CSE 332: Data Structures and Parallelism

Spring 2022
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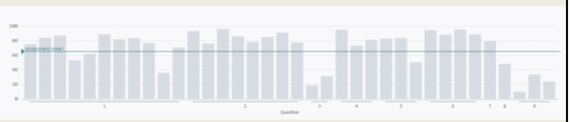
Lecture 20: Analysis of Fork-Join Programs

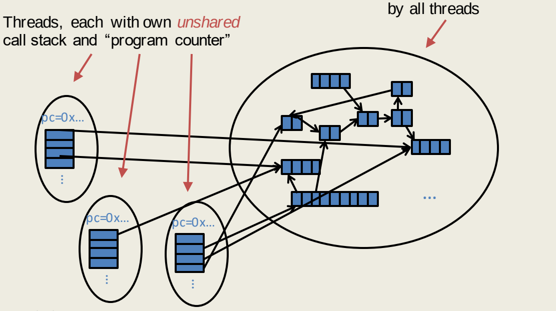
Announcements

- Read parallel computing notes by Dan Grossman 2.1-4.3
- Bring laptop to section this week with IntelliJ set up
  - Work on fork-join parallelism for most of section

Midterm Problems



Shared Memory with Threads

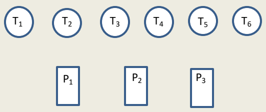


Heap for all objects and static fields, *shared* by all threads

Threads, each with own *unshared* call stack and “program counter”

Threads and Processors

- Simple model
  - Threads are either running or idle
  - Processors select idle threads and execute them for “a while”
- Scheduling of threads is outside of the scope of this course
  - Many different approaches
  - Programmer has limited control on scheduling



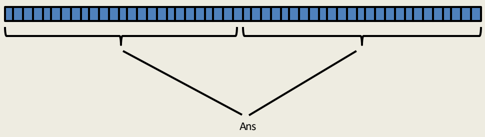
Fork-Join Parallelism

1. Define thread
   - Java: define subclass of `java.lang.Thread`,
   - Override `run` implement operation of the thread
2. Fork: instantiate a thread and start executing
   - Java: create thread object, call `start()`
3. Join: wait for thread to terminate
   - Java: call `join()` method, which returns when thread finishes

Above uses basic thread library build into Java
Later we’ll introduce a better ForkJoin Java library designed for parallel programming

Sum with Threads

For starters: have two threads simultaneously sum half of the array



- Create two *thread objects*, each given half of the array
- Call `start()` on each thread object to run it in parallel
- Wait for threads to finish using `join()`
- Add together their answers for the final result

Part 1: define thread class

```
class SumThread extends java.lang.Thread {

  int lo; // fields, passed to constructor
  int hi; // so threads know what to do.
  int[] arr;

  int ans = 0; // result

  SumThread(int[] a, int l, int h) {
    lo=l; hi=h; arr=a;
  }


  public void run() {
    for(int i=lo; i < hi; i++)
      ans += arr[i];
  }
}
```

Because we must override a no-arguments/no-result run,
we use fields to communicate across threads

Part 2: sum routine

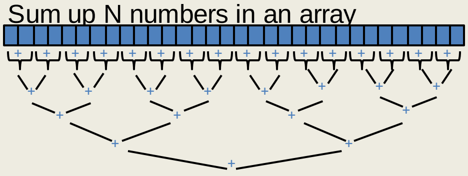
```
int sum(int[] arr){
  int len = arr.length;

  SumThread ts1 = new SumThread(arr,0,len/2);
  SumThread ts2 = new SumThread(arr,len/2,len);

  ts1.start();
  ts2.start();

  ts1.join();
  ts2.join();

  return ts1.ans  + ts2.ans;
}
```

Parameterizing by number of threads

```
int sum(int[] arr, int numTs){
  int ans = 0;
  SumThread[] ts = new SumThread[numTs];
  for(int i=0; i < numTs; i++){
   ts[i] = new SumThread(arr,(i*arr.length)/numTs,
                             ((i+1)*arr.length)/numTs);
   ts[i].start();
  }
  for(int i=0; i < numTs; i++) {
    ts[i].join();
    ans += ts[i].ans;
  }
  return ans;
}
```

Recall: Parallel Sum

- Sum up N numbers in an array



- Let's implement this with threads...

