Graph Algorthms

CSE 332 Summer 2021

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Announcements

- Exercises 13, 14 out!
  - Correct due dates listed on Ed

- Midterm reflection Q1 resubmissions

Lecture Outline

- **Graph Representations**
  - Adjacency Matrix
  - Adjacency List
- Topological Sort
- Traversals
  - Breadth-first
  - Depth-first
  - Conclusion

What is the Data Structure?

- Is a Graph an ADT? Maybe!
  - "Develop an algorithm over the graph, then use whatever data structure is efficient"

- The "best" data structure can depend on:
  - Properties of the graph (e.g., dense versus sparse)
  - Common queries
    - e.g., "is **`(u,v)`** an edge?" vs "what are the neighbors of node **`u`**?"

- There are two standard graph representations:
  - ***Adjacency Matrix*** and ***Adjacency List***
  - Different trade-offs, particularly time vs space

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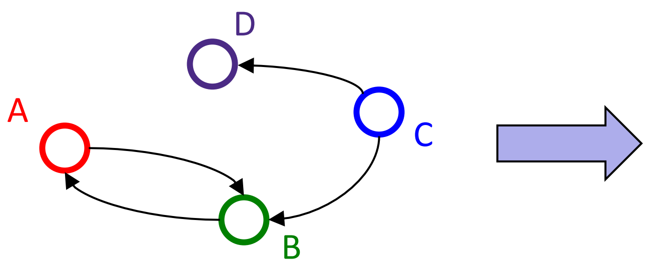
Adjacency Matrix: Representation

- Assign each node a number from `0` to `|V|-1`
- Graph is a `|V|x|V|` matrix (ie, 2-D array) of booleans
  - `M[u][v] == true` means there is an edge from `u` to `v`

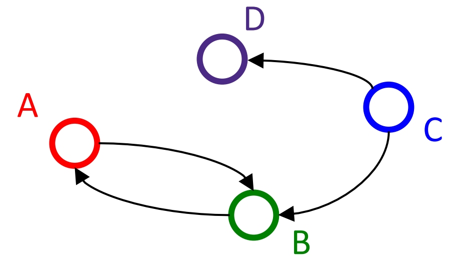


	A	B	C	D
A	F	T	F	F
B	T	F	F	F
C	F	T	F	T
D	F	F	F	F

Adjacency Matrix: Properties (1 of 3)

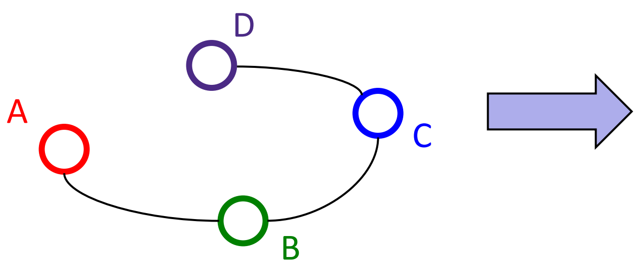
- Running time to:
  - Get a vertex's out-edges:
    - O(|V|)
  - Get a vertex's in-edges:
    - O(|V|)
  - Decide if some edge exists:
    - O(1)
  - Insert an edge:
    - O(1)
  - Delete an edge:
    - O(1)
- Space requirements:
  - $|V|^2$ bits
- Best for sparse or dense graphs?
  - Best for dense graphs

	A	B	C	D
A	F	T	F	F
B	T	F	F	F
C	F	T	F	T
D	F	F	F	F



Adjacency Matrix: Properties (2 of 3)

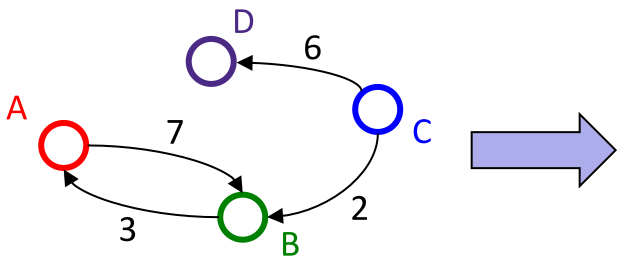
- How does the adjacency matrix vary for an *undirected graph*?
  - *Undirected graphs are symmetric about diagonal axis*
  - *Languages with array-of-array matrix representations can save ½ the space by omitting the symmetric half*
    - *Languages with "proper" 2D matrix representations (eg, C/C++) can't do this*



	A	B	C	D
A	F	T	F	F
B	T	F	T	F
C	F	T	F	T
D	F	F	T	F

Adjacency Matrix: Properties (3 of 3)

- How can we adapt the representation for *weighted graphs*?
  - *Store the weight in each cell*
  - *Need some value to represent "not an edge"*
    - *In some situations, 0 or -1 works*



	A	B	C	D
A	0	**7**	0	0
B	**3**	0	0	0
C	0	**2**	0	**6**
D	0	0	0	0

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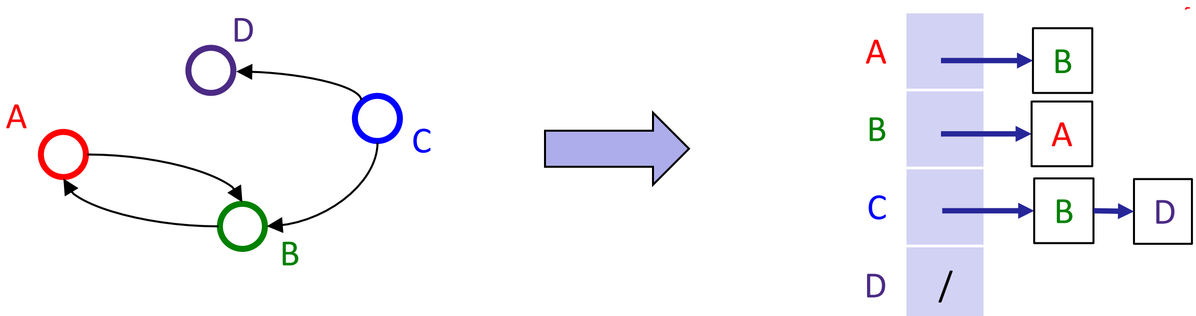
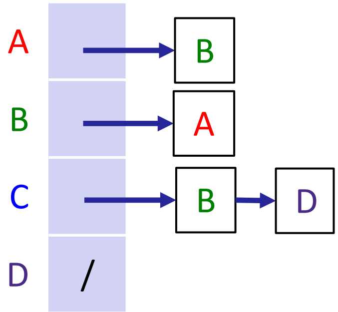
Adjacency List: Representation

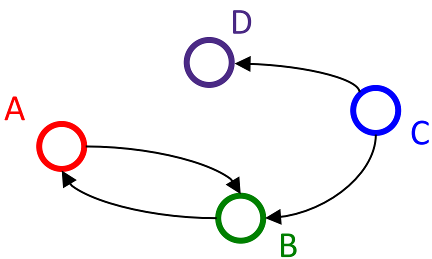
- Assign each node a number from `0` to `|V|-1`
- Graph is an array of length `|V|`; each entry stores a list of all adjacent vertices
  - E.g. linked list



Adjacency List: Properties (1 of 3)

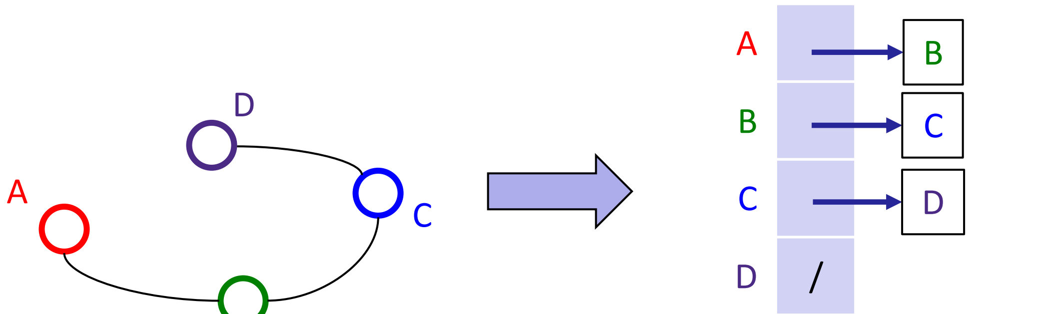
- Running time to:
  - Get a vertex's out-edges:
    - O(*d*) where *d* is out-degree of vertex
  - Get a vertex's in-edges:
    - O(|V| + |E|)
    - (but could keep a second adjacency list for this!)
  - Decide if some edge exists:
    - O(*d*) where *d* is out-degree of source vertex
  - Insert an edge:
    - O(1)
    - (unless you need to check if it's there; then O(*d*))
  - Delete an edge:
    - O(*d*) where *d* is out-degree of source vertex
- Space requirements:
  - O(|V|+|E|)
- Best for sparse or dense graphs?
  - Best for sparse graphs, so usually just stick with linked lists for the buckets



