separate chaining?

Open Addressing: Linear Probing

- Another simple idea: If **`h(key)`** is already full,
  - try **`(h(key) + 1) % TableSize`**. If full,
  - try **`(h(key) + 2) % TableSize`**. If full,
  - try **`(h(key) + 3) % TableSize`**. If full…

- Example: insert 38, 19, 8, 109, 10

✓ find(109) ← find(20)

0	8
1	109
2	10
3	/
4	/
5	/
6	/
7	/
8	38
9	19

Open Addressing: Other Operations

`insert` finds an open table position using a probe function

What about `find`?

- Must use same probe function to "retrace the trail" for the data
- Unsuccessful search when reach empty position

What about `delete`?

- *Must* use "lazy" deletion. Why?
  - Marker indicates "no data here, but don't stop probing"

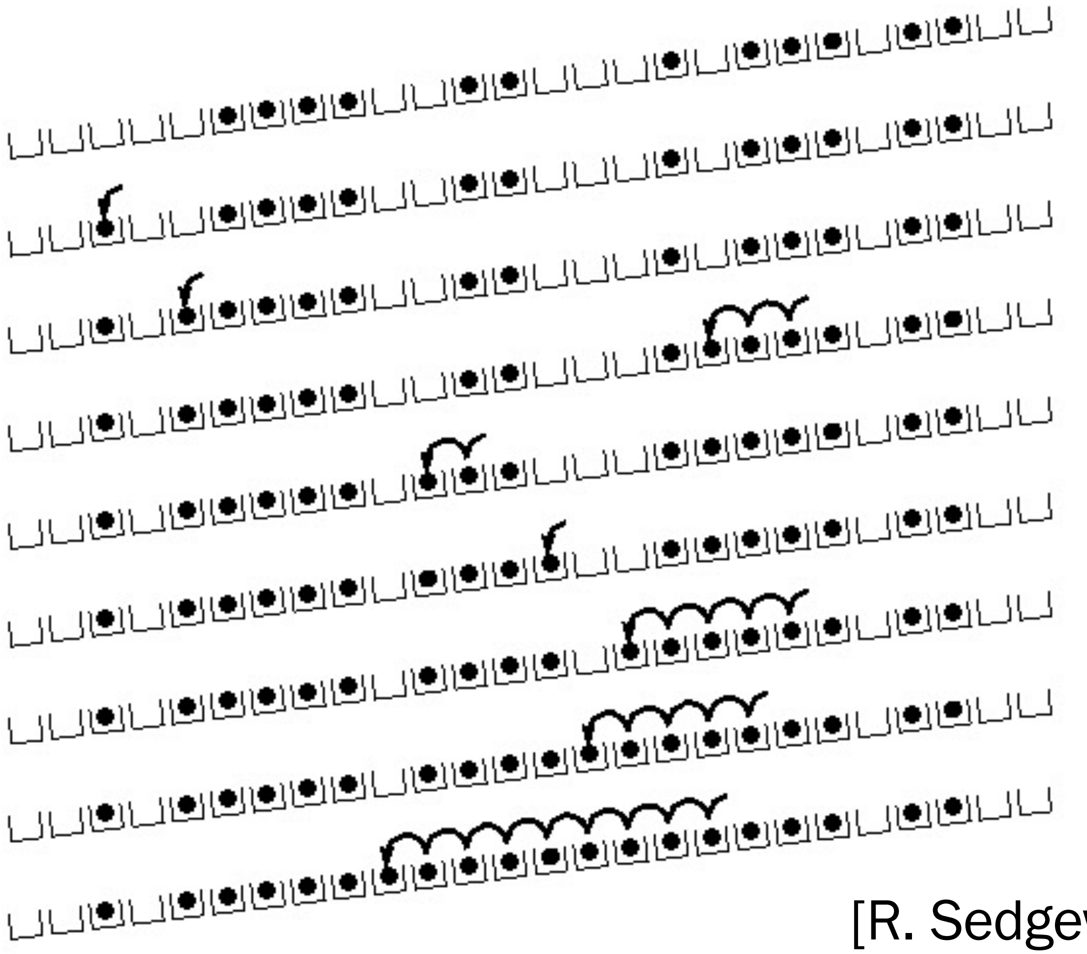
10	×	/	23	/	/	16	×	26

  - As with lazy deletion on other data structures, on insert, spots marked "deleted" can be filled in.
- Note: `delete` with chaining is just calling delete on the bucket (e.g. linked list)

Primary Clustering

It turns out linear probing is a *bad idea*, even though the probe function is quick to compute (a good thing)

- Tends to produce *clusters*, which lead to long probe sequences
- Called primary clustering
- Saw the start of a cluster in our linear probing example



[R. Sedgewick]

Analysis of Linear Probing

- **Trivial fact:** For any $\lambda < 1$, linear probing will find an empty slot
  - It is "safe" in this sense: no infinite loop unless table is full

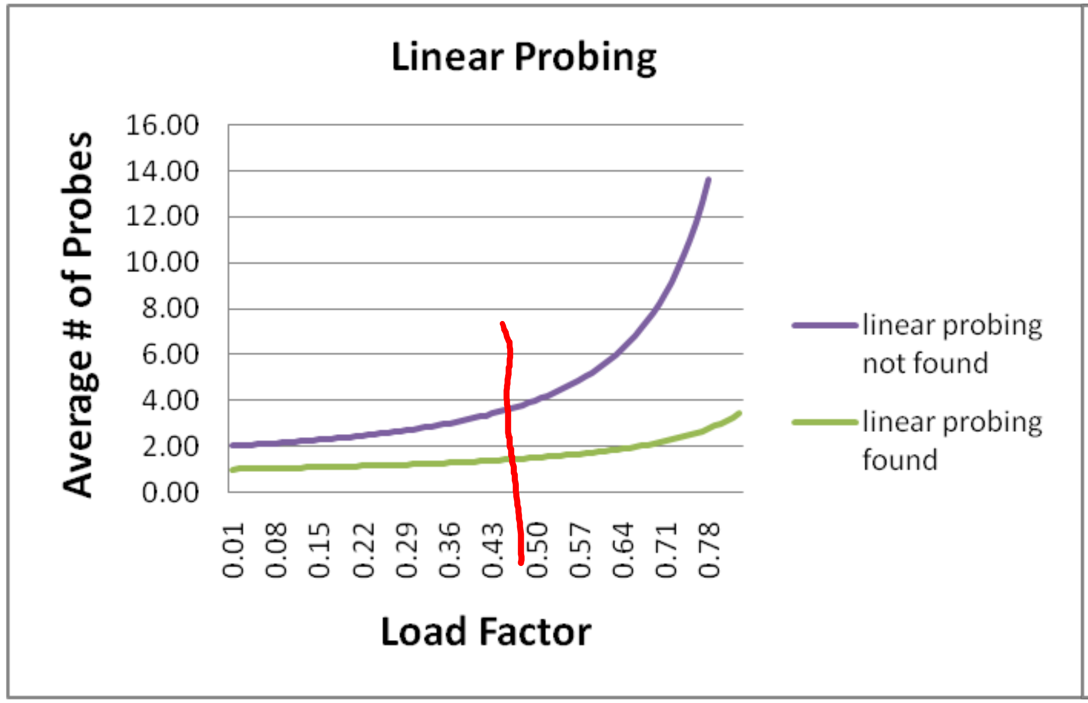
- **Non-trivial facts** we won't prove:

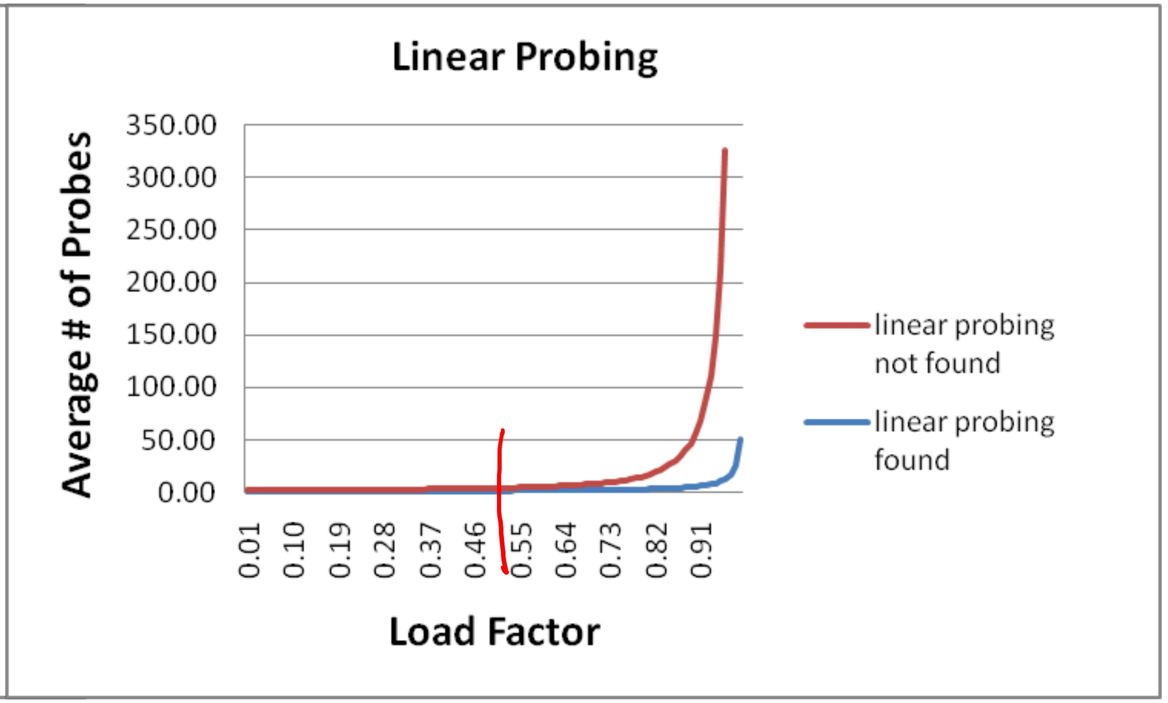
  Average # of probes given λ (in the limit as **`TableSize`** $\rightarrow \infty$)
  - Unsuccessful search: $\frac{1}{2}\left(1+\frac{1}{(1-\lambda)^2}\right)$
  - Successful search: $\frac{1}{2}\left(1+\frac{1}{(1-\lambda)}\right)$

- This is pretty bad: need to leave sufficient empty space in the table to get decent performance (see chart)

Analysis in chart form

- Linear-probing performance degrades rapidly as table gets full
  - (Formula assumes "large table" but point remains)





- By comparison, separate chaining performance is linear in λ and has no trouble with $\lambda>1$

Open Addressing: Linear probing

`(h(key) + f(i)) % TableSize`

- For linear probing:

  `f(i) = i`

- So probe sequence is:
  - 0^{th} probe: `h(key) % TableSize`
  - 1^{st} probe: `(h(key) + 1) % TableSize`
  - 2^{nd} probe: `(h(key) + 2) % TableSize`
  - 3^{rd} probe: `(h(key) + 3) % TableSize`
  - …
  - i^{th} probe: `(h(key) + i) % TableSize`

Open Addressing: Quadratic probing

- We can avoid primary clustering by changing the probe function...

$$\texttt{(h(key) + f(i)) \% TableSize}$$

  - For quadratic probing:

    `f(i) = i`^{2}

  - So probe sequence is:
    - 0th probe: `h(key) % TableSize`
    - 1st probe: `(h(key) + 1) % TableSize`
    - 2nd probe: `(h(key) + 4) % TableSize`
    - 3rd probe: `(h(key) + 9) % TableSize`
    - …
    - ith probe: `(h(key) + i`^{2}`) % TableSize`

- Intuition: Probes quickly "leave the neighborhood" ith probe: (`h(key) + i`^{2}`) % TableSize`

Quadratic Probing Example

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

TableSize=10
Insert:
89
18
49
58
79

Quadratic Probing Example

TableSize = 10

insert(89)

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	89

Quadratic Probing Example

0	
1	
2	
3	
4	
5	
6	
7	
8	18
9	89

TableSize = 10

insert(89)

insert(18)

Quadratic Probing Example

0	49_1
1	
2	
3	
4	
5	
6	
7	
8	18
9	89

49_0

TableSize = 10

insert(89)

insert(18)

insert(49)

$49 \% 10 = 9$

$(49 + 1^2)$

Quadratic Probing Example

0	49	
1		
2	58_2	
3		
4		
5		
6		
7		
8	18	58_0
9	89	58_1

TableSize = 10

insert(89)

insert(18)

insert(49)

49 % 10 = 9 collision!

