CSE 332: Data Structures & Parallelism
Lecture 6: Dictionaries; Binary Search Trees

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Announcements

- Fill out partner form by 6pm tonight!
  - EVERYONE must fill it out, even if same partner or working by yourself
  - You will not receive a repo if you do not fill it out in time
- We will leave feedback on P1 writeup
  - Not graded as harshly, but make sure you look at your feedback to use for P2 writeup!
- Exercise 3 and Exercise 4 due Friday

CS (can be) frustrating

But that is part of being an expert programmer! (And the 3 seconds of feeling awesome after you figure it out)

Expert Programmer duties include:

- Writing expert level bugs
- Removing expert level bugs

Strategies Include:

- Stare and Hope
- ”Shotgun” debugging
- **Other (SEE COURSE WEBSITE HANDOUT!)**

Today – Dictionaries and BST

- Finish Big-Oh
  - Recursion Analysis
  - Amortization
  - Proofs
- Dictionary & BST

Where we are

Studying the absolutely essential ADTs of computer science and classic data structures for implementing them

ADTs so far:

1. Stack: **`push`**, **`pop`**, **`isEmpty`**, …
2. Queue: **`enqueue`**, **`dequeue`**, **`isEmpty`**, …
3. Priority queue: **`insert`**, **`deleteMin`**, …

Next:

4. Dictionary (a.k.a. Map): associate keys with values
   - probably the most common, way more than priority queue

The Dictionary (a.k.a. Map) ADT

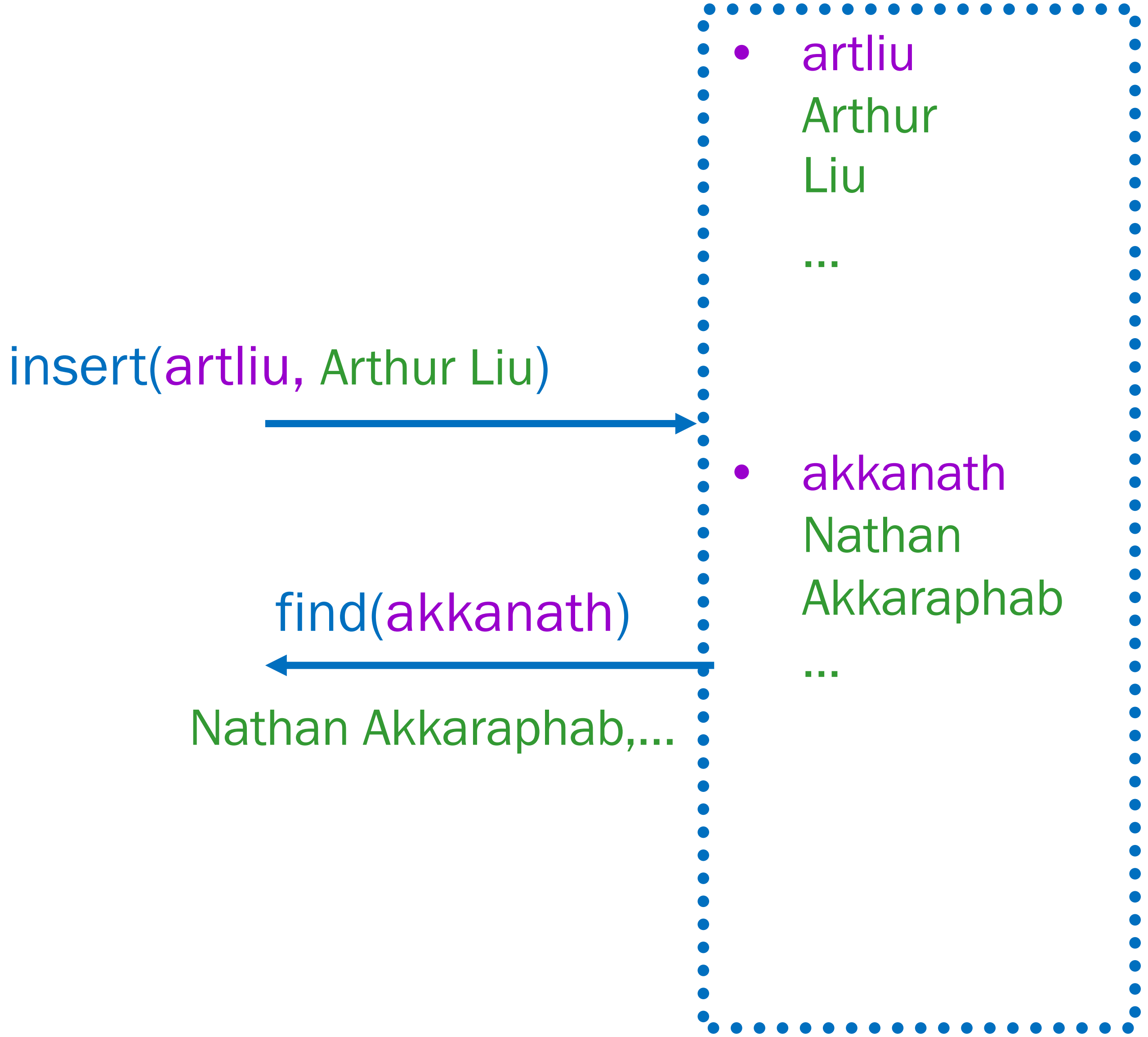
Dictionary ADT

State:

- Set of ***unique*** (key, value) pairs
- Keys can be compared for equality

Operations:

- **insert(key val)** – places (key,val) in map.
  (If key already used, overwrites existing entry)
- **find(key)** – returns val associated with key
- **delete(key)**



We will tend to emphasize the keys, but don't forget about the stored values!

Comparison: Set ADT vs. Dictionary ADT

The *Set* ADT is like a Dictionary without any values

- A key is *present* or not (no repeats)

For **`find`**, **`insert`**, **`delete`**, there is little difference

- In dictionary, values are "just along for the ride"
- So *same data-structure ideas* work for dictionaries and sets
    - Java HashSet implemented using a HashMap, for instance

Set ADT may have other important operations

- **`union`**, **`intersection`**, **`is_subset`**, etc.
- Notice these are binary operators on sets
- We will want different data structures to implement these operators

A Modest Few Uses for Dictionaries

Any time you want to store information according to some key and be able to retrieve it efficiently – a dictionary is the ADT to use!

- Lots of programs do that!

• Networks:	router tables
• Operating systems:	page tables
• Compilers:	symbol tables
• Databases:	dictionaries with other nice properties
• Search:	inverted indexes, phone directories, …
• Biology:	genome maps
• …	

Simple implementations

For dictionary with *n* key/value pairs. (No duplicates allowed!)

	insert	find	delete
Unsorted Linked-List			
Unsorted Array			
Sorted Linked-List			
Sorted Array			

Simple implementations

For dictionary with *n* key/value pairs. (No duplicates allowed!)

	insert	find	delete
Unsorted Linked-List	O(N)	O(N)	O(N)
Unsorted Array	O(N)	O(N)	O(N)
Sorted Linked-List	O(N)	O(N)	O(N)
Sorted Array	O(N)	O(log N)	O(N)

We'll see a Binary Search Tree (BST) probably does better, but not in the worst case unless we keep it balanced

Lazy Deletion (e.g. in a sorted array)

10	12	24	30	41	42	44	45	50
✓	✗	✓	✓	✓	✓	✗	✓	✓

A *general technique* for making **`delete`** as fast as **`find`**:

- Instead of actually removing the item just mark it deleted
- No need to shift values, etc.

Plusses:

- Simpler
- Can do removals later in batches
- If re-added soon thereafter, just unmark the deletion

Minuses:

- Extra *space* for the "is-it-deleted" flag
- Data structure full of deleted nodes wastes *space*
- **`find`** $O($**`log`** $m)$ *time* where *m* is data-structure size (m >= n)
- May complicate other operations

Better Dictionary data structures

Will spend the next several lectures looking at dictionaries with three different data structures

1. AVL trees
   - A special binary search trees with *guaranteed balancing*
2. B-Trees
   - Also always balanced, but different and shallower
   - B!=Binary; B-Trees generally have large branching factor
3. Hashtables
   - Not tree-like at all

Skipping: Other balanced trees (red-black, splay)

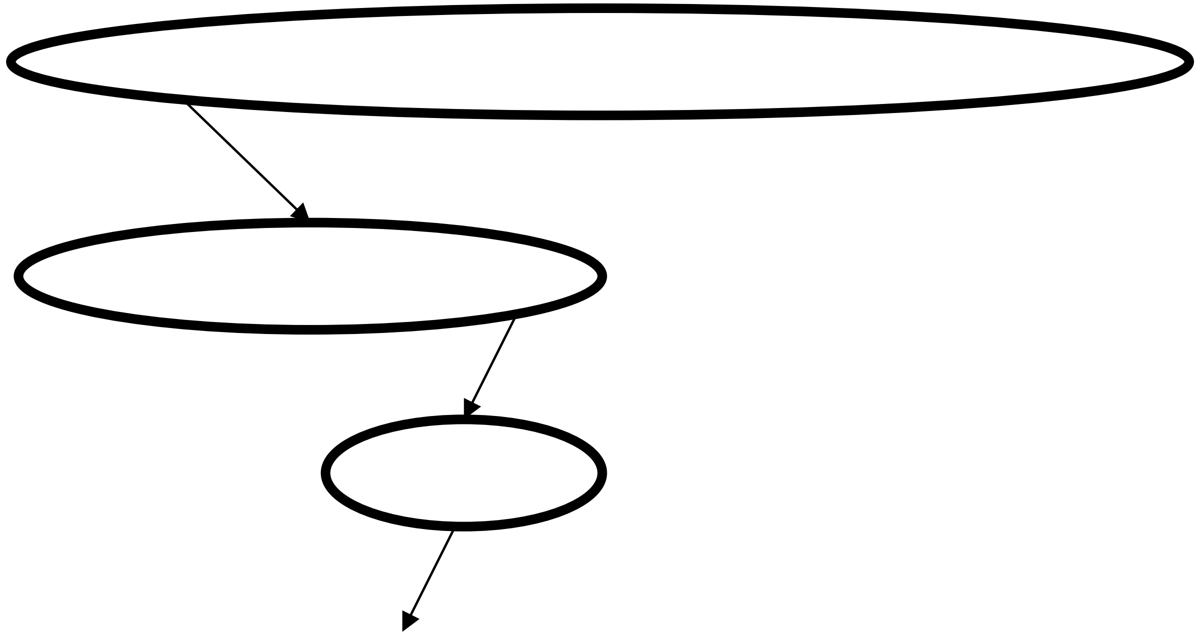
Why Trees?