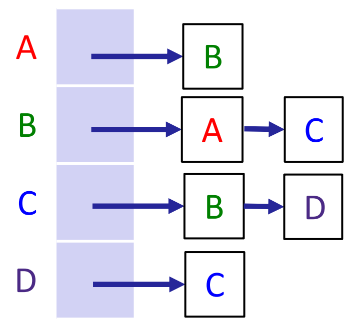


Adjacency List: Properties (2 of 3)

- How does the adjacency list vary for an *undirected graph*?
  - *Optionally, can double the entries to increase edge lookup speed*

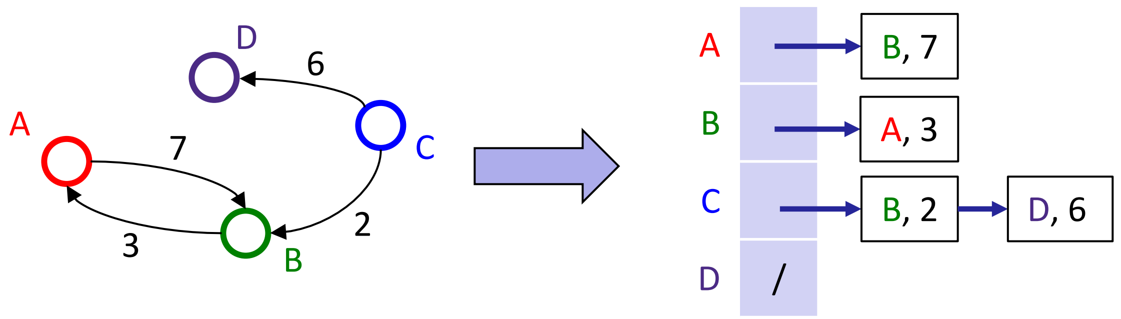


... or ...



Adjacency List: Properties (3 of 3)

- How can we adapt the representation for *weighted graphs*?
  - *Store the weight alongside the destination vertex*
  - *No need for a special value to represent "not an edge"!*



Summary: Which is Better?

- Graphs are often sparse:
  - Road networks are often grids
    - Every corner isn’t connected to every other corner
  - Airlines rarely fly to all possible cities
    - Or if they do it is to/from a hub

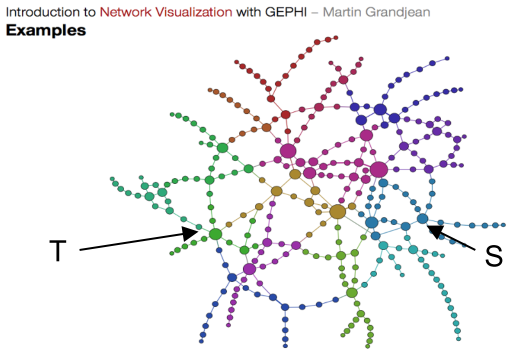
- Adjacency lists should generally be your default choice
  - Slower performance compensated by greater space savings
  - Many graph algorithms rely heavily on getAllEdgesFrom(v)

	getAllEdgesFrom(v)	hasEdge(v, w)	getAllEdges()
Adjacency Matrix	$\Theta(V)$	$\Theta(1)$	$\Theta(V^2)$
Adjacency List	$\Theta(\text{degree}(v))$	$\Theta(\text{degree}(v))$	$\Theta(E + V)$

Quick Detour: Overview of Graph Problems

Graph Problems

- Lots of interesting questions we can ask about a graph:
  - What is the shortest route from S to T? What is the longest route without cycles?
  - Are there cycles in this graph?
  - Is there a cycle that uses each *vertex* exactly once?
  - Is there a cycle that uses each *edge* exactly once?



Graph Problems More Theoretically

- Some well known graph problems and their common names:
  - **s-t Path**. Is there a path between vertices s and t?
  - **Connectivity.** Is the graph connected?
  - **Biconnectivity.** Is there a vertex whose removal disconnects the graph?
  - **Shortest s-t Path.** What is the shortest path between vertices s and t?
  - **Cycle Detection.** Does the graph contain any cycles?
  - **Euler Tour.** Is there a cycle that uses every edge exactly once?
  - **Hamilton Tour.** Is there a cycle that uses every vertex exactly once?
  - **Planarity**. Can you draw the graph on paper with no crossing edges?
  - **Isomorphism**. Are two graphs the same graph (in disguise)?

- Often can’t tell how difficult a graph problem is without very deep consideration.

Graph Problem Difficulty

- Some well known graph problems:
  - **Euler Tour:** Is there a cycle that uses every *edge* exactly once?
  - **Hamilton Tour:** Is there a cycle that uses every *vertex* exactly once?

- Difficulty can be deceiving
  - An efficient Euler tour algorithm O(# edges) was found as early as 1873 [Link].
  - Despite decades of intense study, no efficient algorithm for a Hamilton tour exists. Best algorithms are exponential time.

- Graph problems are among the most mathematically rich areas of CS theory

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- **Topological Sort**
- Traversals
  - Breadth-first
  - Depth-first
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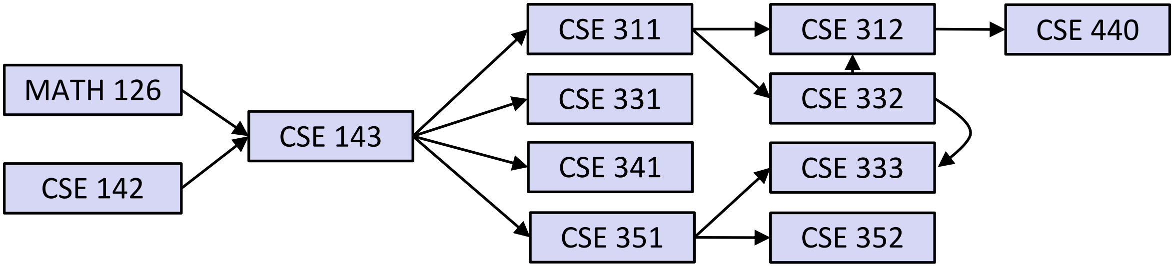
Topological Sort

Disclaimer: Do not use for official advising purposes!
Falsely implies CSE 332 is a prereq for CSE 312, etc.

- Given a DAG, output all the vertices in an order such that no vertex appears before any other vertex that has a path to it

- Example input:



- Example output:
  - 126, 142, 143, 311, 331, 332, 312, 341, 351, 333, 352, 440

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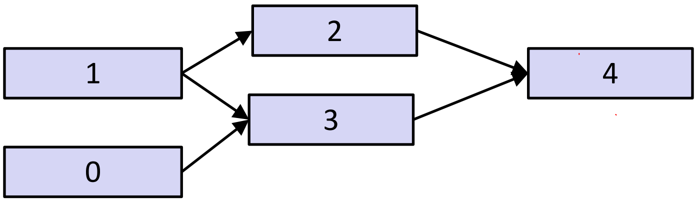
- List 3 valid Topological sorts:

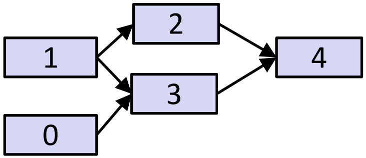


- Why do we perform topological sorts only on DAGs?
  - *A cycle means there is no correct answer*
- Does a DAG always have a unique answer?
  - *No; there can be 1 or more answers, depending on the graph*
- What DAGs have exactly 1 answer?
  - *A list*
- *Terminology*: A DAG represents a ***partial order***, and a topological sort produces a ***total order*** that is consistent with it