(49 + 1) % 10 = 0

insert(58)

$58 + 1^2$

$58 + 2^2 =$

Quadratic Probing Example

0	49	79_1
1		
2	58	
3	79_2	
4		
5		
6		
7		
8	18	
9	89	79_0

TableSize = 10

insert(89)

insert(18)

insert(49)

insert(58)

  58 % 10 = 8 collision!

  (58 + 1) % 10 = 9 collision!

  (58 + 4) % 10 = 2

insert(79)

Quadratic Probing Example

0	49
1	
2	58
3	79
4	
5	
6	
7	
8	18
9	89

TableSize = 10

insert(89)

insert(18)

insert(49)

insert(58)

insert(79)

79 % 10 = 9 collision!

(79 + 1) % 10 = 0 collision!

(79 + 4) % 10 = 3

ith probe: $(h(key) + i^2) \% TableSize$

Another Quadratic Probing Example

0	
1	
2	
3	
4	
5	
6	76

TableSize = 7

Insert:

76	(76 % 7 = 6)
40	(40 % 7 = 5)
48	(48 % 7 = 6)
5	(5 % 7 = 5)
55	(55 % 7 = 6)
47	(47 % 7 = 5)

ith probe: $(h(key) + i^2) \% TableSize$

Another Quadratic Probing Example

0	
1	
2	
3	
4	
5	
6	76

TableSize = 7

Insert:

76	(76 % 7 = 6)
40	(40 % 7 = 5)
48	(48 % 7 = 6)
5	(5 % 7 = 5)
55	(55 % 7 = 6)
47	(47 % 7 = 5)

Another Quadratic Probing Example

0	
1	
2	
3	
4	
5	40
6	76

TableSize = 7

Insert:

76	(76 % 7 = 6)
40	(40 % 7 = 5)
48	(48 % 7 = 6)
5	(5 % 7 = 5)
55	(55 % 7 = 6)
47	(47 % 7 = 5)

ith probe: `(h(key) + i`^{2}`) % TableSize`

Another Quadratic Probing Example

0	48	
1		
2	5_2	
3		
4		
5	40	5_0
6	76	5_1

TableSize = 7

Insert:

76	(76 % 7 = 6)
40	(40 % 7 = 5)
48	(48 % 7 = 6)
5	(5 % 7 = 5)
55	(55 % 7 = 6)
47	(47 % 7 = 5)

Another Quadratic Probing Example

0	48	55_1
1		
2	5	
3	55	
4		
5	40	
6	76	55_0

TableSize = 7

Insert:

76	(76 % 7 = 6)
40	(40 % 7 = 5)
48	(48 % 7 = 6)
5	(5 % 7 = 5)
55	(55 % 7 = 6)
47	(47 % 7 = 5)

1

2^2

3^2

ith probe: (`h(key) + i`^{2}`) % TableSize`

Another Quadratic Probing Example

0	48
1	
2	5
3	55
4	
5	40
6	76

TableSize = 7

Insert:

76	(76 % 7 = 6)
40	(40 % 7 = 5)
48	(48 % 7 = 6)
5	(5 % 7 = 5)
55	(55 % 7 = 6)
47	(47 % 7 = 5)

ith probe: `(h(key) + i`^{2}`) % TableSize`

Another Quadratic Probing Example

0	48	47_3 47_4
1		
2	5	47_2
3	55	
4		
5	40	47
6	76	47_1

TableSize = 7

Insert:

76	(76 % 7 = 6)
40	(40 % 7 = 5)
48	(48 % 7 = 6)
5	(5 % 7 = 5)
55	(55 % 7 = 6)
47	(47 % 7 = 5)

(47 + 1) % 7 = 6 collision!

(47 + 4) % 7 = 2 collision!

(47 + 9) % 7 = 0 collision!

(47 + 16) % 7 = 0 collision!

(47 + 25) % 7 = 2 collision!

Will we ever get a 1 or 4?!?

From bad news to good news

Bad News:

- After **`TableSize`** quadratic probes, we cycle through the same indices

Good News:

- If **`TableSize`** is *prime* and $\lambda < ½$, then quadratic probing will find an empty slot in at most **`TableSize/2`** probes
- So: If you keep $\lambda < ½$ and **`TableSize`** is *prime*, no need to detect cycles

Quadratic Probing: Success guarantee for $\lambda < \frac{1}{2}$

First size/2 probes distinct.
If < half full, one is empty.

- If size is prime and $\lambda < \frac{1}{2}$, then quadratic probing will find an empty slot in size/2 probes or fewer.
  - show for all $0 \le i,j \le size/2$ and $i \neq j$

    (ith probe and jth probe map to distinct locations)

    $$(h(x) + i^2) \bmod size \neq (h(x) + j^2) \bmod size$$

  - by contradiction: suppose that for some $i \neq j$:

    $$(h(x) + i^2) \bmod size = (h(x) + j^2) \bmod size$$
    $$\Rightarrow \quad i^2 \bmod size = j^2 \bmod size$$
    $$\Rightarrow \quad (i^2 - j^2) \bmod size = 0$$
    $$\Rightarrow \quad [(i + j)(i - j)] \bmod size = 0$$

BUT size does not divide $(i - j)$ or $(i + j)$

(Optional for this class, but just know the guarantee for < half full)

How can $i + j = 0$ or $i + j = size$ when:

$i \neq j$ and $0 \le i,j \le size/2$?

Similarly how can $i - j = 0$ or $i - j = size$?

Clustering reconsidered

- Quadratic probing does not suffer from primary clustering:
  As we resolve collisions we are not merely growing "big blobs" by adding one more item to the end of a cluster, we are looking i^2 locations away, for the next possible spot.

- But quadratic probing does not help resolve collisions between keys that initially hash *to the same* ***index***
  - Any 2 keys that initially hash to the same index **will have the same series of moves after that** looking for any empty spot
  - Called secondary clustering

- Can avoid secondary clustering with *a probe function that depends on the key*: double hashing...

