Big Oh: Common Categories

From fastest to slowest

$O(1)$	constant (same as $O(k)$ for constant k)
$O(\log n)$	logarithmic
$O(n)$	linear
$O(n \log n)$	"n log n"
$O(n^2)$	quadratic
$O(n^3)$	cubic
$O(n^k)$	polynomial (where is k is any constant > 1)
$O(k^n)$	exponential (where k is any constant > 1)

Note: *Don't write O(5n)* instead of O(n) – same thing! It's like writing 6/2 instead of 3. Looks weird

Usage note: "exponential" does not mean "grows really fast", it means "grows at rate proportional to k^n for some $k>1$"

More asymptotic analysis

Upper bound: $O($ **f(n)** $)$

$g(n)$ is in $O($ **f(n)** $)$ if there exist constants ***c*** and n_0 such that $g(n) \leq$ ***c*** **f(n)** for all $n \geq n_0$

Lower bound: $\Omega($ **f(n)** $)$

$g(n)$ is in $\Omega($ **f(n)** $)$ if there exist constants ***c*** and n_0 such that $g(n) \geq$ ***c*** **f(n)** for all $n \geq n_0$

Tight bound: $\theta($ **f(n)** $)$

$g(n)$ is in $\theta($ **f(n)** $)$ if it is in $O($ **f(n)** $)$ and it is in $\Omega($ **f(n)** $)$

Big-O, Big-Theta, Big-Omega Relationships

If a function is in Big-Theta, what does it mean for its membership in Big-O and Big-Omega? Vice versa?

Mystery Function	Big-O	Big-Theta	Big-Omega
	$O(N^4)$		$\Omega(N^4)$
		$\Theta(N^3)$	
	$O(N)$		
			$\Omega(N^2)$

Example

Let's show: $10n^2 + 15n$ is in $O(n^2)$

Proving NOT in Big Oh Example: Prove Negation

Let's show: $10n^2$ is NOT $O(n)$

Not in Big OH:

$\forall_{c,n_0}\exists_{n\geq n_0}\ g(n) > c \cdot f(n)$

For any c or n_0 that you pick, there is a valid n where our function g(n) exceeds the Big Oh function f(n)

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Why double size?

The most common strategy for increasing array size is doubling. Why not just increase the size by 10,000 each time we fill up?

Let's say we did n insertions:

Costs of the unlucky insertions:

Costs of the other insertions:

Amortized insert cost:

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1. $f(n)$ is in $O(f(n)^2)$	AT	ST	NT
2. $f(n)$ is in $\Theta(f(n))$	AT	ST	NT
3. $f(n)+g(n)$ is in $\Theta(\max(f(n), g(n))$	AT	ST	NT
4. $f(n) * n$ is in $O(f(n)^2)$	AT	ST	NT

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