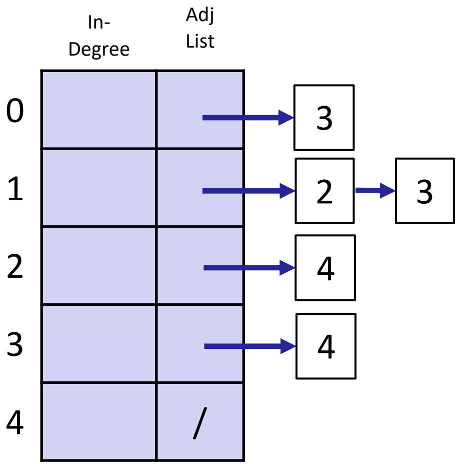
Topological Sort: Applications

- Figuring out how to finish your degree
- Determining the order for recomputing spreadsheet cells
- Computing the order to compile files using a Makefile
- Scheduling jobs in a big data pipeline
- *Often: finding an order of execution for a dependency graph*

TopoSort: A Naïve Algorithm



1. Label ("mark") each vertex with its in-degree
   - Could write directly into a vertex's field or a parallel data structure (e.g., array)
2. While there are vertices not yet output:
   - Choose a vertex v with labeled with in-degree of 0
   - Output v and conceptually remove it from the graph
   - Foreach vertex w adjacent to v:
     - Decrement the in-degree of w

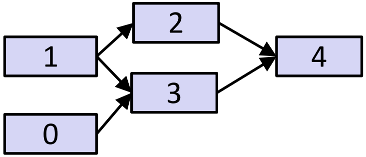


TopoSort: Notes

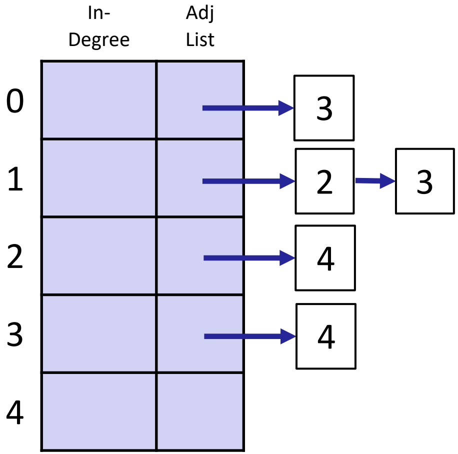
- Needed a vertex with in-degree of 0 to start
  - Remember: graph must be acyclic!

- If >1 vertex with in-degree=0, can break ties arbitrarily
  - Potentially many different correct orders!

Naïve TopoSort: Running Time?




```
labelEachVertexWithItsInDegree();
for (i=0; i < numVertices; i++){
  v = findNewVertexOfDegreeZero();
  put v next in output
  for each w adjacent to v
    w.indegree--;
}
```

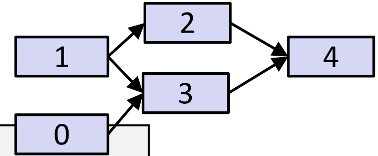


TopoSort's Runtime: Doing Better

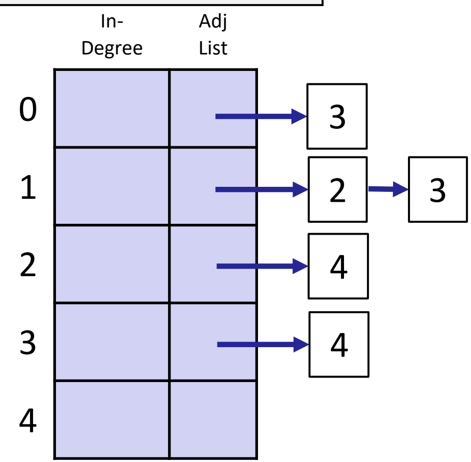
- Avoid searching for a zero-degree node every time!
  - Keep the "pending" 0-degree nodes in a list, stack, queue, table, etc
  - The irder we process them affects output, but not correctness or efficiency (*as long as add/remove are both O(1)*)

- Using a queue:
  - Label each vertex with its in-degree, enqueuing 0-degree nodes
  - While "pending" queue is not empty:
    - v = dequeue()
    - Output v and remove it from the graph
    - For each vertex w adjacent to v (i.e. w such that (v,w) in E):
      - decrement the in-degree of w
      - if new degree is 0, enqueue it

Better TopoSort: Running Time?




```
labelAllAndEnqueueZeros();
for (i=0; i < numVertices; i++){
    v = dequeue();
    put v next in output
    for each w adjacent to v
      w.indegree--;
      if (w.indegree == 0)
        enqueue(w);
}
```



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- You've seen a graph traversal before in 143. List all three.

Tree and Graph Reachability

- Find all nodes ***reachable*** from a starting node v
  - ie, there exists a path
  - Might "do something" at each visited node (an iterator!)
    - "Do something" is called ***visiting*** or ***processing*** a node
      - eg, print to output, set some field, etc.
    - ***Traversing a node*** or ***iterating over a node*** is different!
      - Just fetch adjacent/child nodes

- Related Questions:
  - Is an undirected graph connected?
  - Is a directed graph weakly / strongly connected?
    - For strongly, need a cycle back to starting node

Tree and Graph Traversals

- Can answer reachability with a ***tree traversal*** or ***graph traversal***
  - Iterates over every node in a graph in some defined ordering
  - "Processes" or "visits" its contents

- There are several types of tree traversals
  - Level Order Traversal aka Breadth-First Traversal
  - Depth-First Traversal
    - Pre-order Traversal
    - In-order Traversal
    - Post-order Traversal

Tree/Graph Traversal: High-level Algorithm

- High-level Algorithm:
  - Initialization:
    - Create an empty data structure (often called a "fringe") to track "remaining work"
    - Mark start as visited
  - While we still have work, follow the nodes:
    - Get a node
    - Visit/process that node
    - Update its neighbors (eg, add to "remaining work" if it's not already there)
- ***Memorize this 5-step pattern!***

```
traverseGraph(Node start) {
  pending = emptyFringe()
  pending.add(start)
  mark start as visited

  while (!pending.empty()) {
    next = pending.remove()
    process(next) //marks visited
    foreach u adjacent to next
      if (!u.marked)
        mark u
        pending.add(u)
}
```

Tree/Graph Traversal: Running Time

- Assuming add() and remove() are $O(1)$, traversal is $O(|E|)$
  - Remember: we default to using an adjacency list

Tree/Graph Traversal: Order

- The order we process() depends *entirely* on how pending.add() and pending.remove() are implemented
  - Queue:
    - Tree: Level-order traversal
    - Graph: Breadth-first graph search (BFS)
  - Stack:
    - Tree: Depth-first search (3 flavors!)
    - Graph: Depth-first graph search (DFS)

- DFS and BFS are "big ideas" in computer science
  - Depth: explore one part before exploring other unexplored parts
  - Breadth: explore parts closer to the start before exploring farther parts

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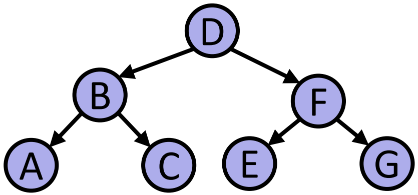
Graphs: Breadth-First Search

- The fringe here is a Queue!

```
BFS(Node start) {
  q.enqueue(start)
  mark start as visited

  while (!q.empty())
    next = q.dequeue()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.enqueue(u)
}
```

Trees: Level-Order



- Process top-to-bottom, left-to-right
  - Like reading in English
  - Goes “broad” instead of “deep”

- Resembles how we converted our binary heap (ie, a complete tree) to its array representation