Trees offer speed ups because of their branching factors

- Binary Search Trees are structured forms of *binary search*

Binary Search

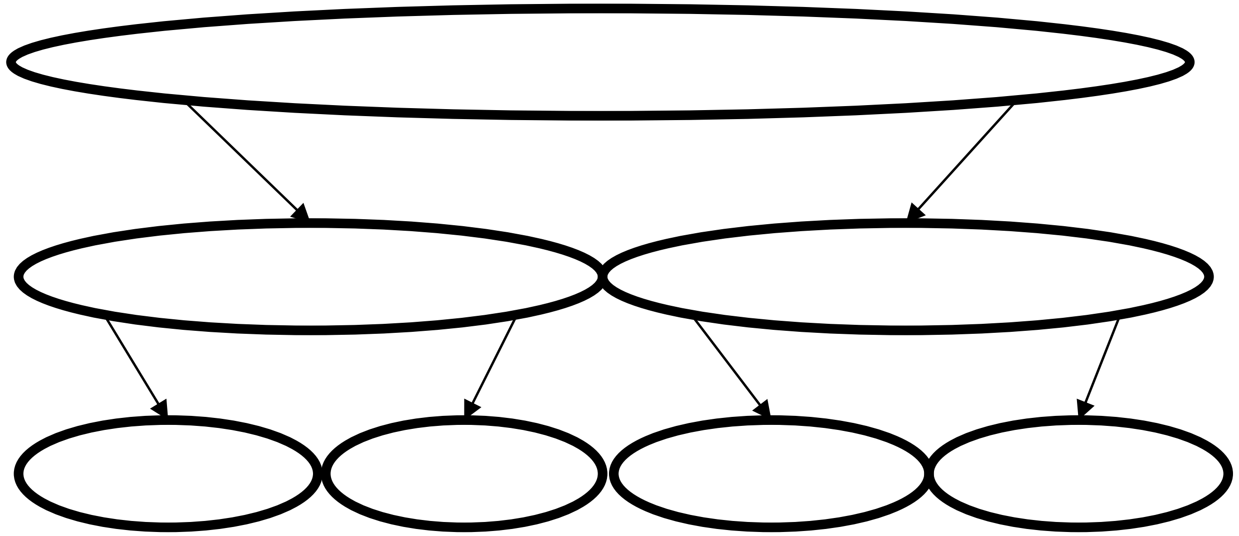
find(4)



1	3	4	5	7	8	9	10

Binary Search Tree

Our goal is the performance of binary search in a tree representation



1	3	4	5	7	8	9	10

Why Trees?

Trees offer speed ups because of their branching factors

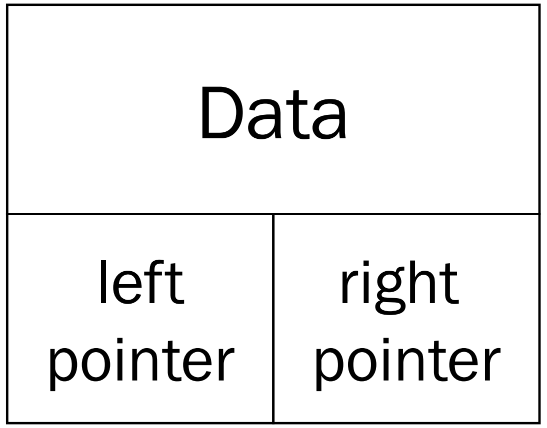
- Binary Search Trees are structured forms of *binary search*

Even a basic BST is fairly good

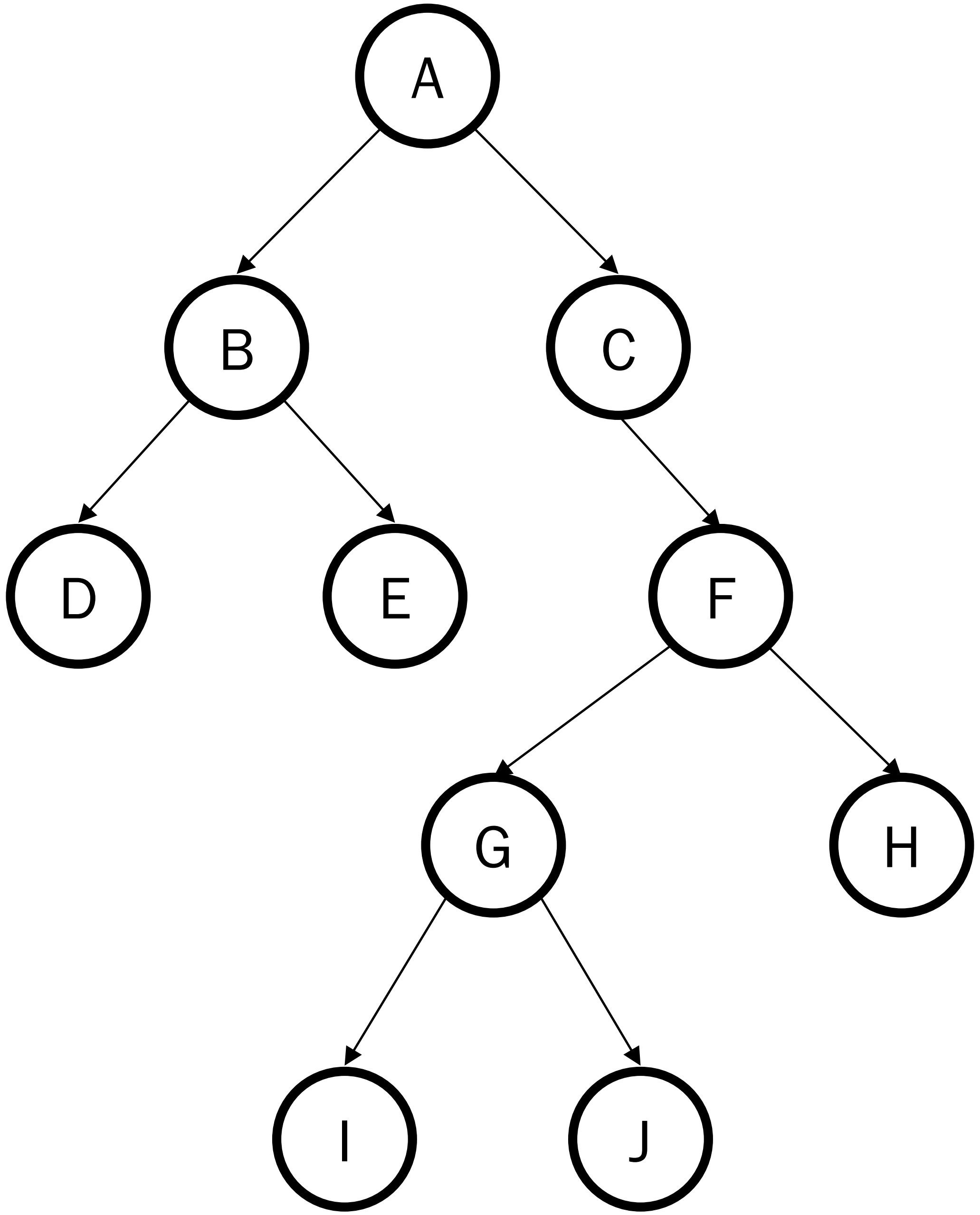
	Insert	Find	Delete
Worse-Case	O(n)	O(n)	O(n)
Average-Case	O(log n)	O(log n)	O(log n)

Binary Trees

- Binary tree is either
  - Empty (ie: null)
  - a root (with left and right subtrees)
- Representation:



For a dictionary, data will include a key and a value



Binary Trees: Some Numbers

Recall: height of a tree = longest path from root to leaf (count edges)

For binary tree of height *h*:

- max # of leaves:
- max # of nodes:
- min # of leaves:
- min # of nodes:

Binary Trees: Some Numbers

Recall: height of a tree = longest path from root to leaf (count edges)

For binary tree of height *h*:

- max # of leaves: 2^h
- max # of nodes: $2^{(h+1)} - 1$
- min # of leaves: 1
- min # of nodes: $h + 1$

For n nodes, we cannot do better than O(`log` n) height,
and we want to avoid O(n) height

Calculating height

What is the height of a tree with root **`r`**?

```
int treeHeight(Node root) {

  ???


}
```

Running time for tree with *n* nodes: *O*(*n*) – single pass over tree

Note: non-recursive is painful – need your own stack of pending nodes;
much easier to use recursion's call stack

Calculating height

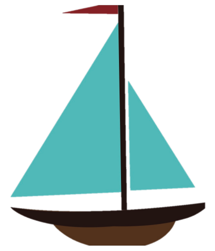
What is the height of a tree with root **`r`**?

```
int treeHeight(Node root) {
 if(root == null)
    return -1;
 return 1 + max(treeHeight(root.left),
                treeHeight(root.right));
}
```