Code looks something like this (using Java Threads)

```
class SumThread extends java.lang.Thread {
  int lo; int hi; int[] arr; // fields to know what to do
  int ans = 0; // result
  SumThread(int[] a, int l, int h) { … }
  public void run(){ // override
    if(hi – lo < SEQUENTIAL_CUTOFF)
      for(int i=lo; i < hi; i++)
        ans += arr[i];
    else {
      SumThread left = new SumThread(arr,lo,(hi+lo)/2);
      SumThread right= new SumThread(arr,(hi+lo)/2,hi);
      left.start();
      right.start();
      left.join(); // don't move this up a line – why?
      right.join();
      ans = left.ans + right.ans;
    }
  }
}
int sum(int[] arr){ // just make one thread!
   SumThread t = new SumThread(arr,0,arr.length);
   t.run();
   return t.ans;
}
```

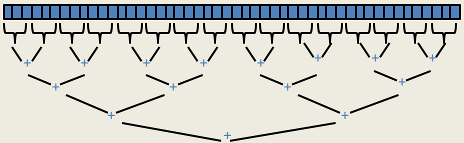
Recursive problem decomposition

```
Thread: sum range [0,10)
  Thread: sum range [0,5)
     Thread: sum range [0,2)
          Thread: sum range [0,1) (return arr[0])
          Thread: sum range [1,2) (return arr[1])
          add results from two helper threads
     Thread: sum range [2,5)
          Thread: sum range [2,3) (return arr[2])
          Thread: sum range [3,5)
               Thread: sum range [3,4) (return arr[3])
               Thread: sum range [4,5) (return arr[4])
               add results from two helper threads
          add results from two helper threads
     add results from two helper threads
  Thread: sum range [5,10)
     Thread: sum range [5,7)
          Thread: sum range [5,6) (return arr[5])
          Thread: sum range [6,7) (return arr[6])
          add results from two helper threads
     Thread: sum range [7,10)
          Thread: sum range [7,8) (return arr[7])
          Thread: sum range [8,10)
               Thread: sum range [8,9) (return arr[8])
               Thread: sum range [9,10) (return arr[9])
               add results from two helper threads
          add results from two helper threads
  add results from two helper threads
```

Divide-and-conquer

Same approach useful for many problems beyond sum
  - *If* you have enough processors, total time *O*(`log` *n*)
  - Next lecture: study reality of **P** << *n* processors

- Will write all our parallel algorithms in this style
  - But using a special fork-join library engineered for this style
    - Takes care of scheduling the computation well
  - Often relies on operations being associative (like +)



Thread Overhead

Creating and managing threads incurs cost

Two optimizations:

1. Use a *sequential cutoff*, typically around 500-1000
   - Eliminates lots of tiny threads
2. Do not create two recursive threads; create one thread and do the other piece of work "yourself"
   - Cuts the number of threads created by another 2x

Half the threads!

order of last 4 lines

Is critical – why?

```
// wasteful: don't
SumThread left  = …
SumThread right = …

left.start();
right.start();



left.join();
right.join();
ans=left.ans+right.ans;
```

```
// better: do!!
SumThread left  = …
SumThread right = …

left.start();
right.run();



left.join();
// no right.join needed
ans=left.ans+right.ans;
```

Note: `run` *is a normal function call! execution won't continue until we* are done with `run`

Better Java Thread Library

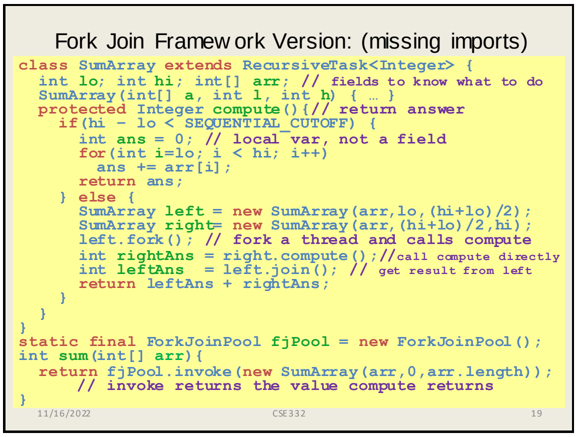
- Even with all this care, Java's threads are too "heavyweight"
  - Constant factors, especially space overhead
  - Creating 20,000 Java threads just a bad idea ☹
- The ForkJoin Framework is designed to meet the needs of divide-and-conquer fork-join parallelism
  - In the Java 8 standard libraries
  - Section will focus on pragmatics/logistics
  - Similar libraries available for other languages
    - C/C++: Cilk (inventors), Intel's Thread Building Blocks
    - C#: Task Parallel Library
    - …