Open Addressing: Double hashing

Idea: Given two good hash functions *h* and *g*, and two different keys *k1* and *k2*, it is very unlikely that: **`h(k1)==h(k2)`** and **`g(k1)==g(k2)`**

`(h(key) + f(i)) % TableSize`

- For double hashing:

  `f(i) = i*g(key)`

- So probe sequence is:
  - 0th probe: **`h(key) % TableSize`**
  - 1st probe: **`(h(key) + g(key)) % TableSize`**
  - 2nd probe: **`(h(key) + 2*g(key)) % TableSize`**
  - 3rd probe: **`(h(key) + 3*g(key)) % TableSize`**
  - …
  - ith probe: **`(h(key) + i*g(key)) % TableSize`**

- Detail: Make sure **`g(key)`** can't be **0**

ith probe: `(h(key) + i*g(key)) % TableSize`

Open Addressing: Double Hashing

0	
1	
2	
3	13
4	
5	
6	
7	
8	
9	

T = 10 (TableSize)

<u>Hash Functions</u>:

h(key) = key mod T

g(key) = 1 + ((key/T) mod (T-1))

Insert these values into the hash table in this order. Resolve any collisions with double hashing:

13
28
33
147
43

ith probe: `(h(key) + i*g(key)) % TableSize`

Double Hashing

0	
1	
2	
3	13
4	
5	
6	
7	
8	28
9	

T = 10 (TableSize)

Hash Functions:

h(key) = key mod T

g(key) = 1 + ((key/T) mod (T-1))

Insert these values into the hash table in this order. Resolve any collisions with double hashing:

13
28
33
147
43

ith probe: `(h(key) + i*g(key)) % TableSize`

Double Hashing

0	
1	
2	
33 3	13
4	
5	
6	
7	33
8	28
9	

T = 10 (TableSize)

<u>Hash Functions</u>:

h(key) = key mod T

g(key) = 1 + ((key/T) mod (T-1))

Insert these values into the hash table in this order. Resolve any collisions with double hashing:

13

28

33

147

43

$g(33) = 1 + 3 \bmod 9 = 4$

ith probe: `(h(key) + i*g(key)) % TableSize`

Double Hashing

0	
1	
2	
3	13
4	
5	
6	
7	33
8	28
9	147

147

147

T = 10 (TableSize)

<u>Hash Functions</u>:

h(key) = key mod T

g(key) = 1 + ((key/T) mod (T-1))

Insert these values into the hash table in this order. Resolve any collisions with double hashing:

13

28

33 → g(33) = 1 + 3 mod 9 = ~~6~~ 4

147

43

g(147) = 1 + 14 mod 9 = 6

ith probe: `(h(key) + i*g(key)) % TableSize`

Double Hashing

0	
1	
2	
3	13
4	
5	
6	
7	33
8	28
9	147

43_2 43_0 43_1

T = 10 (TableSize)

<u>Hash Functions</u>:

h(key) = key mod T

g(key) = 1 + ((key/T) mod (T-1))

Insert these values into the hash table in this order. Resolve any collisions with double hashing:

13

28

33 → g(33) = 1 + 3 mod 9 = 4

147 → g(147) = 1 + 14 mod 9 = 6

43

1 + 4 mod 9 = 5

ith probe: `(h(key) + i*g(key)) % TableSize`

Double Hashing

0	
1	
2	
3	13
4	
5	
6	
7	33
8	28
9	147

T = 10 (TableSize)

<u>Hash Functions</u>:

h(key) = key mod T

g(key) = 1 + ((key/T) mod (T-1))

Insert these values into the hash table in this order. Resolve any collisions with double hashing:

13

28

33 → g(33) = 1 + 3 mod 9 = 6

147 → g(147) = 1 + 14 mod 9 = 6

43 → g(43) = 1 + 4 mod 9 = 5

We have a problem:

3 + 0 = 3 3 + 5 = 8 3 + 10 = 13

3 + 15 = 18 3 + 20 = 23

Double-hashing analysis

Intuition: Since each probe is "jumping" by `g(key)` each time, we "leave the neighborhood" *and* "go different places from other initial collisions"

But, as in quadratic probing, we could still have a problem where we are not "safe" due to an infinite loop despite room in table:

- No guarantee that i*g(key) will let us try all/most indices
- It is known that this cannot happen in at least one case:

For primes p and q such that $2 < q < p$

h(key) = key % p

g(key) = q – (key % q)

More double-hashing facts (Just cool facts)

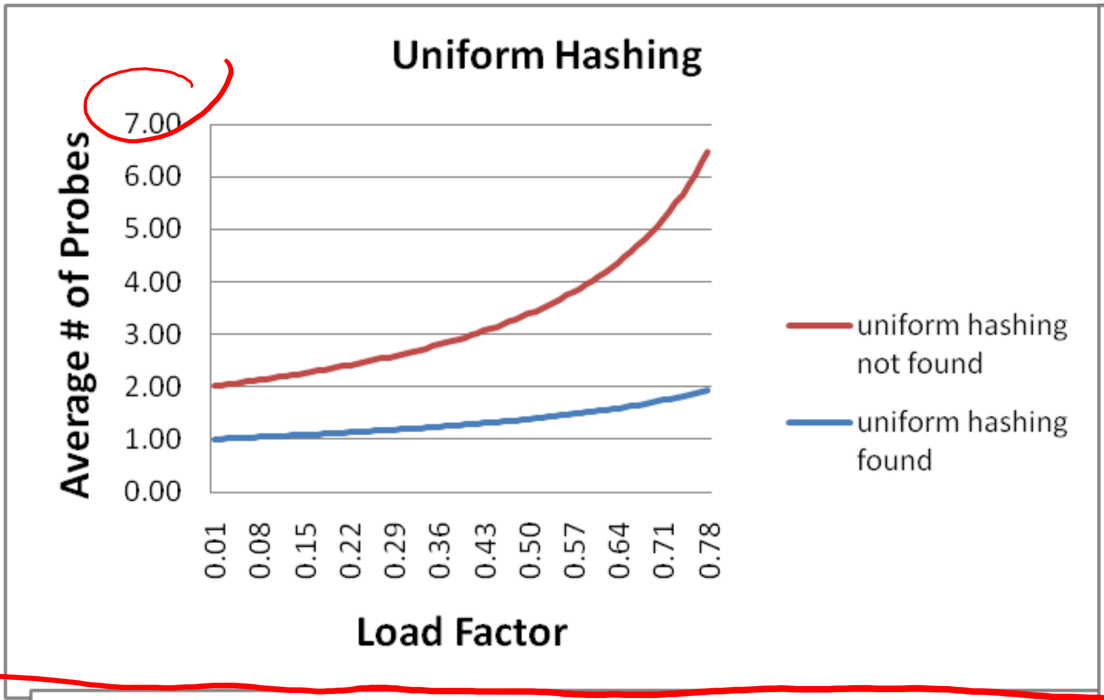
- Assume "uniform hashing"
  - Means probability of **`g(key1) % p == g(key2) % p`** is **`1/p`**