Running time for tree with *n* nodes: *O*(*n*) – single pass over tree

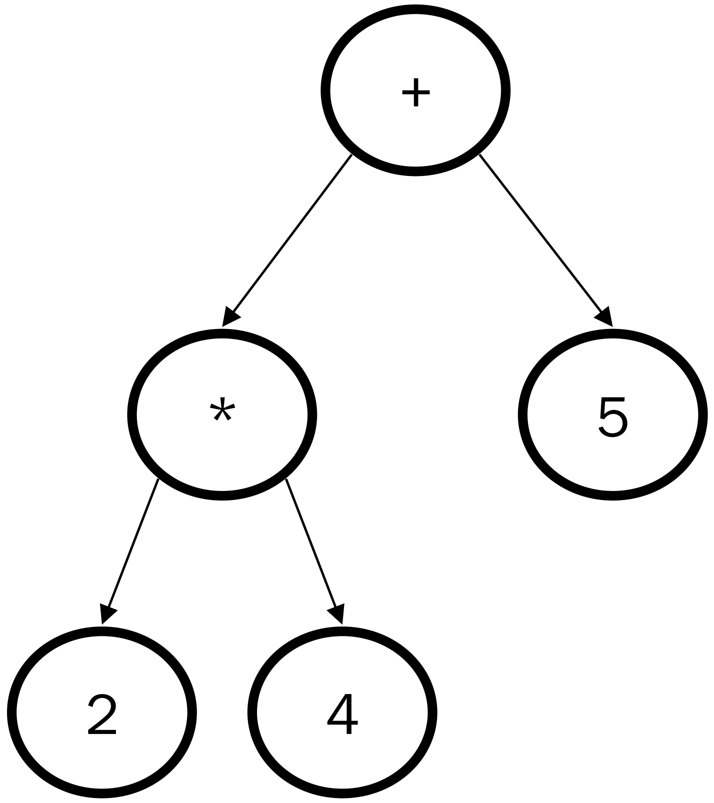
Note: non-recursive is painful – need your own stack of pending nodes; much easier to use recursion's call stack

Tree Traversals



A *traversal* is an order for visiting all the nodes of a tree

- *Pre-order*: root, left subtree, right subtree
- *In-order*: left subtree, root, right subtree
- *Post-order*: left subtree, right subtree, root



(an expression tree)

Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree

- *Pre-order*: root, left subtree, right subtree

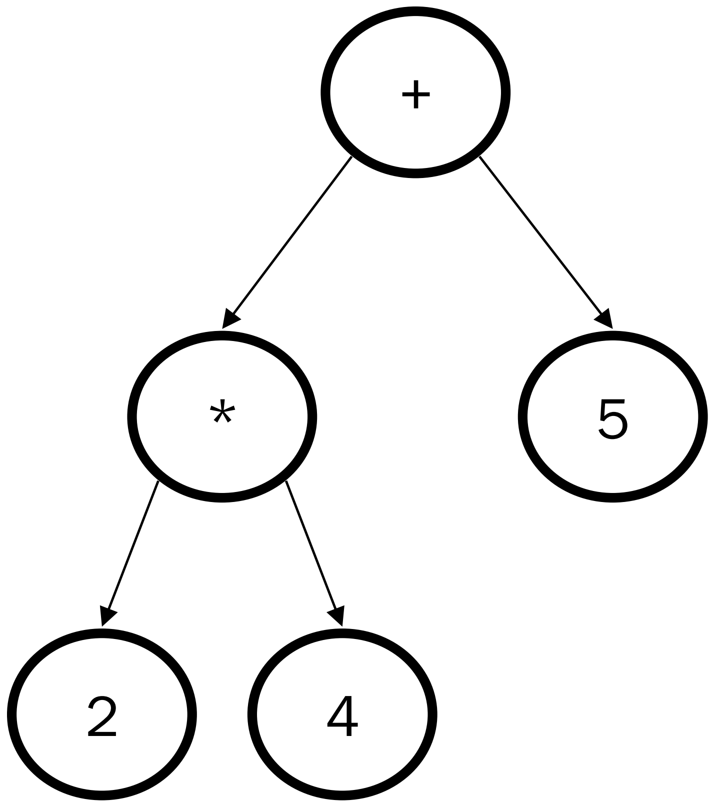
  + * 2 4 5

- *In-order*: left subtree, root, right subtree

  2 * 4 + 5

- *Post-order*: left subtree, right subtree, root

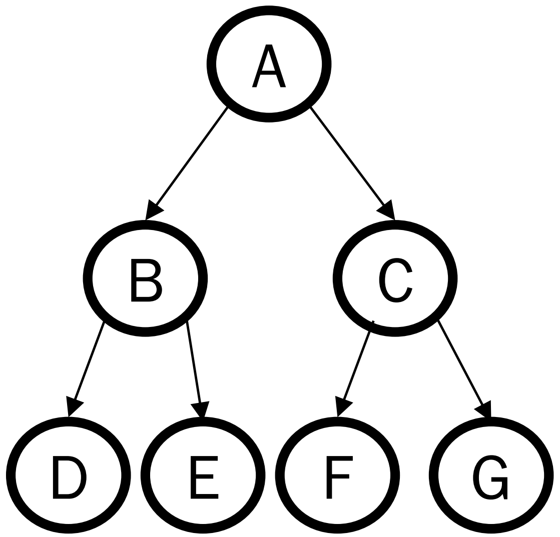
  2 4 * 5 +



(an expression tree)

More on traversals

```
void inOrdertraversal(Node t){
  if(t != null) {
    traverse(t.left);
    process(t.element);
    traverse(t.right);
  }
}
```



The difference between the 3 traversals is when process() gets called

Sometimes order doesn’t matter

- Example: sum all elements

Sometimes order matters

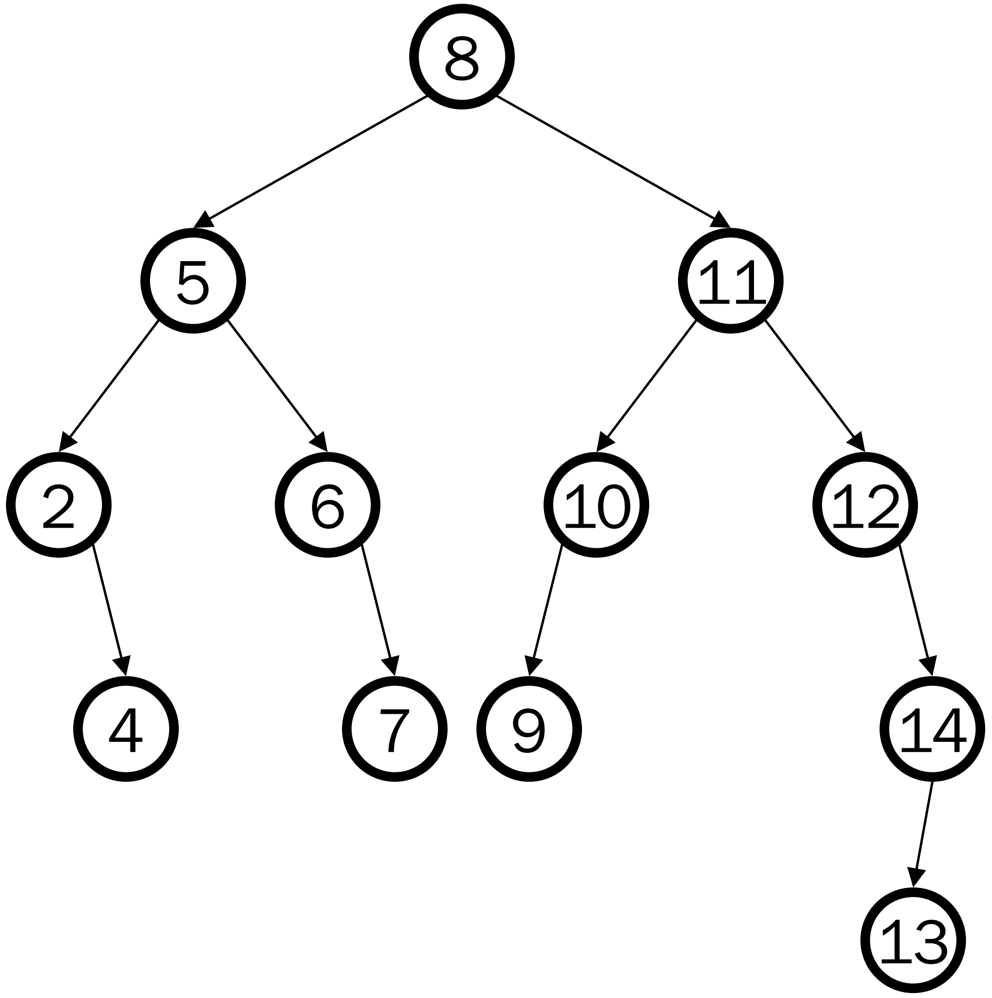
- Example: print tree with parent above indented children (pre-order)
- Example: evaluate an expression tree (post-order)