Different terms, same basic idea

To use the ForkJoin Framework:
- A little standard set-up code (e.g., create a **`ForkJoinPool`**)

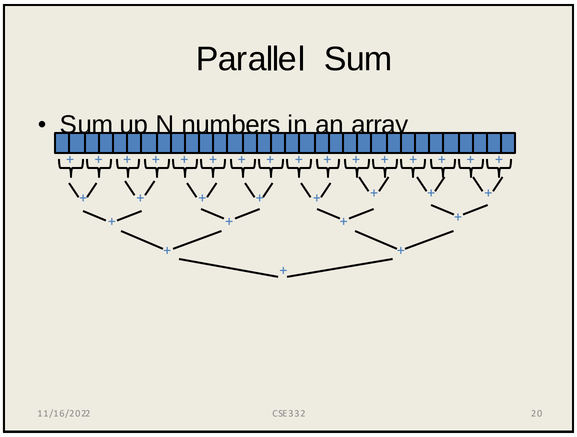
Don't subclass **`Thread`**	Do subclass **`RecursiveTask<V>`**
Don't override **`run`**	Do override **`compute`**
Do not use an **`ans`** field	Do return a **`V`** from **`compute`**
Don't call **`start`**	Do call **`fork`**
Don't *just* call **`join`**	Do call **`join`** (which returns answer)
Don't call **`run`** to hand-optimize	Do call **`compute`** to hand-optimize
Don't have a topmost call to **`run`**	Do create a pool and call **`invoke`**

See the web page for (linked from Handouts page on course website):
"A Beginner's Introduction to the ForkJoin Framework"



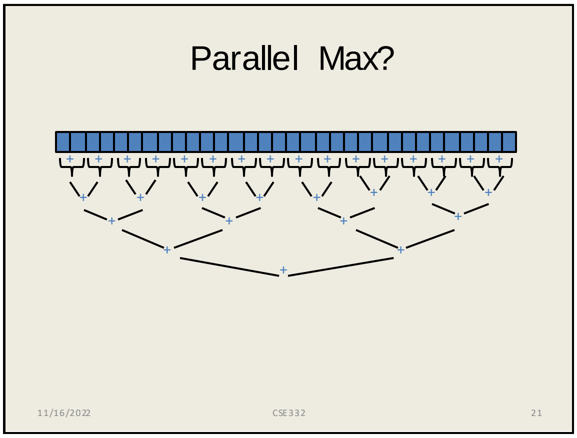
Fork Join Framework Version: (missing imports)

```
class SumArray extends RecursiveTask<Integer> {
  int lo; int hi; int[] arr; // fields to know what to do
  SumArray(int[] a, int l, int h) { … }
  protected Integer compute(){// return answer
    if(hi – lo < SEQUENTIAL_CUTOFF) {
      int ans = 0; // local var, not a field
      for(int i=lo; i < hi; i++)
        ans += arr[i];
      return ans;
    } else {
      SumArray left = new SumArray(arr,lo,(hi+lo)/2);
      SumArray right= new SumArray(arr,(hi+lo)/2,hi);
      left.fork(); // fork a thread and calls compute
      int rightAns = right.compute();//call compute directly
      int leftAns  = left.join(); // get result from left
      return leftAns + rightAns;
    }
  }
}
static final ForkJoinPool fjPool = new ForkJoinPool();
int sum(int[] arr){
  return fjPool.invoke(new SumArray(arr,0,arr.length));
     // invoke returns the value compute returns
}
```

