

CSE 332: Data Structures and Parallelism

Spring 2022
Richard Anderson
Lecture 16: Analysis of Fork-Join Programs

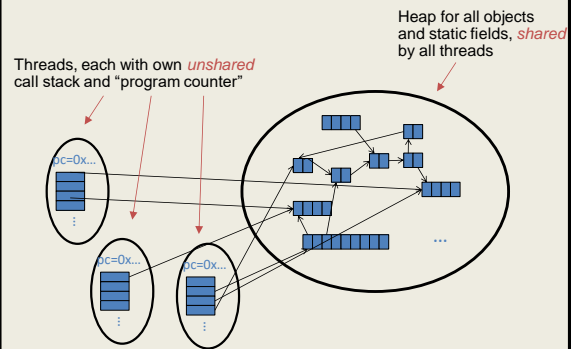
Course Schedule

- Lectures 1- 14: Traditional Data Structures
- Lectures 15-19: Parallelism
- Lectures 20-22: Concurrency
- Lectures 23-27: Graph Algorithms
- Lectures 28-29: Theory of NP-Completeness

Announcements

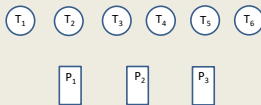
- Read parallel computing notes by Dan Grossman 2.1-4.3
- Midterm – grading in progress
- Bring laptop to section this week with IntelliJ set up
 - Work on fork-join parallelism for most of section

Shared Memory with Threads



Threads and Processors

- Simple model
 - Threads are either running or idle
 - Processors select idle threads and execute them for “a while”
- Scheduling of threads is outside of the scope of this course
 - Many different approaches
 - Programmer has limited control on scheduling



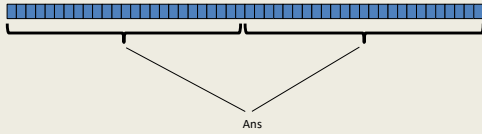
Fork-Join Parallelism

- 1. Define thread**
 - Java: define subclass of `java.lang.Thread`,
 - Override `run` implement operation of the thread
- 2. Fork:** instantiate a thread and start executing
 - Java: create thread object, call `start()`
- 3. Join:** wait for thread to terminate
 - Java: call `join()` method, which returns when thread finishes

Above uses basic thread library build into Java
Later we'll introduce a better **ForkJoin Java library** designed for parallel programming

Sum with Threads

For starters: have two threads simultaneously sum half of the array



- Create two *thread objects*, each given half of the array
- Call `start()` on each thread object to run it in parallel
- Wait for threads to finish using `join()`
- Add together their answers for the final result

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Part 1: define thread class

```
class SumThread extends java.lang.Thread {  
    int lo; // fields, passed to constructor  
    int hi; // so threads know what to do.  
    int[] arr;  
  
    int ans = 0; // result  
  
    SumThread(int[] a, int l, int h) {  
        lo=l; hi=h; arr=a;  
    }  
  
    public void run() {  
        for(int i=lo; i < hi; i++)  
            ans += arr[i];  
    }  
}
```

Because we must override a no-arguments/no-result run, we use fields to communicate across threads

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Part 2: sum routine

```
int sum(int[] arr){  
    int len = arr.length;  
  
    SumThread ts1 = new SumThread(arr,0,len/2);  
    SumThread ts2 = new SumThread(arr,len/2,len);  
  
    ts1.start();  
    ts2.start();  
  
    ts1.join();  
    ts2.join();  
  
    return ts1.ans + ts2.ans;  
}
```

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Now with four threads

```
int sum(int[] arr){// can be a static method  
    int len = arr.length;  
    int ans = 0;  
    SumThread[] ts = new SumThread[4];  
    for(int i=0; i < 4; i++){// do parallel computations  
        ts[i] = new SumThread(arr,i*len/4,(i+1)*len/4);  
        ts[i].start();  
    }  
    for(int i=0; i < 4; i++) { // combine results  
        ts[i].join(); // wait for helper to finish!  
        ans += ts[i].ans;  
    }  
    return ans;  
}
```