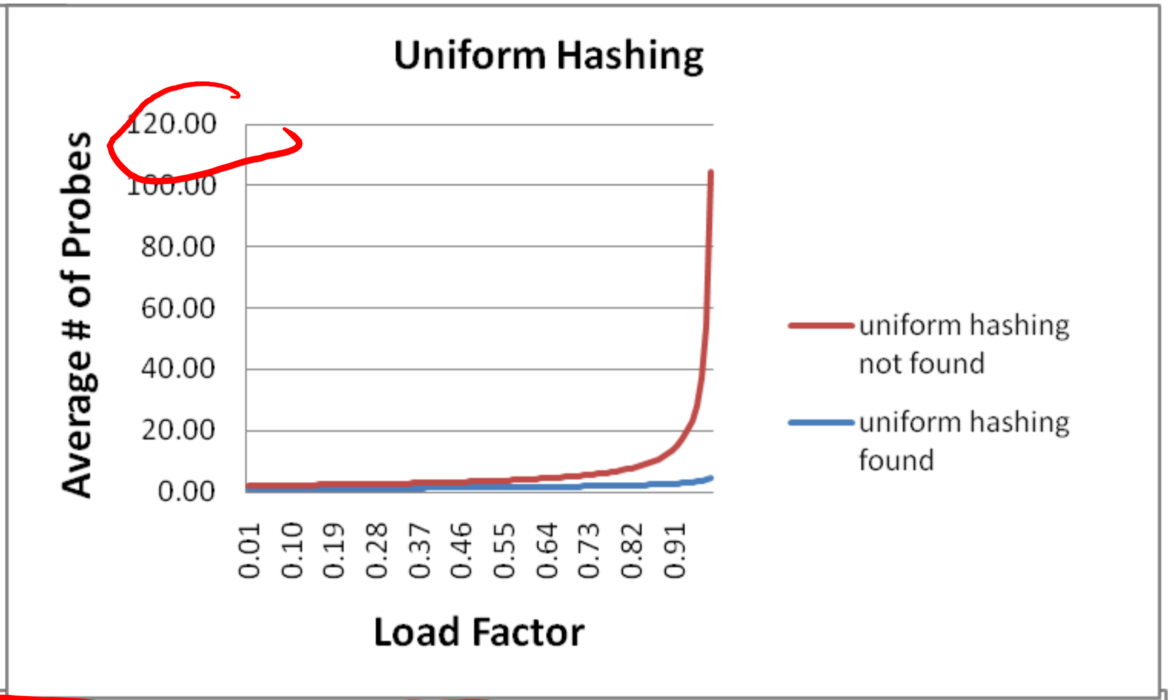
- Non-trivial facts we won't prove:

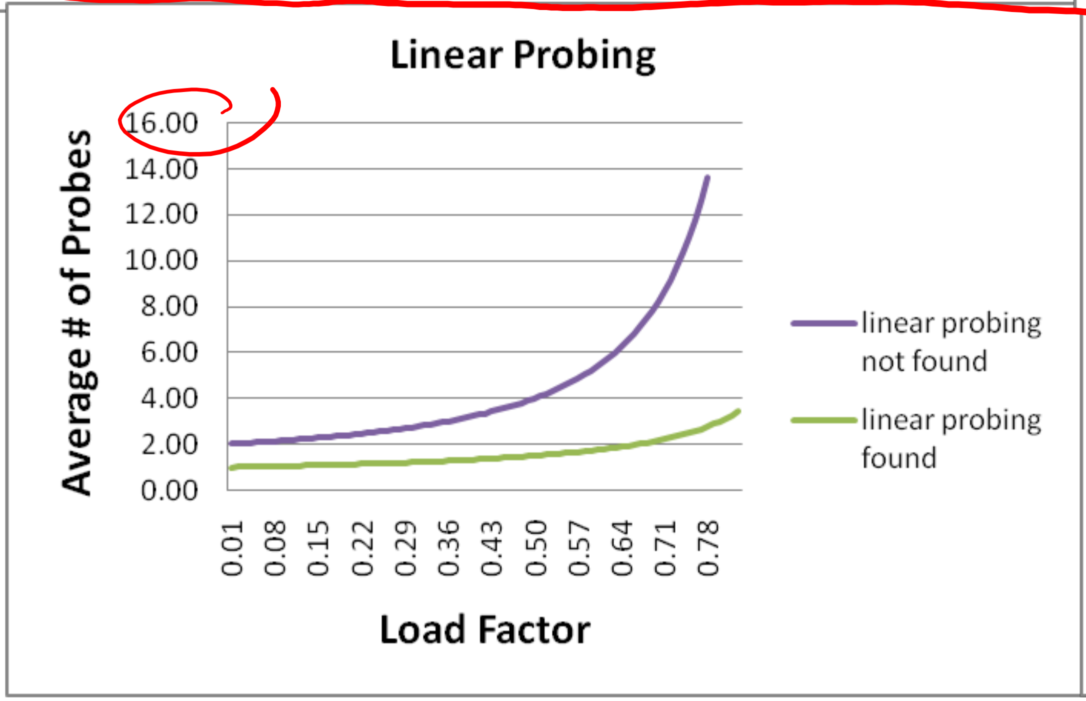
  Average # of probes given λ (in the limit as **`TableSize`** $\rightarrow \infty$)
  - Unsuccessful search (intuitive): $\dfrac{1}{1-\lambda}$
  - Successful search (less intuitive): $\dfrac{1}{\lambda}\log_e\left(\dfrac{1}{1-\lambda}\right)$