A
  B
    D
    E
  C
    F
    G

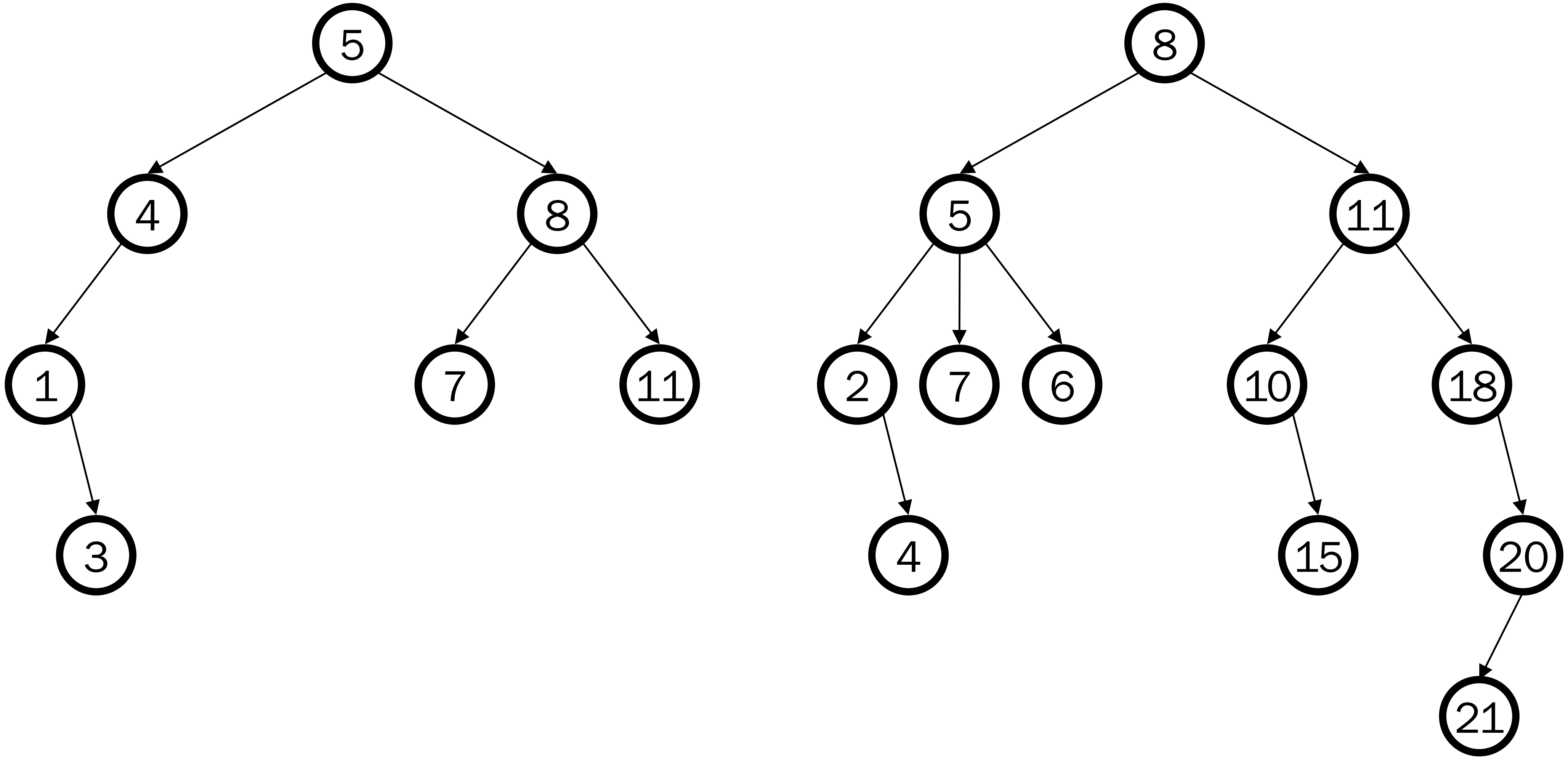
Dictionary data structure: Binary ***<u>Search</u>*** Tree

Defined by these properties:

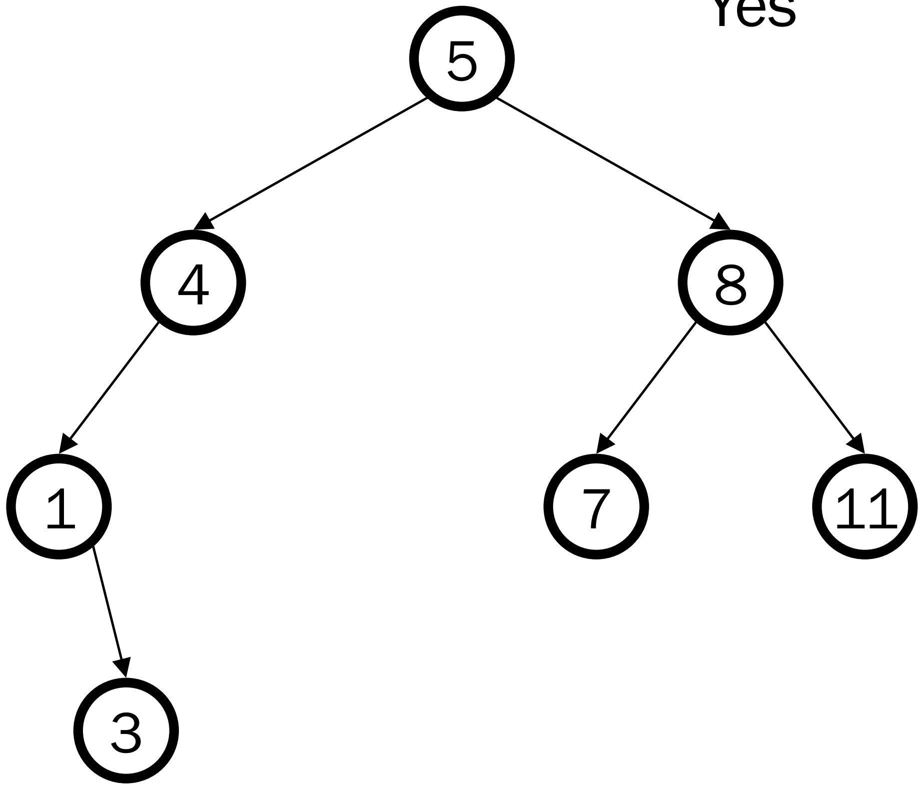
- Structural property ("binary")
  - each node has ≤ 2 children
  - result: keeps operations simple
- Order property
  - all keys in left subtree smaller than node's key
  - all keys in right subtree larger than node's key
  - result: easy to find any given key

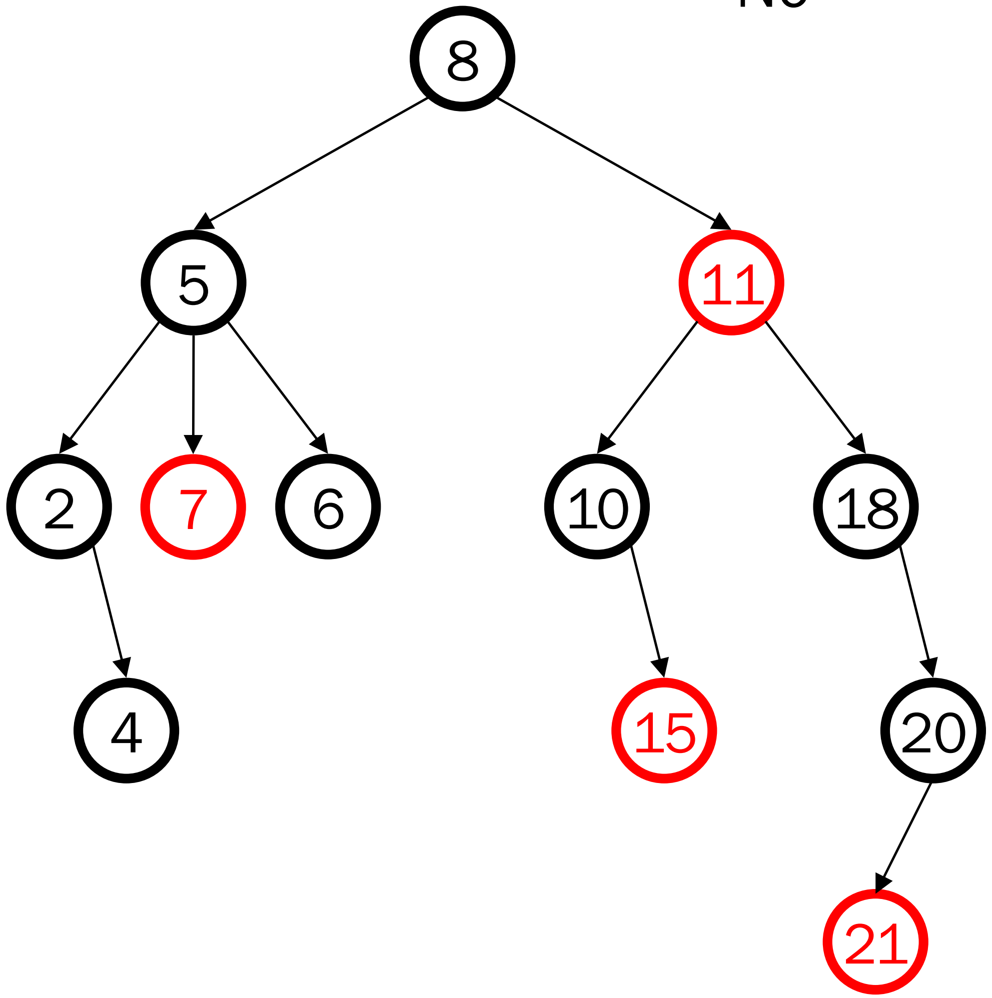


Are these BSTs?

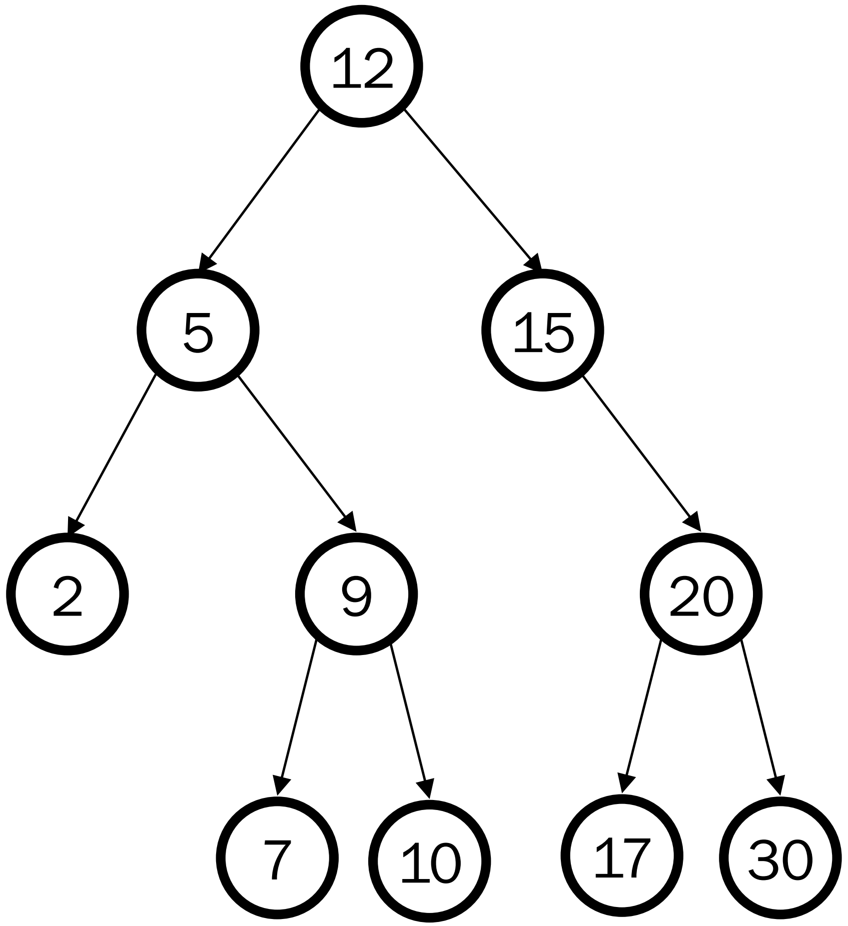


Are these BSTs?