Breadth-First Search on a Graph

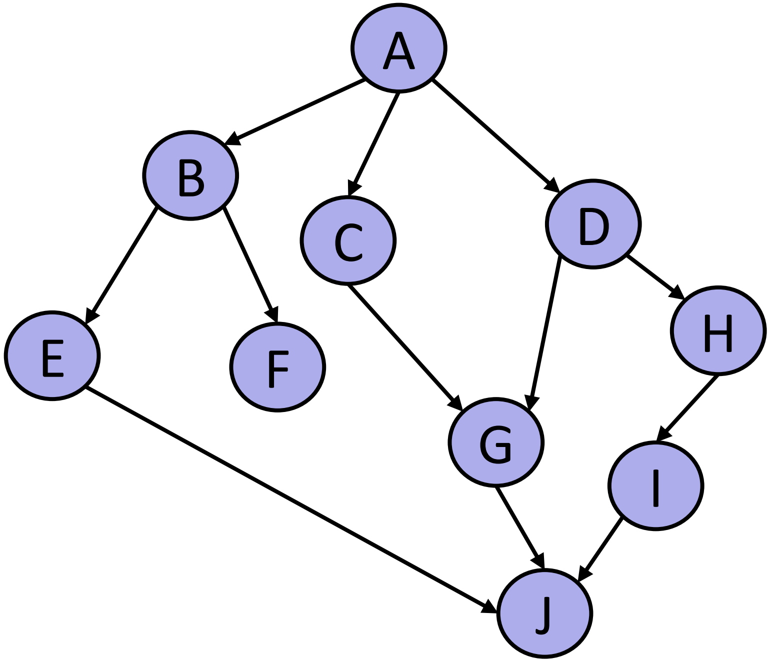
Queue:

Marked:

Order Processed:

```
BFS(Node start) {
  q.enqueue(start)
  mark start as visited

  while (!q.empty())
    next = q.dequeue()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.enqueue(u)
}
```



Breadth-First Search on a Graph

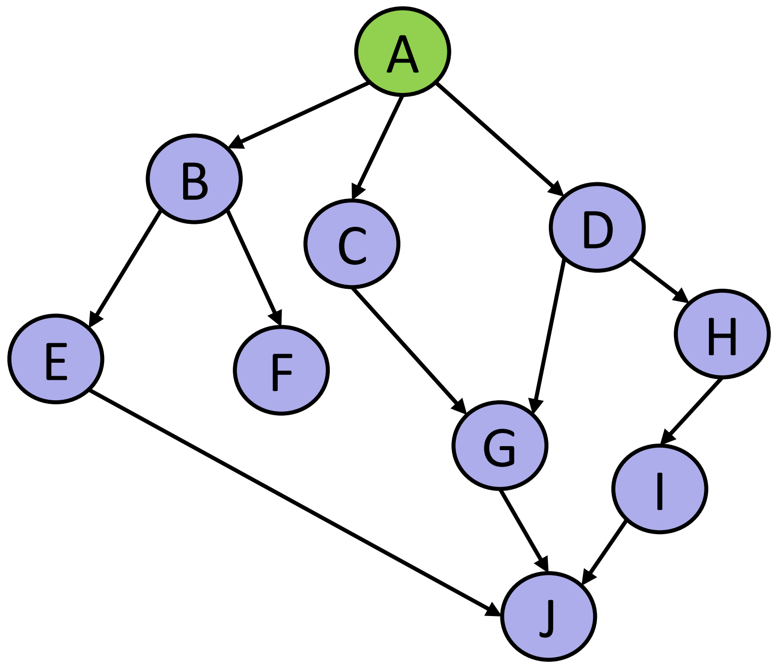
Queue:
A

Marked:
A

Order Processed:

```
BFS(Node start) {
  q.enqueue(start)
  mark start as visited

  while (!q.empty())
    next = q.dequeue()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.enqueue(u)
}
```



Breadth-First Search on a Graph

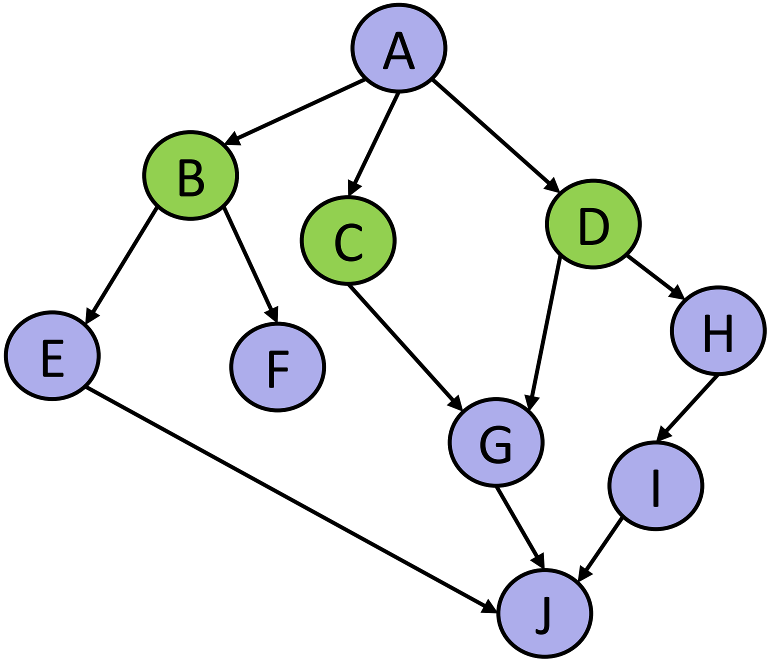
Queue:
B, C, D

Marked:
A, B, C, D

Order Processed:
A

```
BFS(Node start) {
  q.enqueue(start)
  mark start as visited

  while (!q.empty())
    next = q.dequeue()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.enqueue(u)
}
```



Breadth-First Search on a Graph

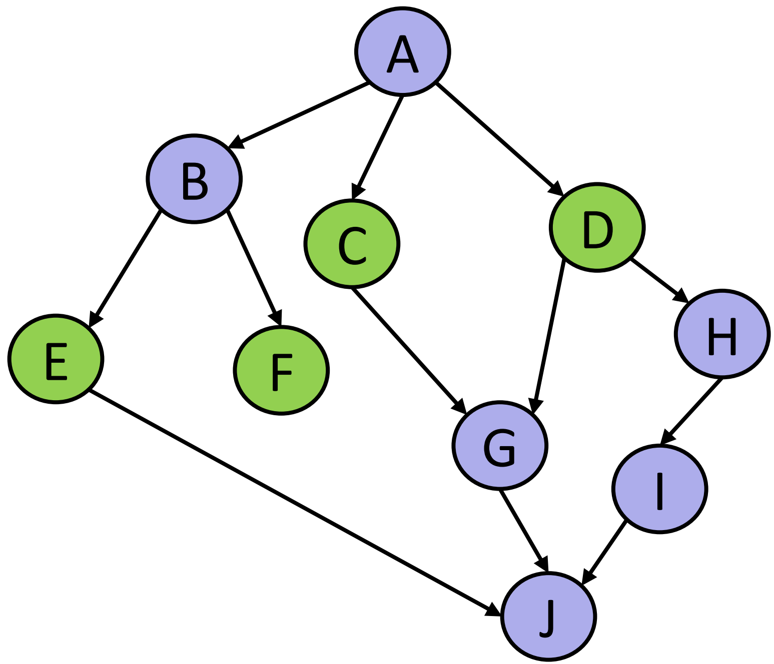
Queue:
C, D, E, F

Marked:
A, B, C, D, E, F

Order Processed:
A, B

```
BFS(Node start) {
  q.enqueue(start)
  mark start as visited

  while (!q.empty())
    next = q.dequeue()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.enqueue(u)
}
```



Breadth-First Search on a Graph

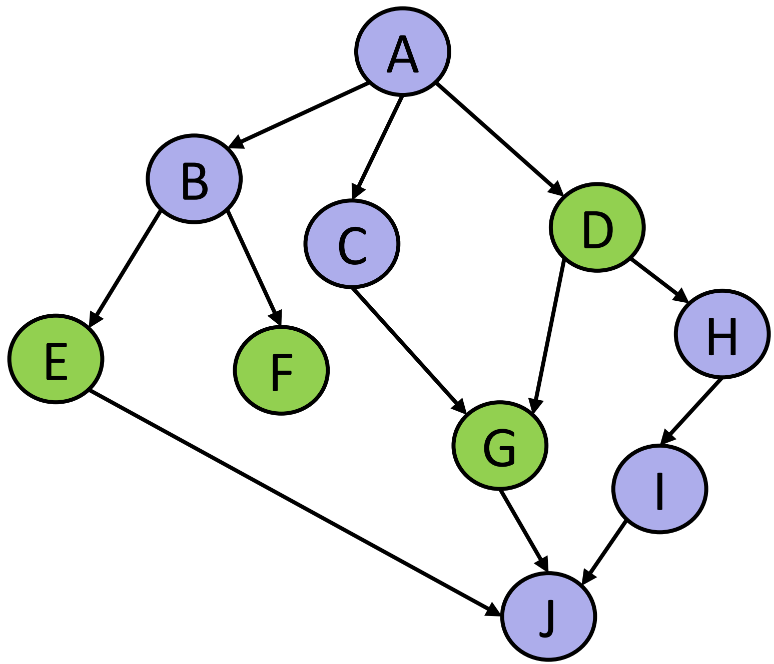
Queue:
D, E, F, G

Marked:
A, B, C, D, E, F, G

Order Processed:
A, B, C

```
BFS(Node start) {
  q.enqueue(start)
  mark start as visited

  while (!q.empty())
    next = q.dequeue()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.enqueue(u)
}
```