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Parameterizing by number of threads

```
int sum(int[] arr, int numTs){  
    int ans = 0;  
    SumThread[] ts = new SumThread[numTs];  
    for(int i=0; i < numTs; i++){  
        ts[i] = new SumThread(arr,(i*arr.length)/numTs,  
            (i+1)*arr.length)/numTs);  
        ts[i].start();  
    }  
    for(int i=0; i < numTs; i++) {  
        ts[i].join();  
        ans += ts[i].ans;  
    }  
    return ans;  
}
```

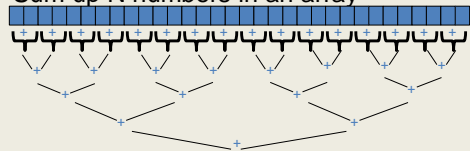
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Recall: Parallel Sum

- Sum up N numbers in an array



- Let's implement this with threads...

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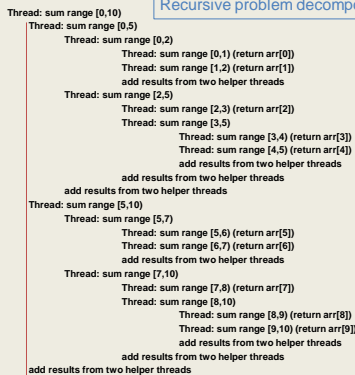
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Code looks something like this (using Java Threads)

```
class SumThread extends java.lang.Thread {
    int lo; int hi; int[] arr; // fields to know what to do
    int ans = 0; // result
    SumThread(int[] a, int l, int h) { ... }
    public void run() { // override
        if(hi - lo < SEQUENTIAL CUTOFF)
            for(int i=lo; i < hi; i++)
                ans += arr[i];
        else {
            SumThread left = new SumThread(arr, lo, (hi+lo)/2);
            SumThread right = new SumThread(arr, (hi+lo)/2, hi);
            left.start();
            right.start();
            left.join(); // don't move this up a line - why?
            right.join();
            ans = left.ans + right.ans;
        }
    }
}
int sum(int[] arr){ // just make one thread!
    SumThread t = new SumThread(arr,0,arr.length);
    t.run();
    return t.ans;
}
```

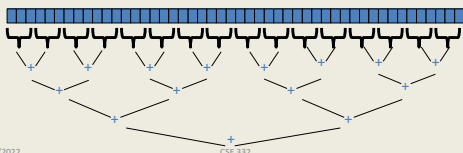
Recursive problem decomposition



Divide-and-conquer

Same approach useful for many problems beyond sum

- If you have enough processors, total time $O(\log n)$
- Next lecture: study reality of $P \ll n$ processors
- Will write all our parallel algorithms in this style
 - But using a special fork-join library engineered for this style
 - Takes care of scheduling the computation well
 - Often relies on operations being associative (like +)



Thread Overhead

Creating and managing threads incurs cost

Two optimizations:

1. Use a *sequential cutoff*, typically around 500-1000
 - Eliminates lots of tiny threads
2. Do not create two recursive threads; create one thread and do the other piece of work "yourself"
 - Cuts the number of threads created by another 2x

Half the threads!

order of last 4 lines is critical - why?

```
// wasteful: don't
SumThread left = ...
SumThread right = ...

left.start();
right.start();

left.join();
right.join();
ans=left.ans+right.ans;
```

```
// better: do!!
SumThread left = ...
SumThread right = ...

left.start();
right.run();

left.join();
// no right.join needed
ans=left.ans+right.ans;
```

Note: run is a normal function call execution won't continue until we are done with run

Better Java Thread Library

- Even with all this care, Java's threads are too "heavyweight"
 - Constant factors, especially space overhead
 - Creating 20,000 Java threads just a bad idea ☹
- The **ForkJoin Framework** is designed to meet the needs of divide-and-conquer fork-join parallelism
 - In the Java 8 standard libraries
 - Section will focus on pragmatics/logistics
 - Similar libraries available for other languages
 - C/C++: Cilk (inventors), Intel's Thread Building Blocks
 - C#: Task Parallel Library
 - ...