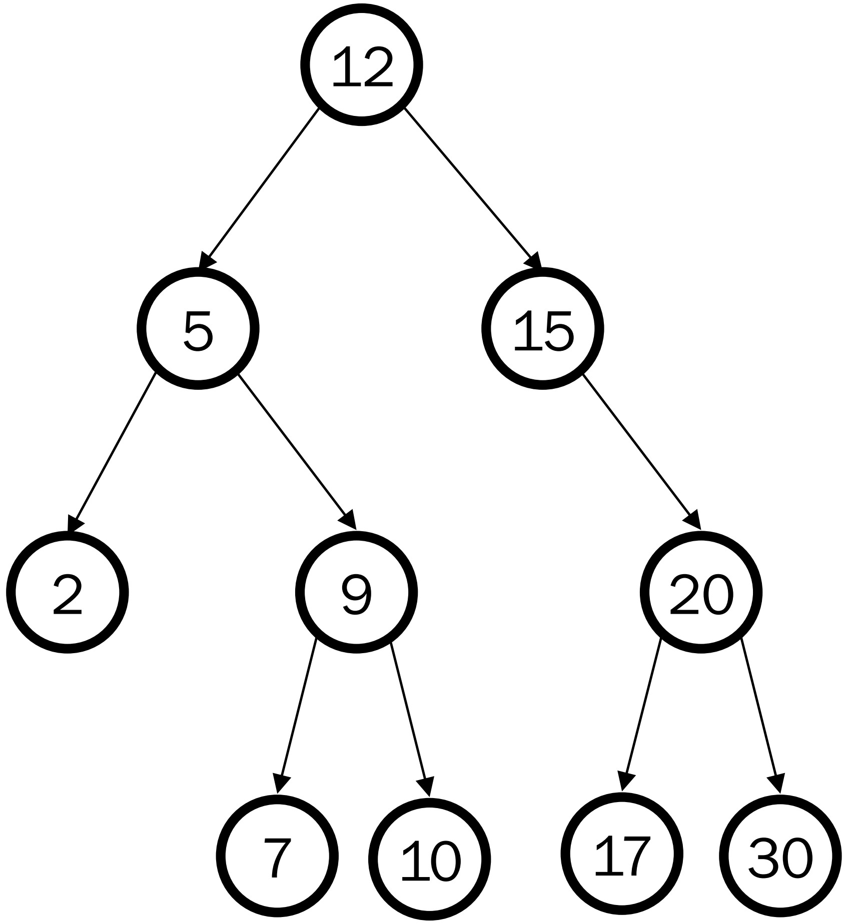


Find in BST, Recursive




```
Data find(Key key, Node root){
 if(root == null)
   return null;
 if(key < root.key)
   return find(key,root.left);
 if(key > root.key)
   return find(key,root.right);
 return root.data;
}
```

Find in BST, Iterative

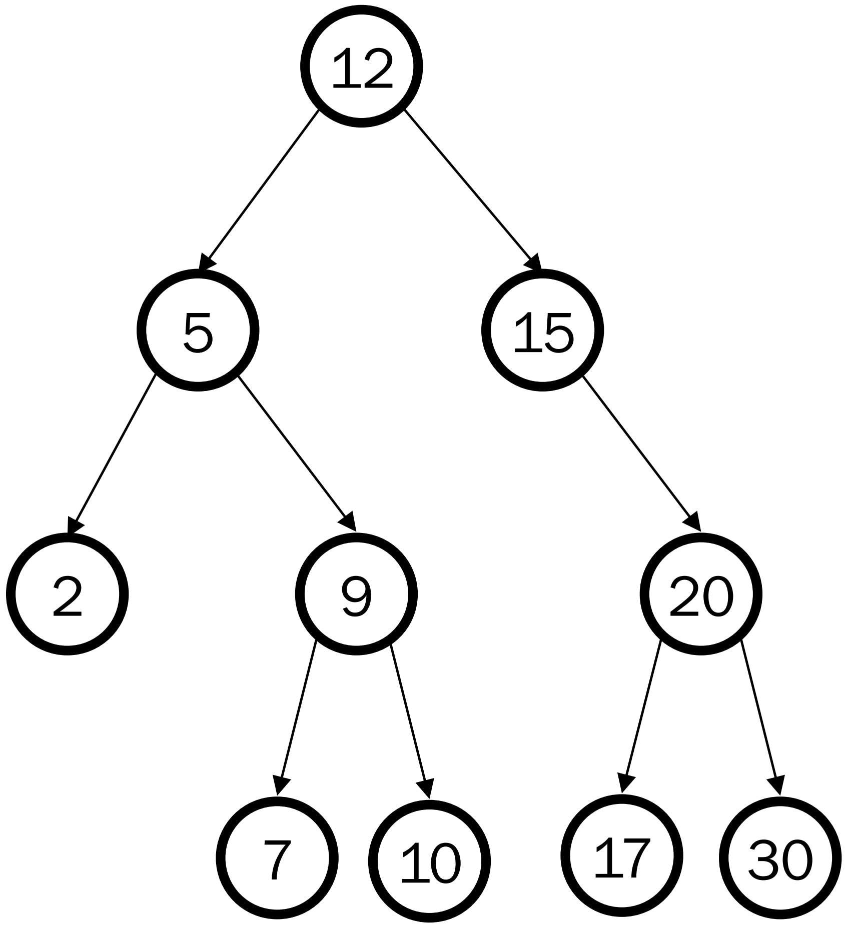



```
Data find(Key key, Node root){
 while(root != null
        && root.key != key) {
  if(key < root.key)
    root = root.left;
  else(key > root.key)
    root = root.right;
 }
 if(root == null)
    return null;
 return root.data;
}
```

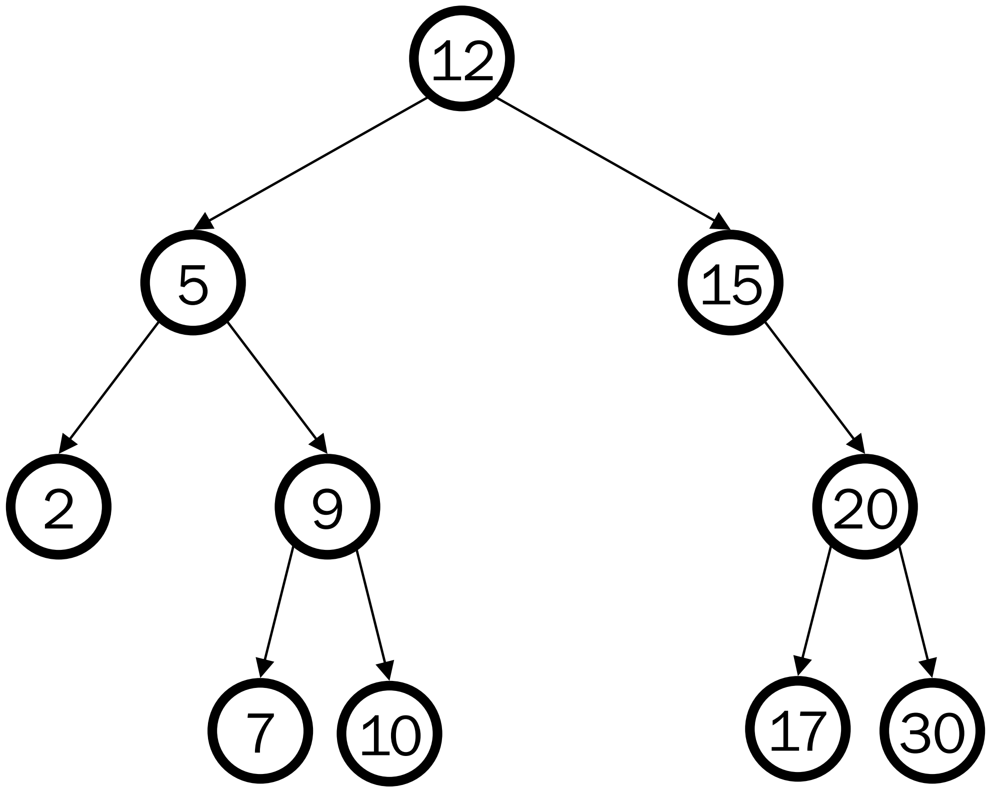
Other “finding operations”