Breadth-First Search on a Graph

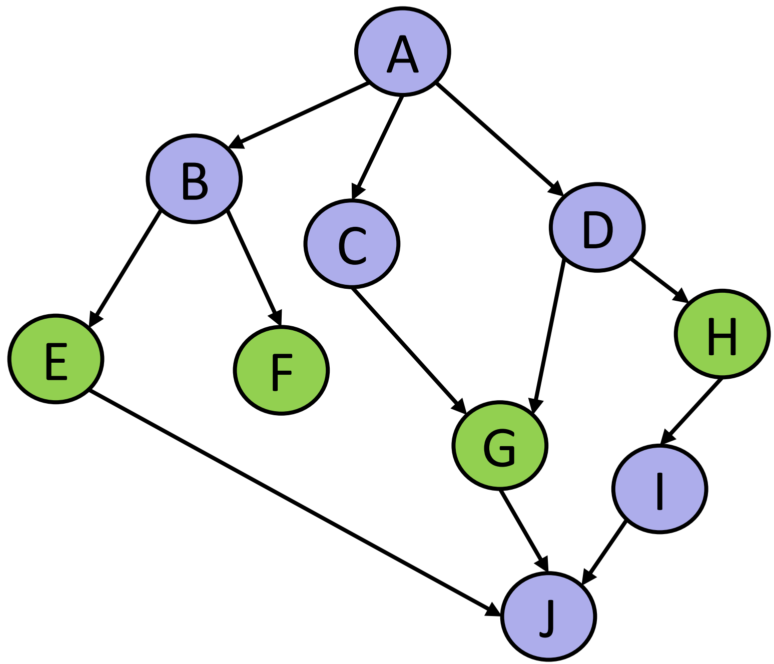
Queue:
E, F, G, H

Marked:
A, B, C, D, E, F, G, H

Order Processed:
A, B, C, D

```
BFS(Node start) {
  q.enqueue(start)
  mark start as visited

  while (!q.empty())
    next = q.dequeue()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.enqueue(u)
}
```



Breadth-First Search on a Graph

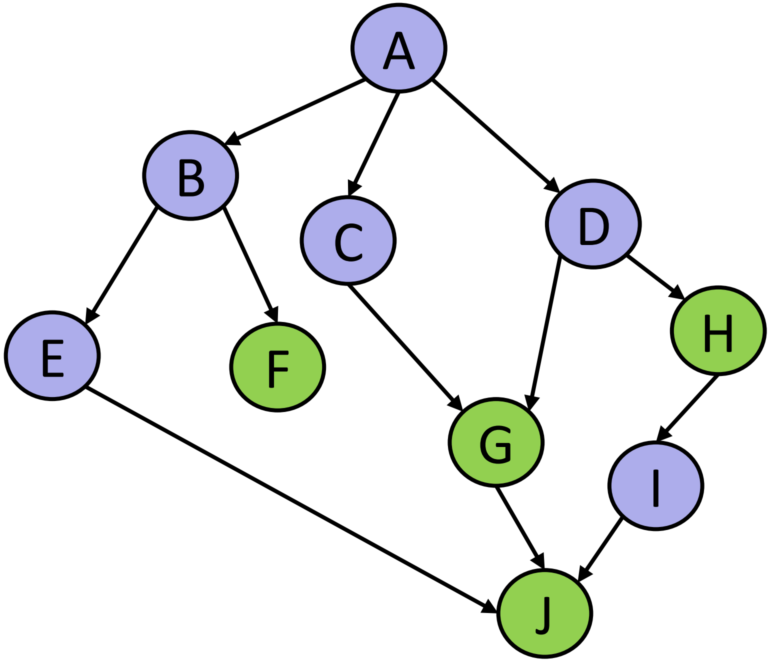
Queue:
F, G, H, J

Marked:
A, B, C, D, E, F, G, H, J

Order Processed:
A, B, C, D, E

```
BFS(Node start) {
  q.enqueue(start)
  mark start as visited

  while (!q.empty())
    next = q.dequeue()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.enqueue(u)
}
```



Breadth-First Search on a Graph

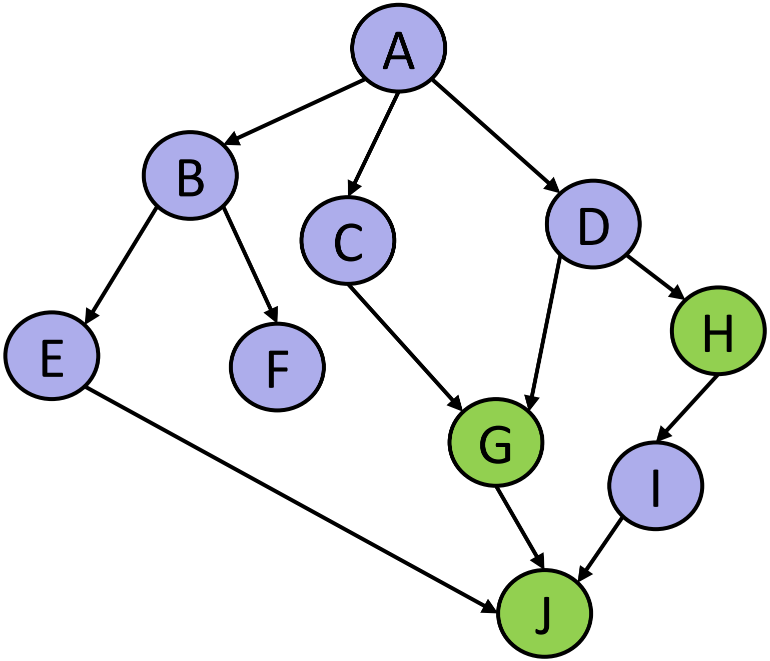
Queue:
G, H, J

Marked:
A, B, C, D, E, F, G, H, J

Order Processed:
A, B, C, D, E, F

```
BFS(Node start) {
  q.enqueue(start)
  mark start as visited

  while (!q.empty())
    next = q.dequeue()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.enqueue(u)
}
```



Breadth-First Search on a Graph

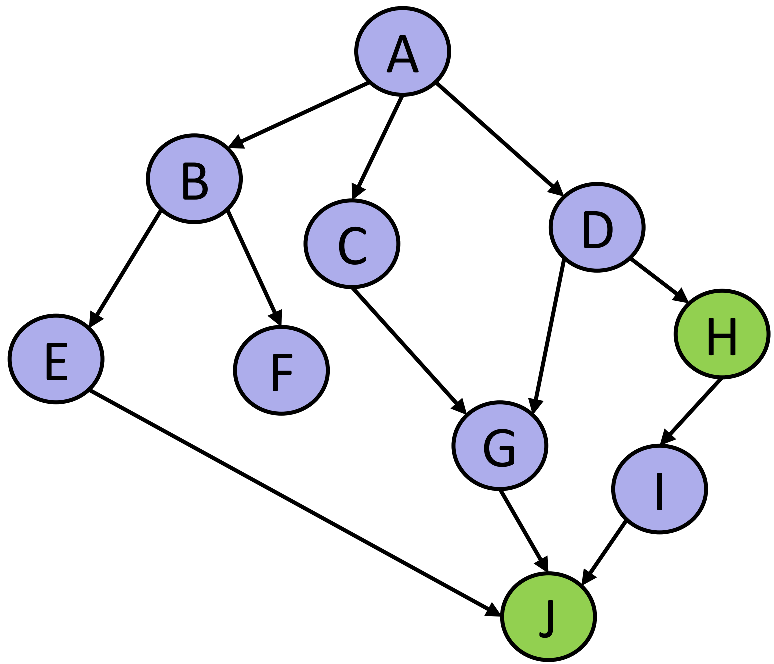
Queue:
H, J

Marked:
A, B, C, D, E, F, G, H, J

Order Processed:
A, B, C, D, E, F, G

```
BFS(Node start) {
  q.enqueue(start)
  mark start as visited

  while (!q.empty())
    next = q.dequeue()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.enqueue(u)
}
```