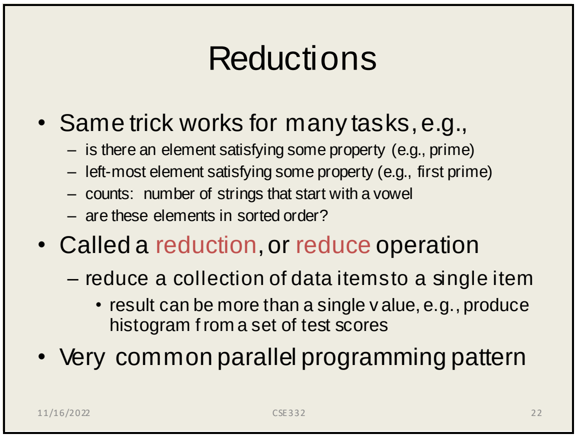


Parallel Sum

- Sum up N numbers in an array

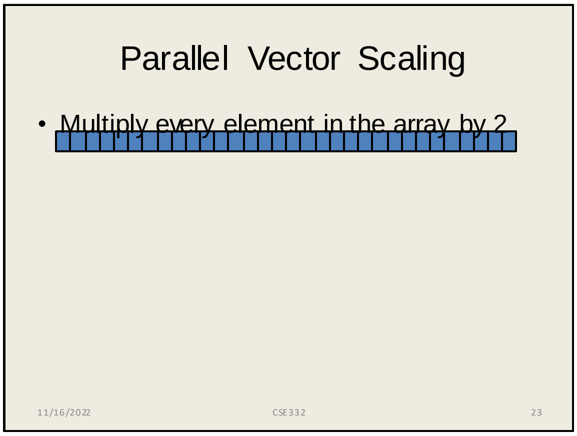


Parallel Max?



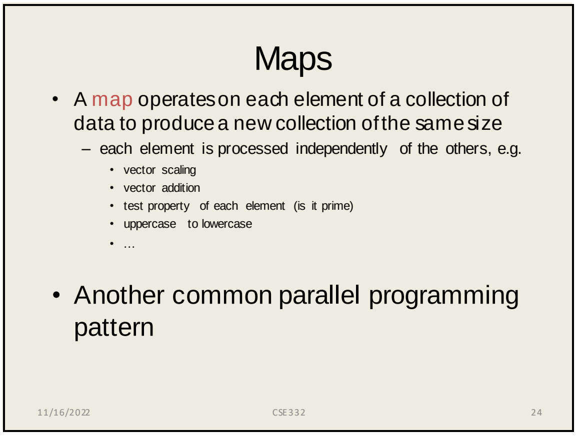
Reductions

- Same trick works for many tasks, e.g.,
  - is there an element satisfying some property (e.g., prime)
  - left-most element satisfying some property (e.g., first prime)
  - counts: number of strings that start with a vowel
  - are these elements in sorted order?
- Called a reduction, or reduce operation
  - reduce a collection of data items to a single item
    - result can be more than a single value, e.g., produce histogram from a set of test scores
- Very common parallel programming pattern



Parallel Vector Scaling

- Multiply every element in the array by 2



Maps

- A map operates on each element of a collection of data to produce a new collection of the same size
  - each element is processed independently of the others, e.g.
    - vector scaling
    - vector addition
    - test property of each element (is it prime)
    - uppercase to lowercase
    - …
- Another common parallel programming pattern

Maps in ForkJoin Framework

```
class VecAdd extends RecursiveAction {
  int lo; int hi; int[] res; int[] arr1; int[] arr2;
  VecAdd(int l,int h,int[] r,int[] a1,int[] a2){ … }
  protected void compute(){
    if(hi – lo < SEQUENTIAL_CUTOFF) {
      for(int i=lo; i < hi; i++)
        res[i] = arr1[i] + arr2[i];
    } else {
      int mid = (hi+lo)/2;
      VecAdd left = new VecAdd(lo,mid,res,arr1,arr2);
      VecAdd right= new VecAdd(mid,hi,res,arr1,arr2);
      left.fork();
      right.compute();
      left.join();
    }
  }
}
static final ForkJoinPool fjPool = new ForkJoinPool();
int[] add(int[] arr1, int[] arr2){
  assert (arr1.length == arr2.length);
  int[] ans = new int[arr1.length];
  fjPool.invoke(new VecAdd(0,arr.length,ans,arr1,arr2);
  return ans;
}
```

Maps and Reductions

Maps and reductions: the "workhorses" of parallel programming

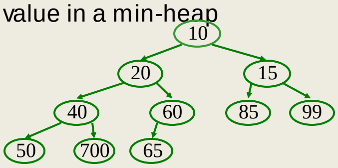
- By far the most important and common patterns
- Learn to recognize when an algorithm can be written in terms of maps and reductions
- makes parallel programming easy (plug and play)

Distributed Map Reduce

- You may have heard of Google's map/reduce
  - or open-source version called Hadoop
  - powers much of Google's infrastructure
- Idea: maps/reductions using many machines
  - same principles, applied to distributed computing
  - system takes care of distributing data, fault-tolerance
  - you just write code to handle one element, reduce a collection
- Co-developed by Jeff Dean (UW alum!)