- Find *minimum* node

- Find *maximum* node



Insert in BST

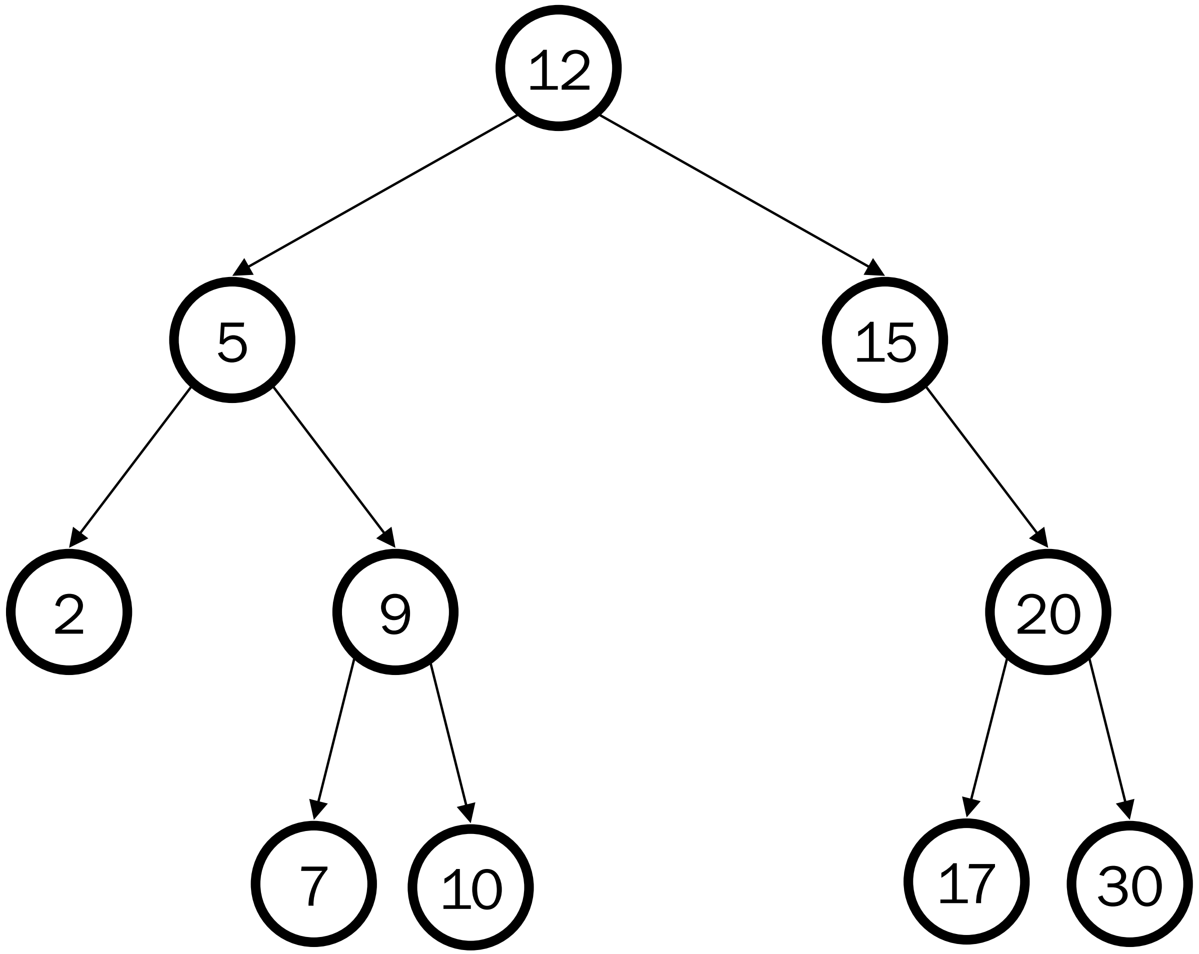



```
insert(13)
insert(8)
insert(31)
```

(New) insertions happen only at leaves – easy!

1. Find
2. Create a new node

Deletion in BST



Why might deletion be harder than insertion?

Deletion

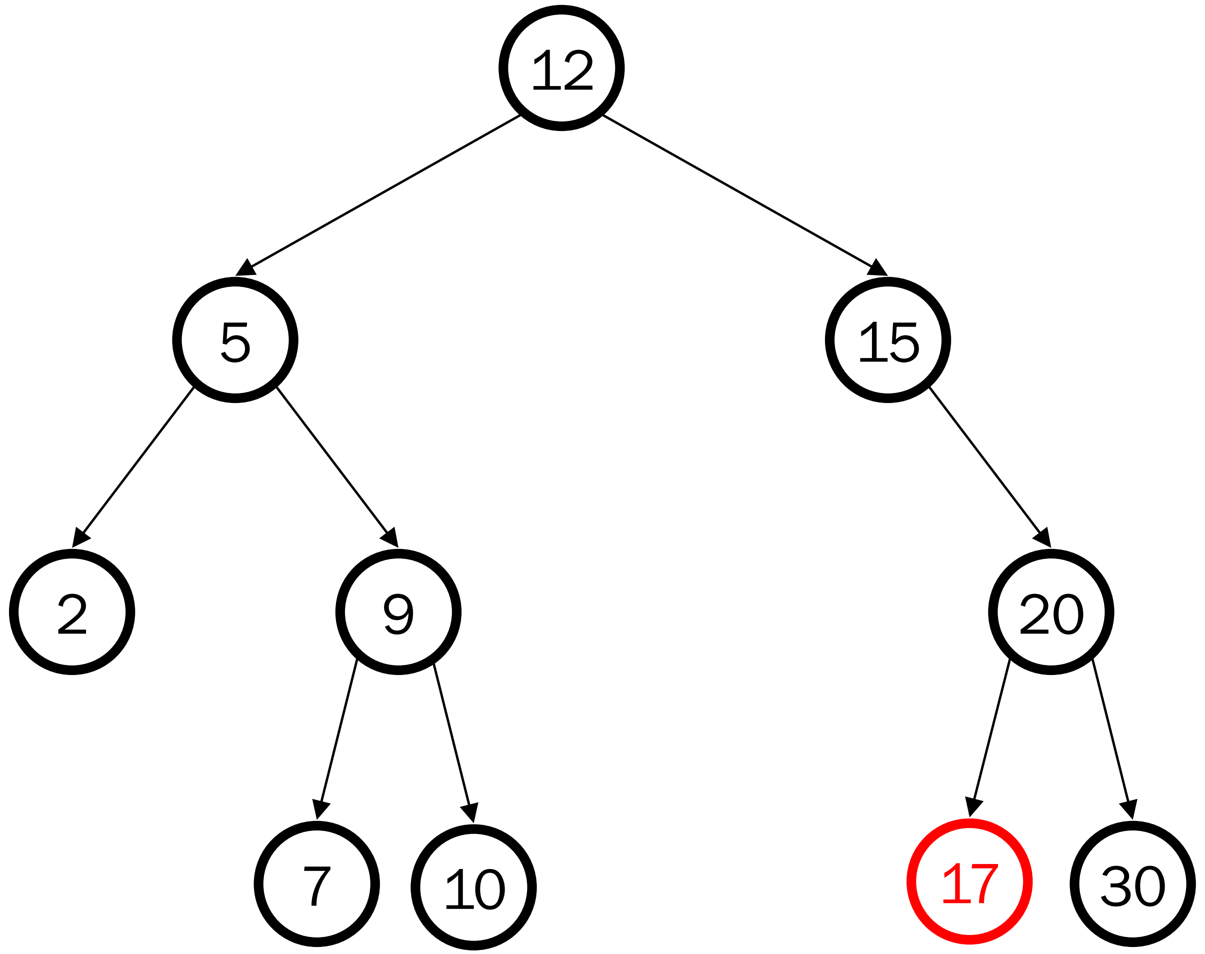
- Removing an item disrupts the tree structure

- Basic idea:
  - `find` the node to be removed,
  - Remove it
  - “fix” the tree so that it is still a binary search tree

- Three cases:
  - node has no children (leaf)
  - node has one child
  - node has two children