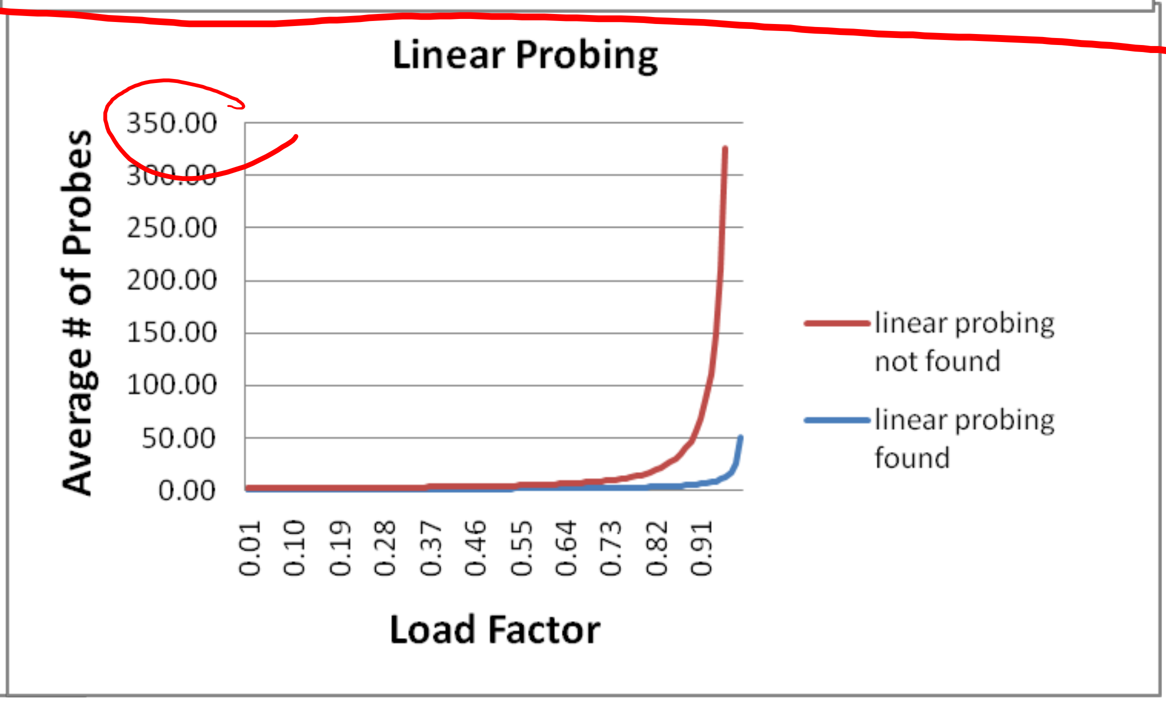
- Bottom line: unsuccessful bad (but not as bad as linear probing), but successful is not nearly as bad

Charts


Uniform Hashing
Average # of Probes
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
0.01 0.08 0.15 0.22 0.29 0.36 0.43 0.50 0.57 0.64 0.71 0.78
Load Factor
uniform hashing not found
uniform hashing found


Uniform Hashing
Average # of Probes
120.00
100.00
80.00
60.00
40.00
20.00
0.00
0.01 0.10 0.19 0.28 0.37 0.46 0.55 0.64 0.73 0.82 0.91
Load Factor
uniform hashing not found
uniform hashing found


Linear Probing
Average # of Probes
16.00
14.00
12.00
10.00
8.00
6.00
4.00
2.00
0.00
0.01 0.08 0.15 0.22 0.29 0.36 0.43 0.50 0.57 0.64 0.71 0.78
Load Factor
linear probing not found
linear probing found


Linear Probing
Average # of Probes
350.00
300.00
250.00
200.00
150.00
100.00
50.00
0.00
0.01 0.10 0.19 0.28 0.37 0.46 0.55 0.64 0.73 0.82 0.91
Load Factor
linear probing not found
linear probing found

Outline - Where are we?

- Separate Chaining is easy
  - **`find, insert, delete`** proportional to load factor on average if using unsorted linked list nodes
  - If using another data structure for buckets (e.g. AVL tree) , runtime is proportional to runtime for that structure.
- Open addressing uses probing, has clustering issues as table fills Why use it:
  - Less memory allocation?
    - Some run-time overhead for allocating linked list (or whatever) nodes; open addressing could be faster
  - Easier data representation?
- Now:
  - Growing the table when it gets too full (aka “rehashing”)
  - Relation between hashing/comparing and connection to Java

Rehashing

- As with array-based stacks/queues/lists, if table gets too full, create a bigger table and copy everything over

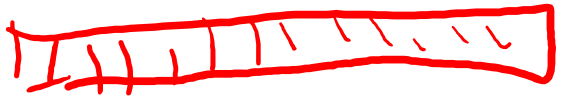
- With **separate chaining,** we get to decide what "too full" means
  - Keep load factor reasonable (e.g., < 1)?
  - Consider average or max size of non-empty chains?
- For **open addressing**, half-full is a good rule of thumb

- New table size
  - Twice-as-big is a good idea, except, uhm, that won't be prime!
  - So go *about* twice-as-big
  - Can have a list of prime numbers in your code since you probably won't grow more than 20-30 times, and then calculate after that

2 · tablesize + 1

More on rehashing

- Can we just copy all data to the same indices in the new table?
  - Will not work; we calculated the index based on **`TableSize`**



- Go through table, do standard insert for each into new table
  - Iterate over old table: O(n)
  - n inserts / calls to the hash function: n · O(1) = O(n)

- Is there some way to avoid all those hash function calls?
  - Space/time tradeoff: Could store **`h(key)`** with each data item
  - Growing the table is still *O*(*n*); saving **`h(key)`** only helps by a constant factor

A Generally Good hashCode()


```
int result = 17; // start at a prime

foreach field f
   int fieldHashcode =
     boolean: (f ? 1: 0)
     byte, char, short, int: (int) f
     long: (int) (f ^ (f >>> 32))
     float: Float.floatToIntBits(f)
     double: Double.doubleToLongBits(f), then above
     Object: object.hashCode( )

     result = 31 * result + fieldHashcode;
return result;
```