Maps and Reductions on Trees

- Max value in a min-heap



- How to parallelize?
- Is this a map or a reduce?
- Complexity?

Analyzing Parallel Programs

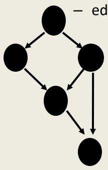
Let T_P be the running time on **P** processors

Two key measures of run-time:

- Work: How long it would take 1 processor = T_1
- Span: How long it would take infinity processors = T_∞
  - The hypothetical ideal for parallelization
  - This is the longest "dependence chain" in the computation
  - Example: $O($`log` $n)$ for summing an array
  - Also called "critical path length" or "computational depth"

The DAG

- Fork-join programs can be modeled with a DAG
  - nodes: pieces of work
  - edges: order dependencies



A `fork` creates two children
- new thread
- continuation of current thread

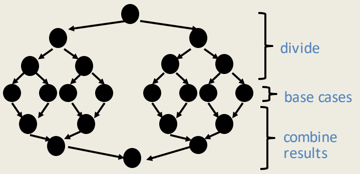
A `join` makes a node with two incoming edges
- terminated thread
- continuation of current thread

What's T_1 (work):

What's T_∞ (span):

Divide and Conquer Algorithms

Our **`fork`** and **`join`** frequently look like this:



In this context, the span (T_∞) is:

- The longest dependence-chain; longest 'branch' in parallel 'tree'
- Example: *O*(log *n*) for summing an array; we halve the data down to our cut-off, then add back together; *O*(log *n*) steps, O(1) time for each
- Also called "critical path length" or "computational depth"

Parallel Speed-up

- Speed-up on **P** processors: $\mathbf{T_1 / T_P}$
- If speed-up is **P**, we call it perfect linear speed-up
  - e.g., doubling **P** halves running time
  - hard to achieve in practice
- Parallelism is the maximum possible speed-up: $\mathbf{T_1 / T_\infty}$
  - if you had infinite processors

Estimating T_p

- How to estimate $\mathbf{T_P}$ (e.g., P = 4)?
- Lower bounds on $\mathbf{T_P}$ (ignoring memory, caching...)
  1. $\mathbf{T_\infty}$
  2. $\mathbf{T_1 / P}$
  - which one is the tighter (higher) lower bound?
- The ForkJoin Java Framework achieves the following expected time asymptotic bound:

$$\mathbf{T_P \in O(T_\infty + T_1 / P)}$$

  - this bound is optimal

Amdahl's Law

- Most programs have
  1. parts that parallelize well
  2. parts that don't parallelize at all
- The latter become bottlenecks

Amdahl's Law

- Let T_1 = 1 unit of time
- Let S = proportion that can't be parallelized

$$1 = \mathbf{T_1 = S + (1 - S)}$$

- Suppose we get perfect linear speedup on the parallel portion:

$$\mathbf{T_P} =$$

- So the overall speed-up on P processors is (Amdahl's Law):

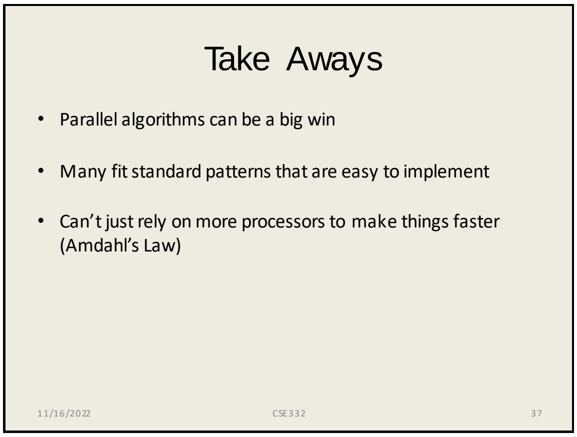
$$\mathbf{T_1 / T_P} =$$

$$\mathbf{T_1 / T_\infty} =$$

- If 1/3 of your program is parallelizable, max speedup is:

Pretty Bad News

- Suppose 25% of your program is sequential.
  - Then a billion processors won't give you more than a 4x speedup!
- What portion of your program must be parallelizable to get 10x speedup on a 1000 core GPU?
  - 10 <= 1 / (S + (1-S)/1000)
- Motivates minimizing sequential portions of your programs



Take Aways

- Parallel algorithms can be a big win
- Many fit standard patterns that are easy to implement
- Can’t just rely on more processors to make things faster (Amdahl’s Law)