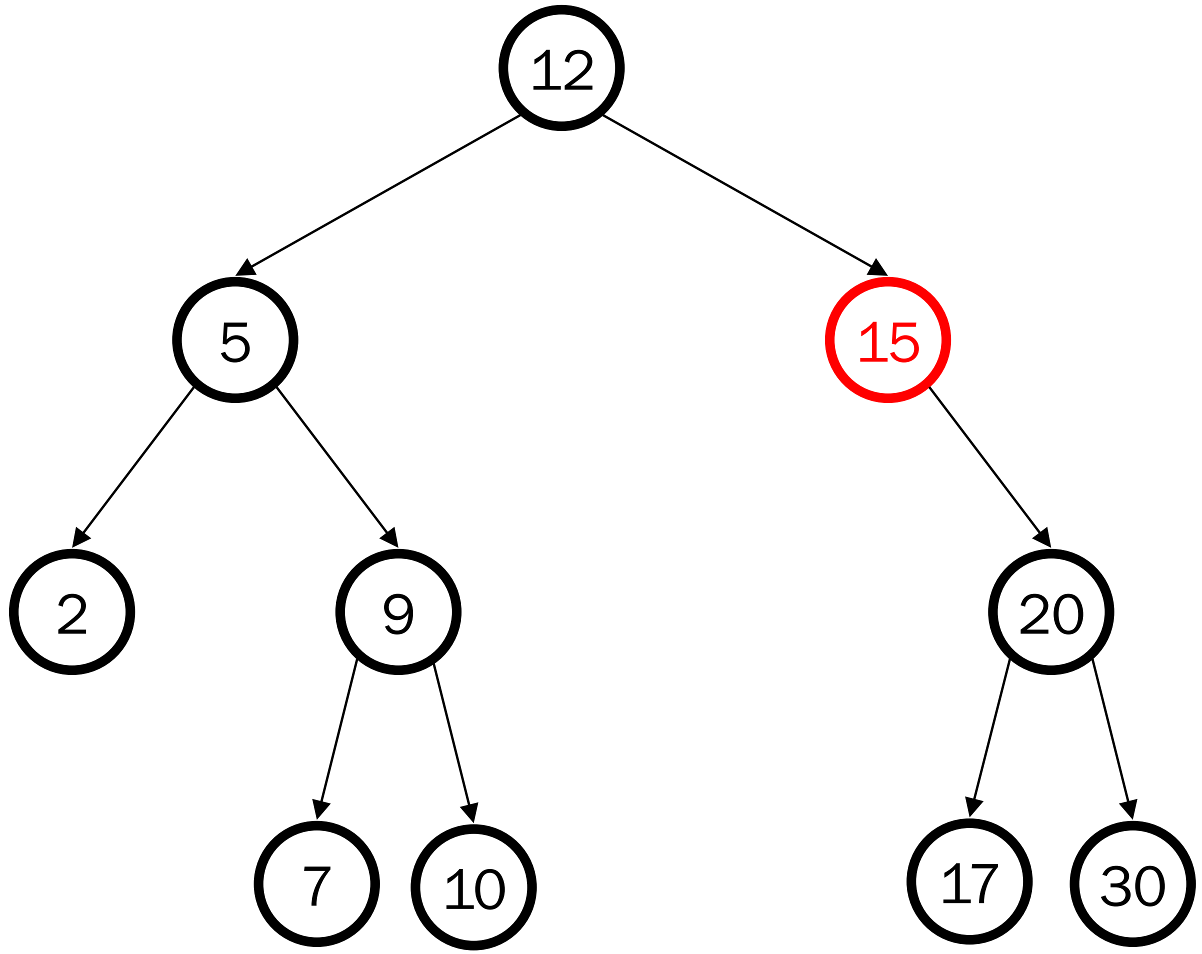
Deletion – The Leaf Case

`delete(17)`



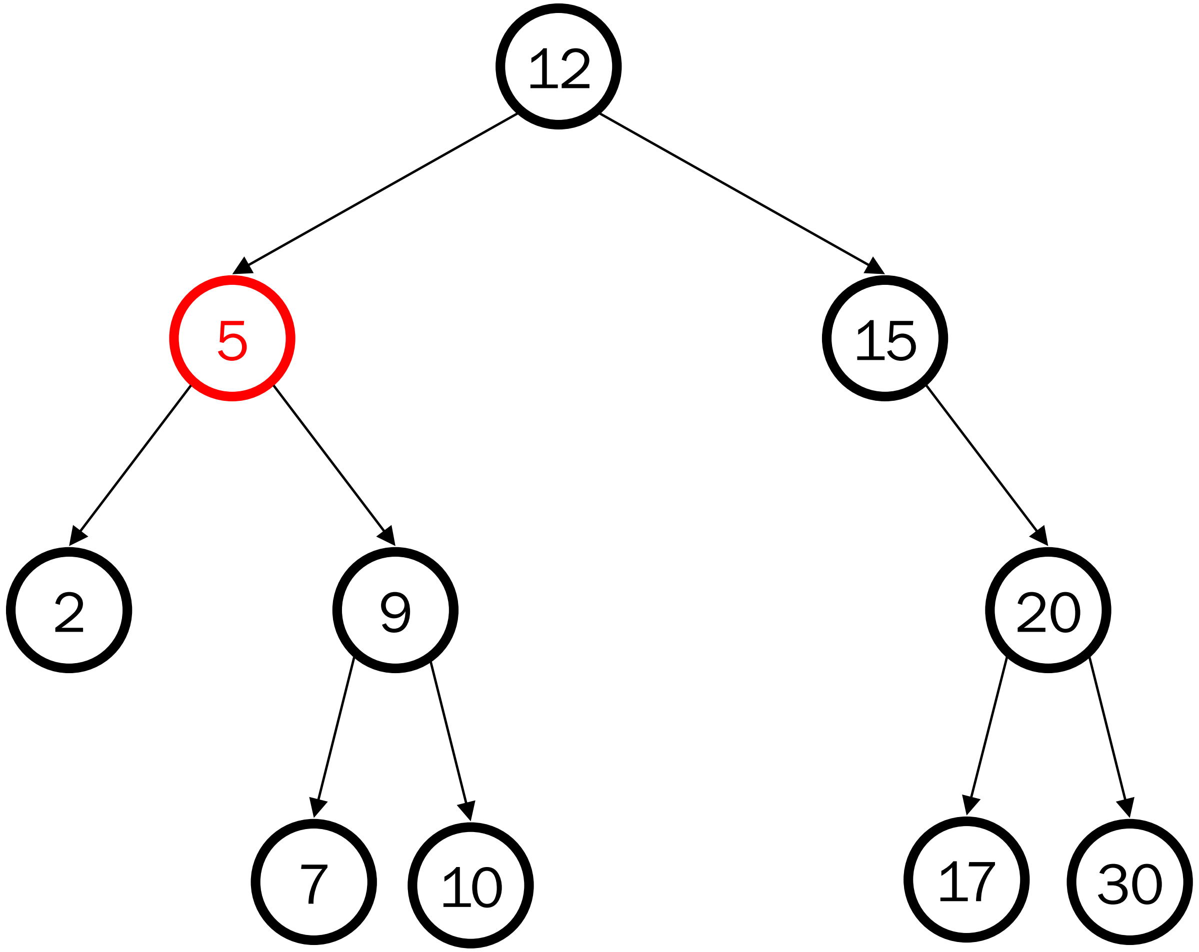
Deletion – The One Child Case

`delete(15)`



Deletion – The Two Child Case

`delete(5)`



What can we replace 5 with?

Deletion – The Two Child Case

Idea: Replace the deleted node with a value guaranteed to be between the two child subtrees

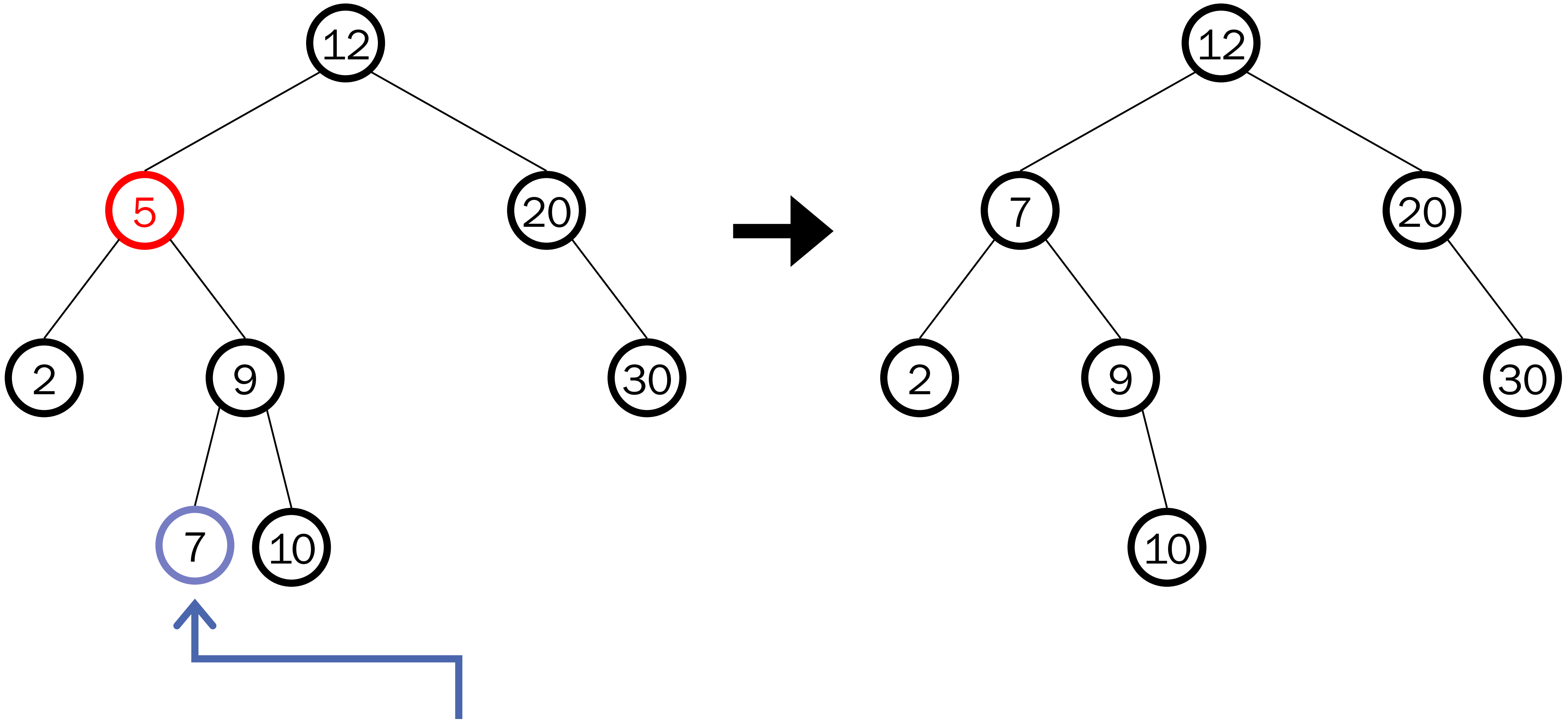
Two Options:

- *successor* from right subtree: **`findMin(node.right)`**
- *predecessor* from left subtree: **`findMax(node.left)`**
  - These are the easy cases of predecessor/successor

Now delete the original node containing *successor* or *predecessor*

- Leaf or one child case – easy cases of delete!