Check it out!

https://github.com/openjdk/jdk/blob/master/src/java.base/share/classes/java/lang/Double.java

```java
     */
    @Override
    public int hashCode() {
        return Double.hashCode(value);
    }

    /**
     * Returns a hash code for a {@code double} value; compatible with
     * {@code Double.hashCode()}.
     *
     * @param value the value to hash
     * @return a hash code value for a {@code double} value.
     * @since 1.8
     */
    public static int hashCode(double value) {
        return Long.hashCode(doubleToLongBits(value));
    }

```

Final word on hashing

- The hash table is one of the most important data structures
  - Efficient find, insert, and delete
  - Operations based on sorted order are not so efficient!
  - Useful in many, many real-world applications
  - Popular topic for job interview questions
- Important to use a good hash function
  - Good distribution, Uses enough of key's components
  - Not overly expensive to calculate (bit shifts good!)
- Important to keep hash table at a good size
  - Prime #
  - Preferable λ depends on type of table
- Side-comment: hash functions have uses beyond hash tables
  - Examples: Cryptography, check-sums

Outline

- Hashing
  - Open Addressing
  - Rehashing
  - Hashing in Practice

- Comparison Sorting

Sorting

Great general pre-processing step

- Binary Search
- Let's us find the k^{th} element in $O(1)$ time for any k.

Also, a convenient way to discuss algorithm design principles.

Three goals

Three things you might want in a sorting algorithm:

- In-Place
  - Only use $O(1)$ extra memory.
  - Sorted array given back in the input array.
- Stable
  - If a appears before b in the initial array and a.compareTo(b) == 0
  - Then a appears before b in the final array.
  - Example: sort by first name, then by last name.
- Fast

$[0\ 3\ 7_0\ 11\ 7_1]$

$[0\ 3\ 7_0\ 7_1\ 11]$

Insertion Sort

How you sort a hand of cards.

Maintain a sorted subarray at the front.

Start with one element.

While(your subarray is not the full array)

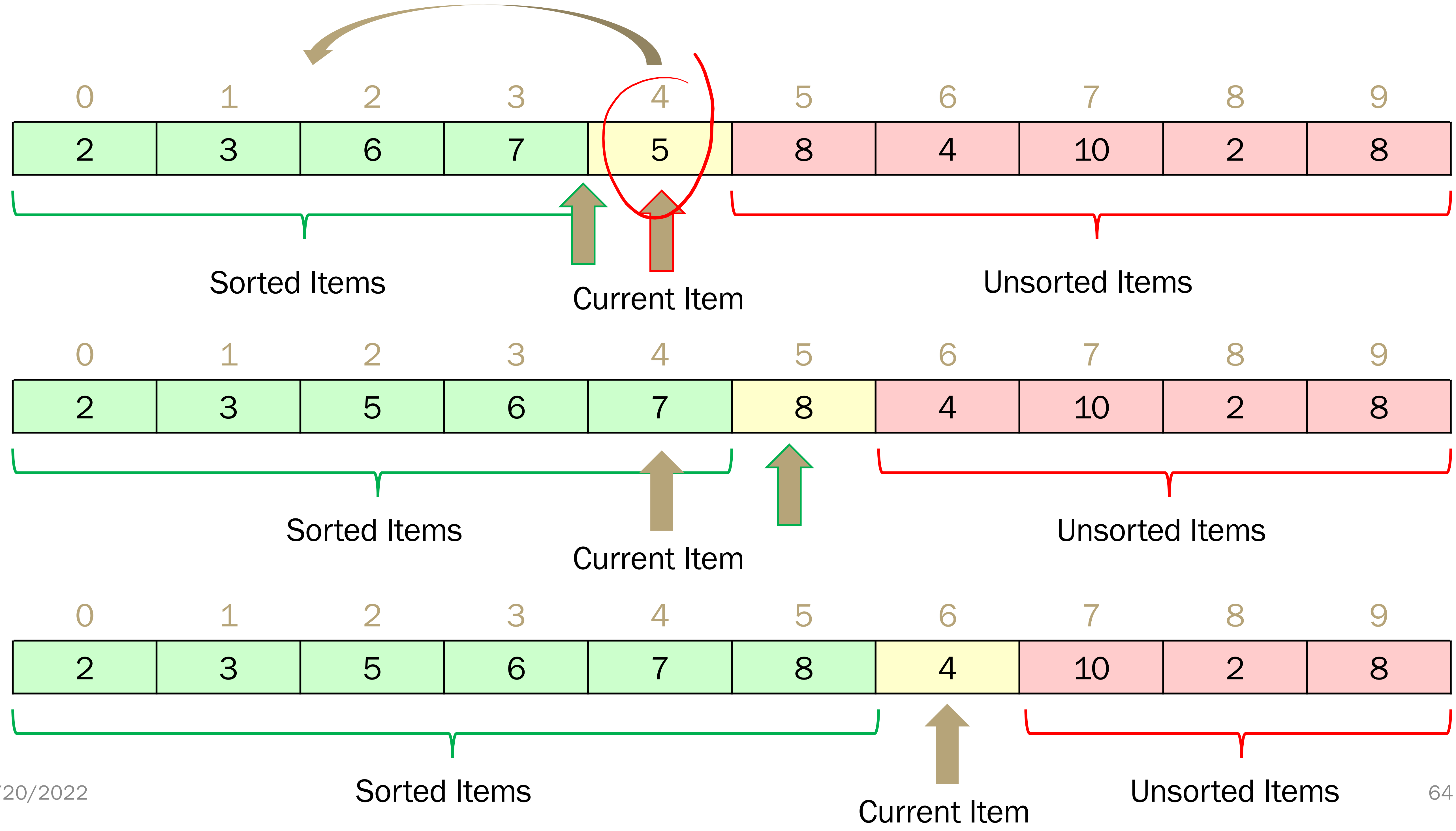
  Take the next element not in your subarray

  Insert it into the sorted subarray

Insertion Sort

```
for(i from 1 to n-1){
    int index = i
    while(a[index-1] > a[index]){
        swap(a[index-1], a[index])
        index = index-1
    }
}
```

Insertion Sort



Insertion Sort Analysis

Stable? Yes! (If you're careful)

In Place? Yes!