Different terms, same basic idea

To use the ForkJoin Framework:

- A little standard set-up code (e.g., create a `ForkJoinPool`)

Don't subclass <code>Thread</code>	Do subclass <code>RecursiveTask<V></code>
Don't override <code>run</code>	Do override <code>compute</code>
Do not use an <code>ans</code> field	Do return a <code>V</code> from <code>compute</code>
Don't call <code>start</code>	Do call <code>fork</code>
Don't just call <code>join</code>	Do call <code>join</code> (which returns answer)
Don't call <code>run</code> to hand-optimize	Do call <code>compute</code> to hand-optimize
Don't have a topmost call to <code>run</code>	Do create a pool and call <code>invoke</code>

See the web page for (linked in to project 3 description):
 "A Beginner's Introduction to the ForkJoin Framework"

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Fork Join Framework Version: (missing imports)

```
class SumArray extends RecursiveTask<Integer> {
    int lo; int hi; int[] arr; // fields to know what to do
    SumArray(int[] a, int l, int h) { ... }
    protected Integer compute() { // return answer
        if (hi - lo < SEQUENTIAL_CUTOFF) {
            int ans = 0; // local var, not a field
            for (int i=lo; i < hi; i++)
                ans += arr[i];
            return ans;
        } else {
            SumArray left = new SumArray(arr, lo, (hi+lo)/2);
            SumArray right = new SumArray(arr, (hi+lo)/2, hi);
            left.fork(); // fork a thread and calls compute
            int rightAns = right.compute(); // call compute directly
            int leftAns = left.join(); // get result from left
            return leftAns + rightAns;
        }
    }
}
static final ForkJoinPool fjPool = new ForkJoinPool();
int sum(int[] arr) {
    return fjPool.invoke(new SumArray(arr, 0, arr.length));
} // invoke returns the value compute returns
```

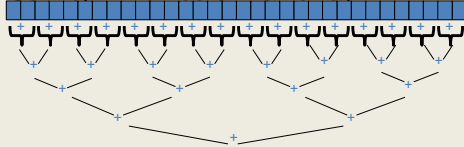
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Parallel Sum

- Sum up N numbers in an array

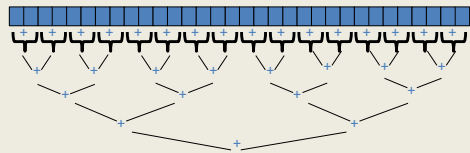


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Parallel Max?



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Reductions

- Same trick works for many tasks, e.g.,
 - is there an element satisfying some property (e.g., prime)
 - left-most element satisfying some property (e.g., first prime)
 - smallest rectangle encompassing a set of points (proj3)
 - counts: number of strings that start with a vowel
 - are these elements in sorted order?
- Called a **reduction**, or **reduce** operation
 - reduce a collection of data items to a single item
 - result can be more than a single value, e.g., produce histogram from a set of test scores
- Very common parallel programming pattern

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Parallel Vector Scaling

- Multiply every element in the array by 2



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Maps

- A **map** operates on each element of a collection of data to produce a new collection of the same size
 - each element is processed independently of the others, e.g.
 - vector scaling
 - vector addition
 - test property of each element (is it prime)
 - uppercase to lowercase
 - ...
- Another common parallel programming pattern

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Maps in ForkJoin Framework

```
class VecAdd extends RecursiveAction {
    int lo; int hi; int[] res; int[] arr1; int[] arr2;
    VecAdd(int l, int h, int[] r, int[] a1, int[] a2) { ... }
    protected void compute() {
        if (hi - lo < SEQUENTIAL_CUTOFF) {
            for (int i = lo; i < hi; i++)
                res[i] = arr1[i] + arr2[i];
        } else {
            int mid = (hi + lo) / 2;
            VecAdd left = new VecAdd(lo, mid, res, arr1, arr2);
            VecAdd right = new VecAdd(mid, hi, res, arr1, arr2);
            left.fork();
            right.compute();
            left.join();
        }
    }
}

static final ForkJoinPool fjPool = new ForkJoinPool();
int[] add(int[] arr1, int[] arr2) {
    assert (arr1.length == arr2.length);
    int[] ans = new int[arr1.length];
    fjPool.invoke(new VecAdd(0, arr.length, ans, arr1, arr2));
    return ans;
}
```