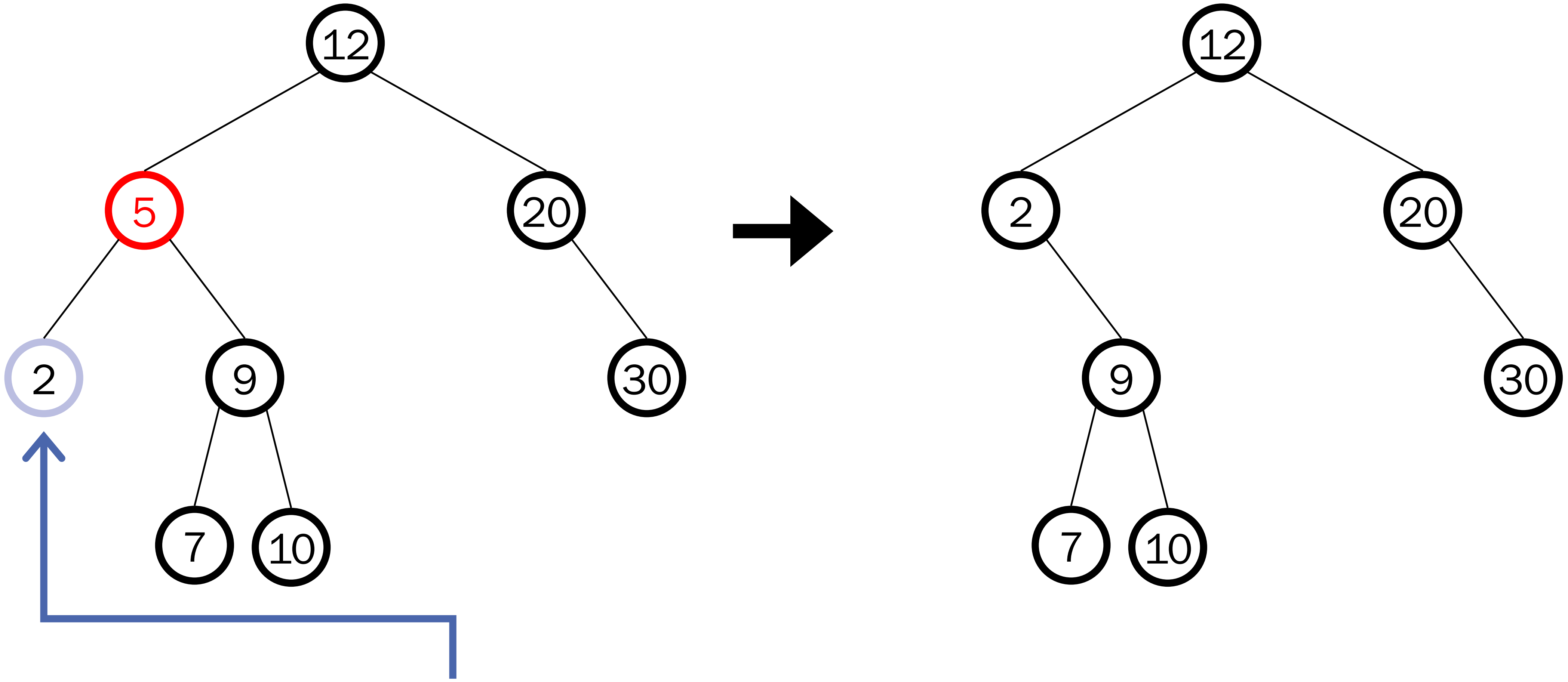
Delete Using Successor



findMin(right sub tree) → 7

`delete(5)`

Delete Using Predecessor



findMax(left sub tree) → 2

`delete(5)`

BuildTree for BST

We had **`buildHeap`**, so let's consider **`buildTree`**

Insert keys 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST

- If inserted in given order, what is the tree?

- What big-O runtime for this kind of sorted input?

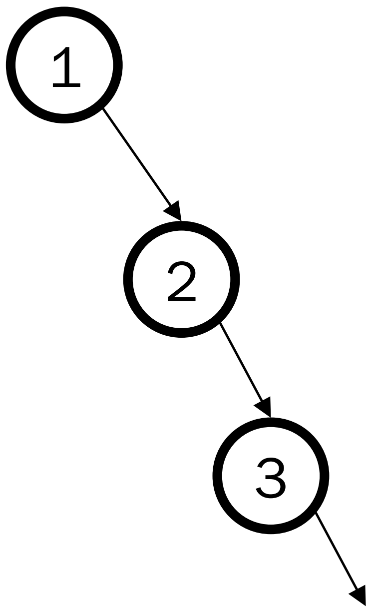
- Is inserting in the reverse order any better?

BuildTree for BST

We had **`buildHeap`**, so let's consider **`buildTree`**

Insert keys 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST

If inserted in given order,
what is the tree?



What big-O runtime for
this kind of sorted input?

$O(n^2)$
Not a happy place

Is inserting in the reverse order
any better?

Balanced BST

Observation

BST: the shallower the better!

For a BST with n nodes inserted in arbitrary order

- Average height is $O(\mathtt{log}\ n)$ – see text for proof
- Worst case height is $O(n)$

Simple cases such as inserting in key order lead to the worst-case scenario

Solution: Require a Balance Condition that

1. ensures depth is always $O(\mathtt{log}\ n)$ – strong enough!
2. is easy to maintain – not too strong!

Potential Balance Conditions

pollev.com/artliu

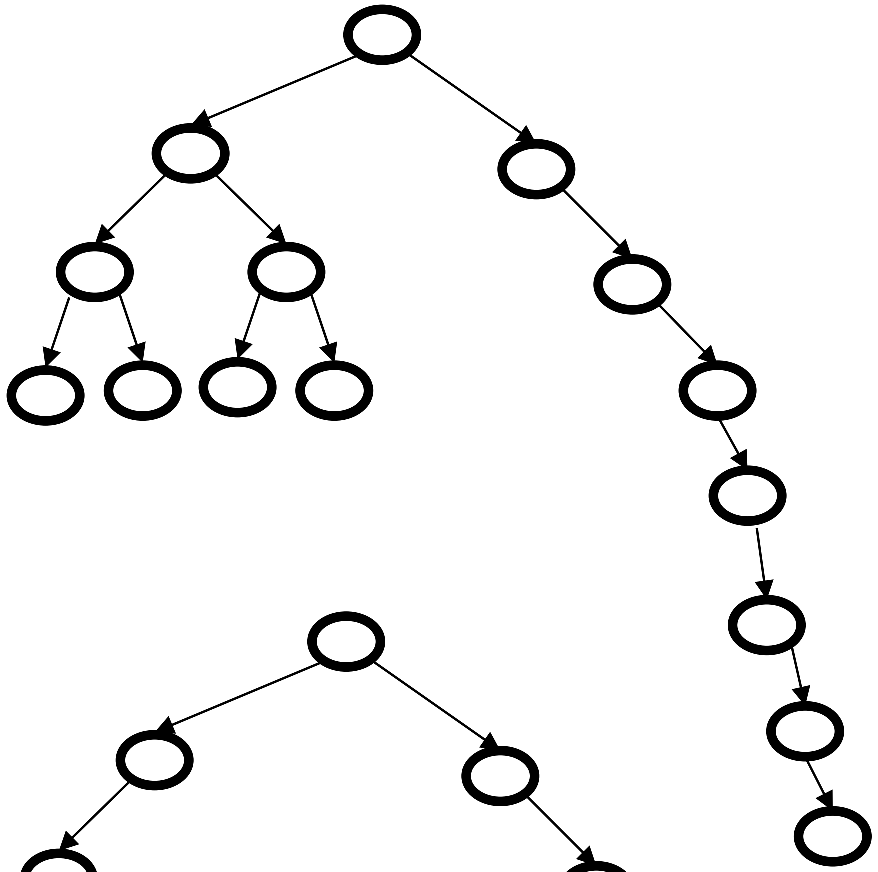
1. Left and right subtrees of the ***root*** have equal ***number of nodes***

2. Left and right subtrees of the ***root*** have equal ***height***

Potential Balance Conditions

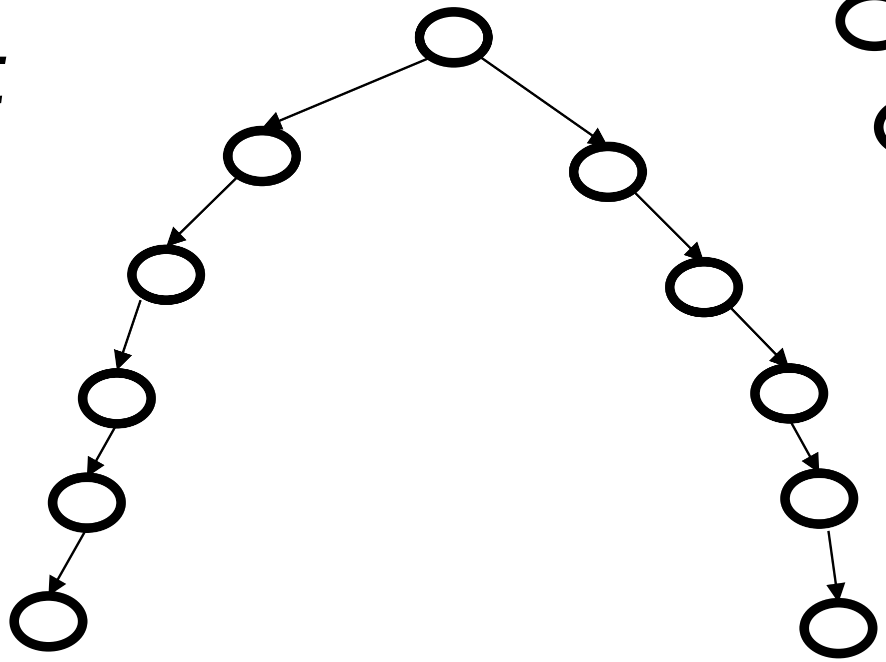
1. Left and right subtrees of the ***root*** have equal ***number of nodes***

Too weak!
Height mismatch example:



2. Left and right subtrees of the ***root*** have equal ***height***

Too weak!
Double chain example:



Potential Balance Conditions

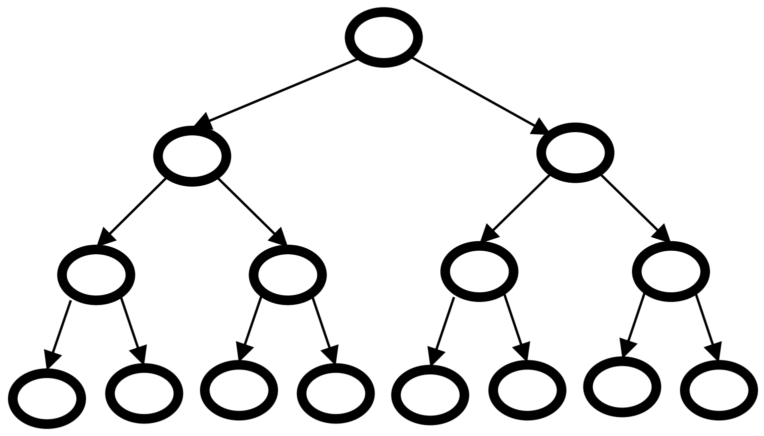
3. Left and right subtrees of ***every node*** have equal ***number of nodes***

4. Left and right subtrees of ***every node*** have equal ***height***

Potential Balance Conditions

3. Left and right subtrees of ***every node*** have equal ***number of nodes***

Too strong!
Only perfect trees (2^n – 1 nodes)



4. Left and right subtrees of ***every node*** have equal ***height***

Too strong!
Only perfect trees (2^n – 1 nodes)

The AVL Balance Condition

Left and right subtrees of *every node*
have *heights* **differing by at most 1**

Definition: **balance**($node$) = height($node$.left) – height($node$.right)

AVL *property*: **for every node x, $-1 \le$ balance(x) ≤ 1**

- Ensures small depth
  - Will prove this by showing that an AVL tree of height h must have a number of nodes *exponential* in h
- Easy (well, efficient) to maintain
  - Using single and double rotations