Running time:

- Best Case: $O(n)$
- Worst Case: $O(n^2)$
- Average Case: $O(n^2)$

$$\sum_{i=0}^{n} i = O(n^2)$$

_________ Sort

Here's another idea for a sorting algorithm:

- Maintain a sorted subarray
- While(subarray is not full array)
  - Find the smallest element remaining in the unsorted part.
  - Insert it at the end of the sorted part.

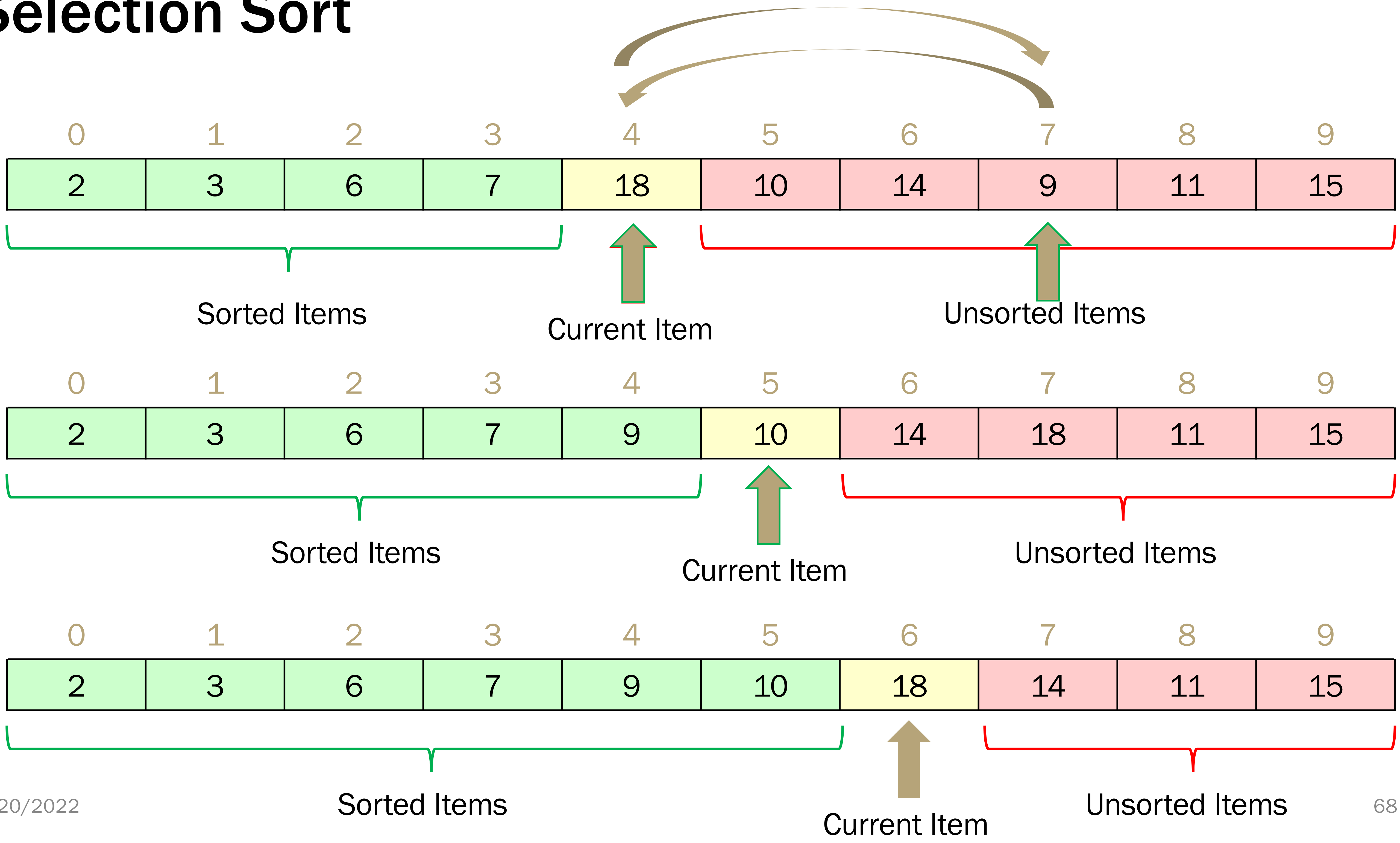
Selection Sort

Here's another idea for a sorting algorithm:

- Maintain a sorted subarray
- While(subarray is not full array)
  - Find the smallest element remaining in the unsorted part.
    - *By scanning the remainder of the array*
  - Insert it at the end of the sorted part.

Running time $O(n^2)$

Selection Sort



Selection Sort

Here's another idea for a sorting algorithm:

- Maintain a sorted subarray
- While(subarray is not full array)
  - Find the smallest element remaining in the unsorted part.
    - *By scanning the remainder of the array*
  - Insert it at the end of the sorted part.

Running time $O(n^2)$

Can we do better? With a data structure?

Heap Sort

Here's another idea for a sorting algorithm:

- Maintain a sorted subarray; Make the unsorted part a min-heap
- While(subarray is not full array)
  - Find the smallest element remaining in the unsorted part.
    - *By calling `removeMin` on the heap*
  - Insert it at the end of the sorted part.

Running time $O(n \log n)$

Heap Sort

0	1	2	3	4	5	6	7	8	9
1	4	2	14	15	18	16	17	20	22

Current Item

Heap

Sorted Items

0	1	2	3	4	5	6	7	8	9
22	4	2	14	15	18	16	17	20	1

percolateDown(22)

Current Item

Heap

Sorted Items

0	1	2	3	4	5	6	7	8	9
2	4	16	14	15	18	22	17	20	1

Current Item

Heap

Sorted Items

Heap Sort (Better)

- We're sorting in the wrong order!
  - Could reverse at the end.
- Our heap implementation will implicitly assume that the heap is on the left of the array.
- Switch to a max-heap and keep the sorted stuff on the right.

- What's our running time? $O(n \log n)$

Heap Sort

- Our first step is to make a heap. Does using `buildHeap` instead of inserts improve the running time?
- Not in a big-O sense (though we did by a constant factor).

- In place: Yes
- Stable: No

Next time

- MergeSort, QuickSort
- Beyond Comparison Sorting