Breadth-First Search on a Graph

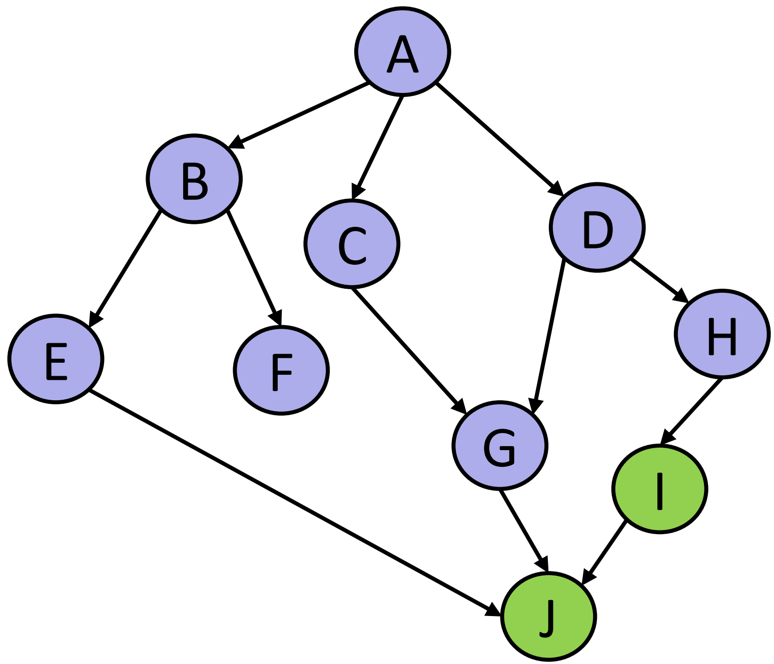
Queue:
J, I

Marked:
A, B, C, D, E, F, G, H, J, I

Order Processed:
A, B, C, D, E, F, G, H

```
BFS(Node start) {
  q.enqueue(start)
  mark start as visited

  while (!q.empty())
    next = q.dequeue()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.enqueue(u)
}
```



Breadth-First Search on a Graph

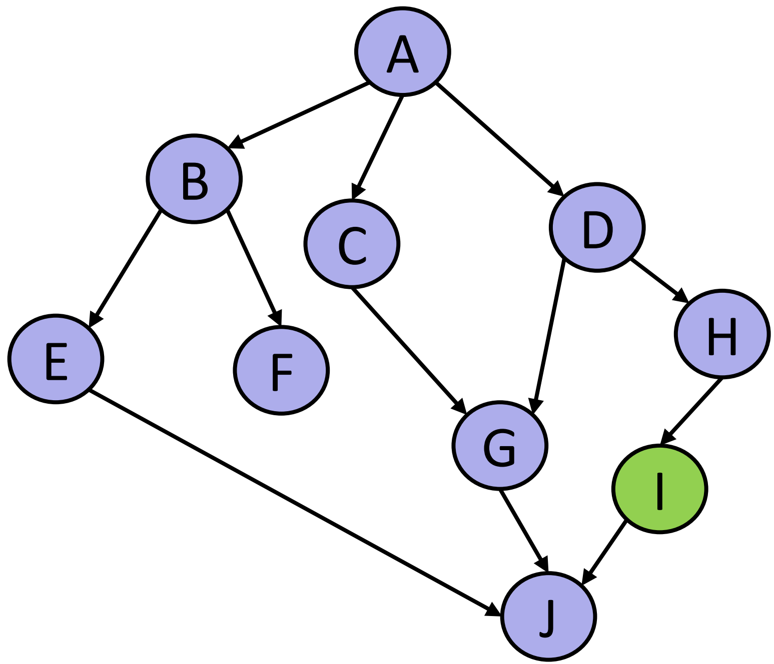
Queue:
I

Marked:
A, B, C, D, E, F, G, H, J, I

Order Processed:
A, B, C, D, E, F, G, H, J

```
BFS(Node start) {
  q.enqueue(start)
  mark start as visited

  while (!q.empty())
    next = q.dequeue()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.enqueue(u)
}
```



Breadth-First Search on a Graph

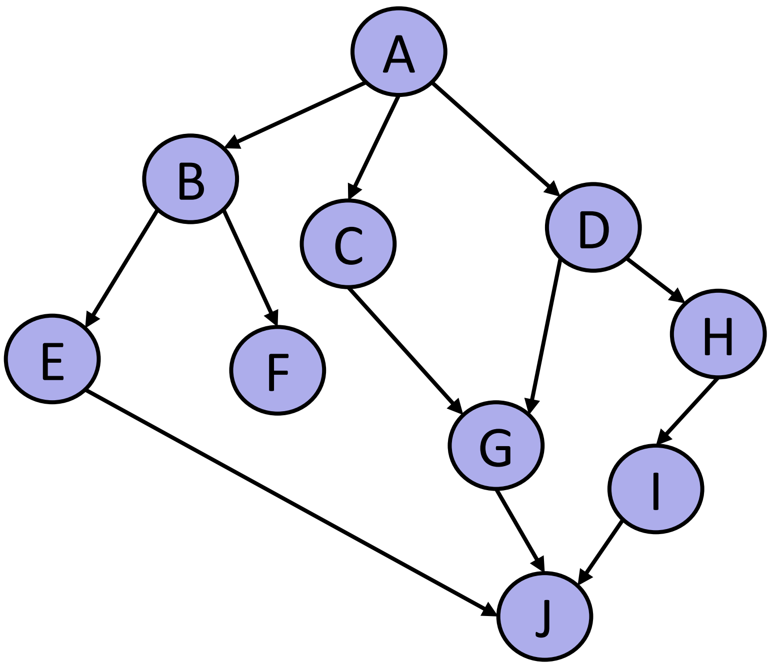
Queue:

Marked:
A, B, C, D, E, F, G, H, J, I

Order Processed:
A, B, C, D, E, F, G, H, J, I

```
BFS(Node start) {
  q.enqueue(start)
  mark start as visited

  while (!q.empty())
    next = q.dequeue()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.enqueue(u)
}
```



Lecture Outline

- Graph Representations
  - Adjacency Matrix
  - Adjacency List
- Topological Sort
- Traversals
  - Breadth-first
  - **Depth-first**
  - Conclusion

Graphs: Depth-First Search

- The fringe here is a Stack!
- Note: many algorithms that use a stack have an Iterative and a Recursive solution…

```
DFSIterative(Node start) {
  s.push(start)
  mark start as visited

  while (!s.empty())
    next = s.pop()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.push(u)
}
```

Depth-First Search on a Graph

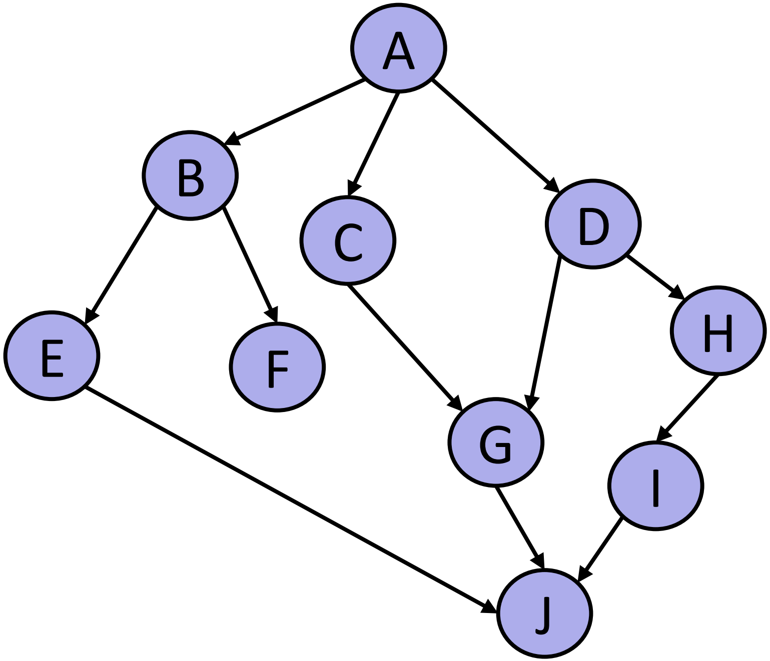
Stack:

Marked:

Order Processed:

```
DFSIterative(Node start) {
  s.push(start)
  mark start as visited

  while (!s.empty())
    next = s.pop()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.push(u)
}
```

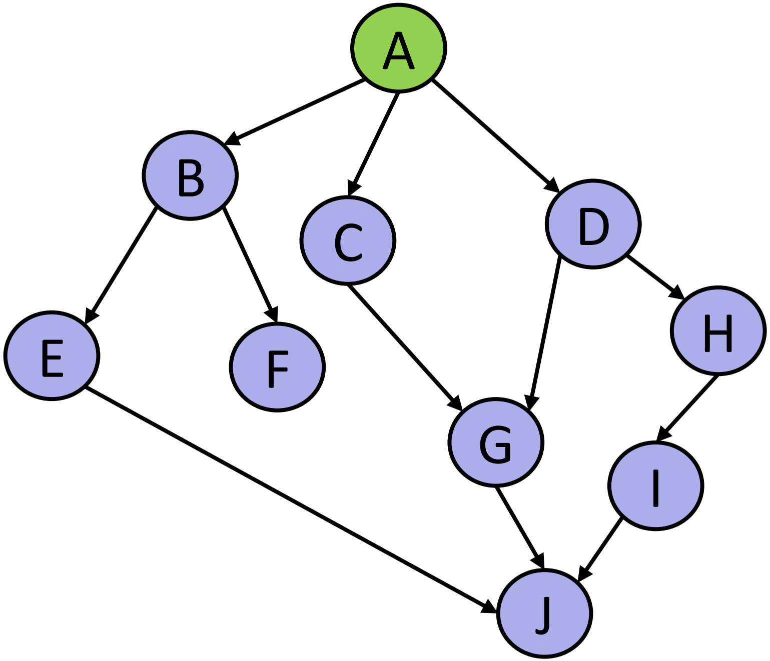


Depth-First Search on a Graph

Stack:
A

Marked:
A

Order Processed:




```
DFSIterative(Node start) {
  s.push(start)
  mark start as visited

  while (!s.empty())
    next = s.pop()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.push(u)
}
```

Depth-First Search on a Graph

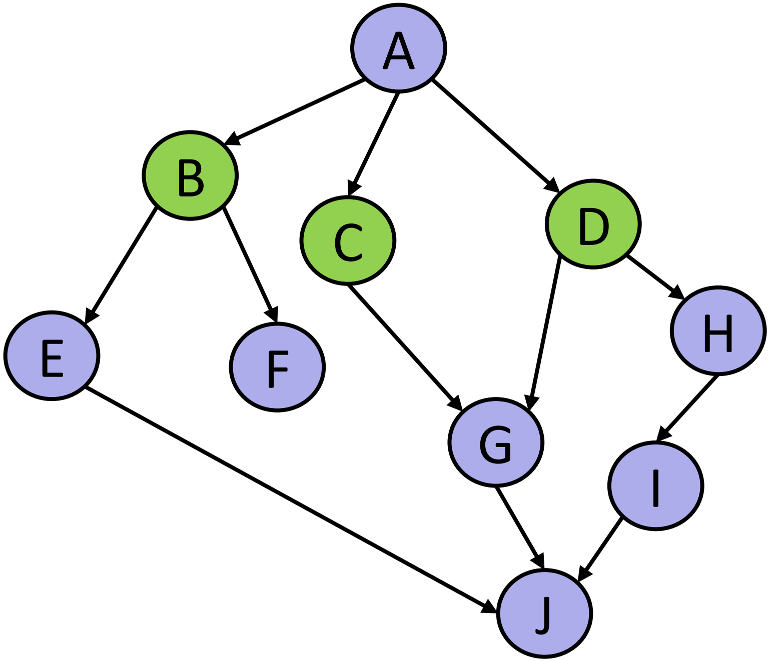
Stack:
B, C, D

Marked:
A, B, C, D

Order Processed:
A

```
DFSIterative(Node start) {
  s.push(start)
  mark start as visited

  while (!s.empty())
    next = s.pop()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.push(u)
}
```



Depth-First Search on a Graph

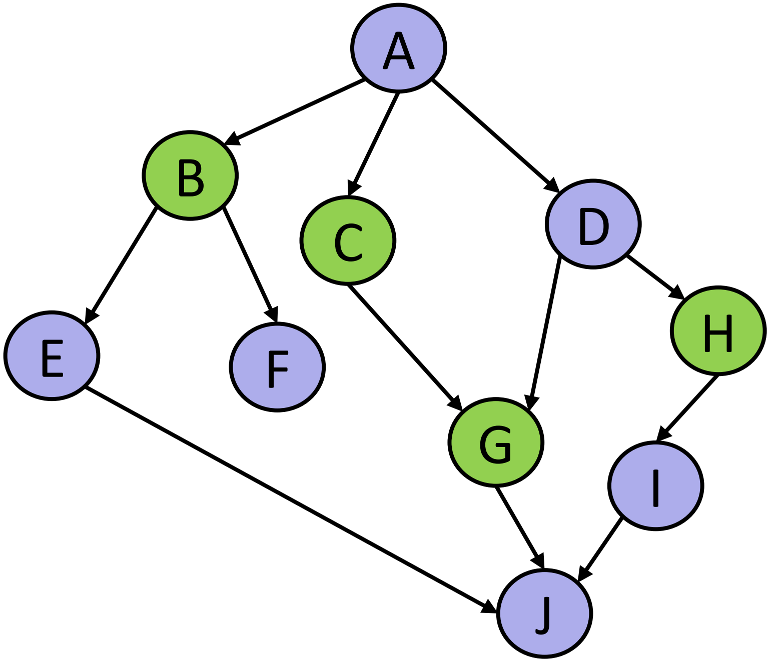
Stack:
B, C, G, H

Marked:
A, B, C, D, G, H

Order Processed:
A, D

```
DFSIterative(Node start) {
  s.push(start)
  mark start as visited

  while (!s.empty())
    next = s.pop()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.push(u)
}
```



Depth-First Search on a Graph

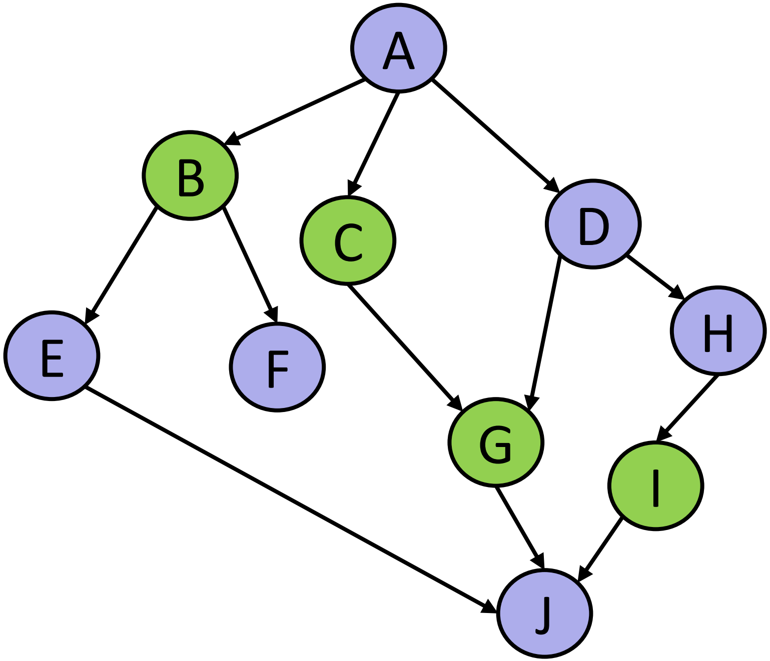
Stack:
B, C, G, I

Marked:
A, B, C, D, G, H, I

Order Processed:
A, D, H

```
DFSIterative(Node start) {
  s.push(start)
  mark start as visited

  while (!s.empty())
    next = s.pop()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.push(u)
}
```