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Maps and Reductions

Maps and reductions: the “workhorses” of parallel programming

- By far the most important and common patterns
- Learn to recognize when an algorithm can be written in terms of maps and reductions
- makes parallel programming easy (plug and play)

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Distributed Map Reduce

- You may have heard of Google’s map/reduce
 - or open-source version called Hadoop
 - powers much of Google’s infrastructure
- Idea: maps/reductions using many machines
 - same principles, applied to distributed computing
 - system takes care of distributing data, fault-tolerance
 - you just write code to handle one element, reduce a collection
- Co-developed by Jeff Dean (UW alum!)

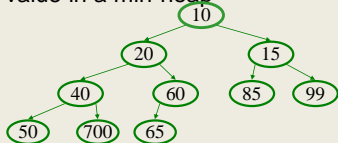
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Maps and Reductions on Trees

- Max value in a min-heap



- How to parallelize?
- Is this a map or a reduce?
- Complexity?

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Analyzing Parallel Programs

Let T_P be the running time on P processors

Two key measures of run-time:

- **Work:** How long it would take 1 processor = T_1
- **Span:** How long it would take infinity processors = T_∞
 - The hypothetical ideal for parallelization
 - This is the longest “dependence chain” in the computation
 - Example: $O(\log n)$ for summing an array
 - Also called “critical path length” or “computational depth”

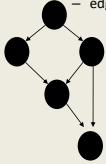
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The DAG

- Fork-join programs can be modeled with a DAG
 - nodes: pieces of work
 - edges: order dependencies



- A **fork** creates two children
- new thread
 - continuation of current thread
- A **join** makes a node with two incoming edges
- terminated thread
 - continuation of current thread

What's T_1 (work):

What's T_∞ (span):

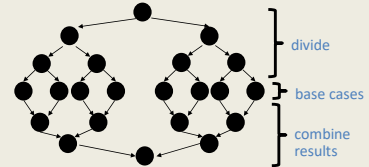
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Divide and Conquer Algorithms

Our **fork** and **join** frequently look like this:



In this context, the span (T_∞) is:

- The longest dependence-chain; longest 'branch' in parallel 'tree'
- Example: $O(\log n)$ for summing an array; we halve the data down to our cut-off, then add back together; $O(\log n)$ steps, $O(1)$ time for each
- Also called "critical path length" or "computational depth"

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Parallel Speed-up

- Speed-up on P processors: T_1 / T_P
- If speed-up is P , we call it **perfect linear speed-up**
 - e.g., doubling P halves running time
 - hard to achieve in practice
- Parallelism** is the maximum possible speed-up: T_1 / T_∞
 - if you had infinite processors

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Estimating T_p

- How to estimate T_p (e.g., $P = 4$)?
- Lower bounds on T_p (ignoring memory, caching...)
 - T_∞
 - T_1 / P
 - which one is the tighter (higher) lower bound?
- The ForkJoin Java Framework achieves the following expected time asymptotic bound:

$$T_p \in O(T_\infty + T_1 / P)$$
 - this bound is optimal

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Amdahl's Law

- Most programs have
 - parts that parallelize well
 - parts that don't parallelize at all
- The latter become bottlenecks

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Amdahl's Law

- Let $T_1 = 1$ unit of time
- Let S = proportion that can't be parallelized

$$1 = T_1 = S + (1 - S)$$
- Suppose we get perfect linear speedup on the parallel portion:

$$T_p =$$
- So the overall speed-up on P processors is (Amdahl's Law):

$$T_1 / T_p =$$

$$T_1 / T_\infty =$$
- If 1/3 of your program is parallelizable, max speedup is:

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Pretty Bad News

- Suppose 25% of your program is sequential.
 - Then a billion processors won't give you more than a 4x speedup!
- What portion of your program must be parallelizable to get 10x speedup on a 1000 core GPU?
 - $10 \leq 1 / (S + (1-S)/1000)$
- Motivates minimizing sequential portions of your programs

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Take Aways

- Parallel algorithms can be a big win
- Many fit standard patterns that are easy to implement
- Can't just rely on more processors to make things faster (Amdahl's Law)

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