Depth-First Search on a Graph

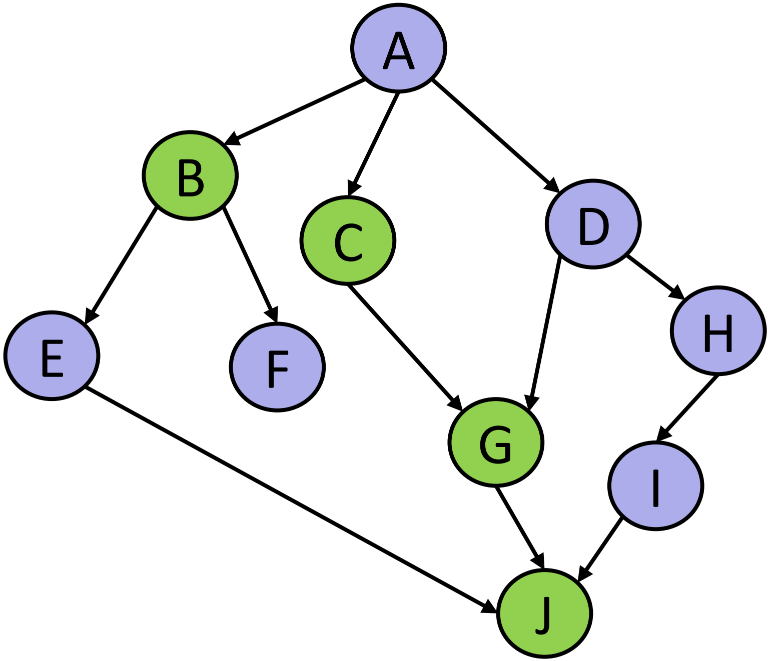
Stack:
B, C, G, J

Marked:
A, B, C, D, G, H, I, J

Order Processed:
A, D, H, I

```
DFSIterative(Node start) {
  s.push(start)
  mark start as visited

  while (!s.empty())
    next = s.pop()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.push(u)
}
```



Depth-First Search on a Graph

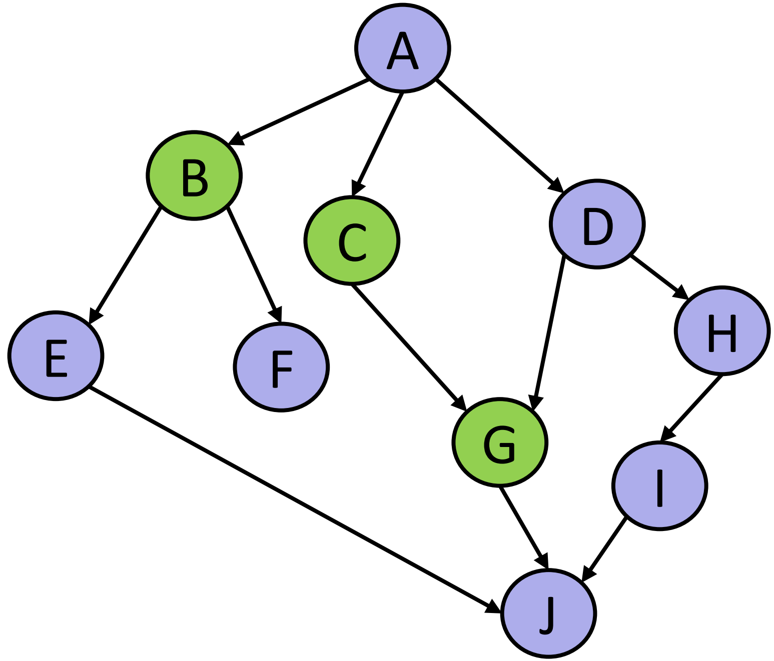
Stack:
B, C, G

Marked:
A, B, C, D, G, H, I, J

Order Processed:
A, D, H, I, J

```
DFSIterative(Node start) {
  s.push(start)
  mark start as visited

  while (!s.empty())
    next = s.pop()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.push(u)
}
```



Depth-First Search on a Graph

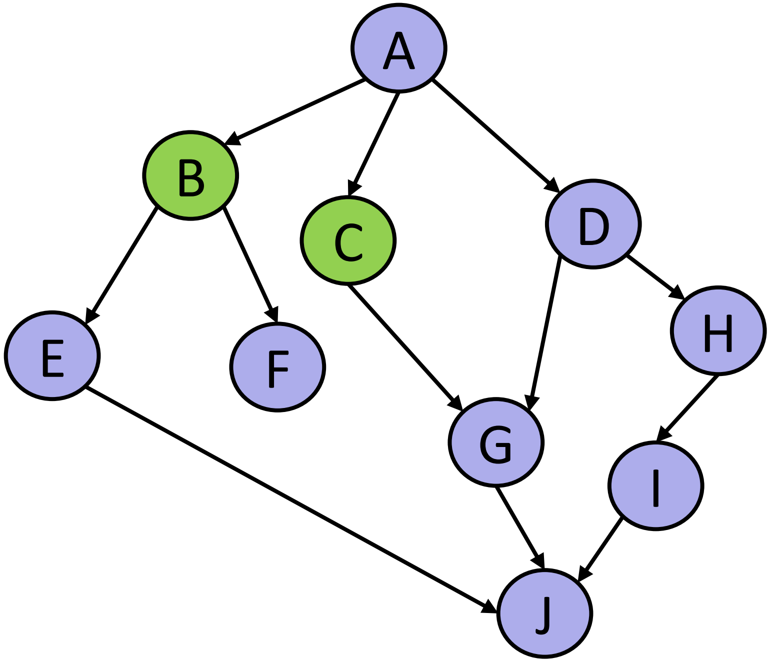
Stack:
B, C,

Marked:
A, B, C, D, G, H, I, J

Order Processed:
A, D, H, I, J, G

```
DFSIterative(Node start) {
  s.push(start)
  mark start as visited

  while (!s.empty())
    next = s.pop()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.push(u)
}
```



Depth-First Search on a Graph

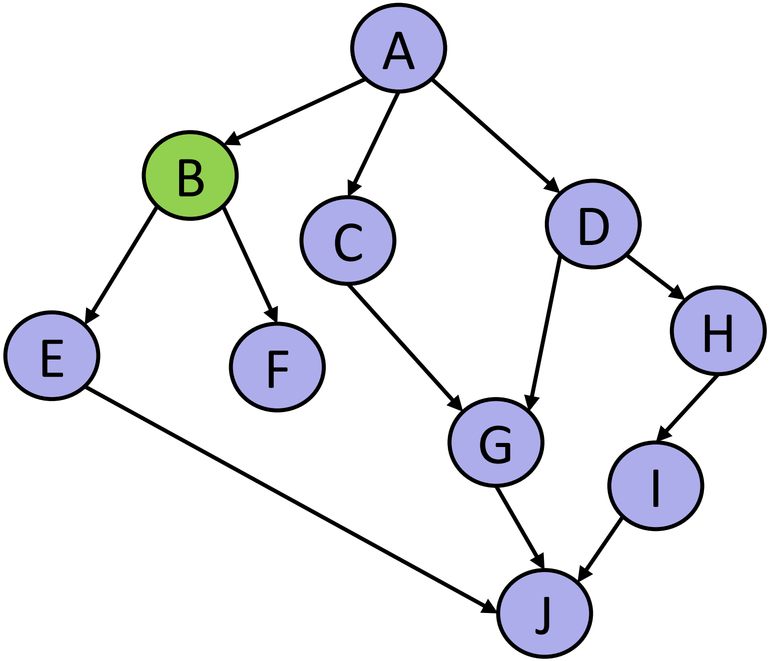
Stack:
B,

Marked:
A, B, C, D, G, H, I, J

Order Processed:
A, D, H, I, J, G, C

```
DFSIterative(Node start) {
  s.push(start)
  mark start as visited

  while (!s.empty())
    next = s.pop()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.push(u)
}
```



Depth-First Search on a Graph

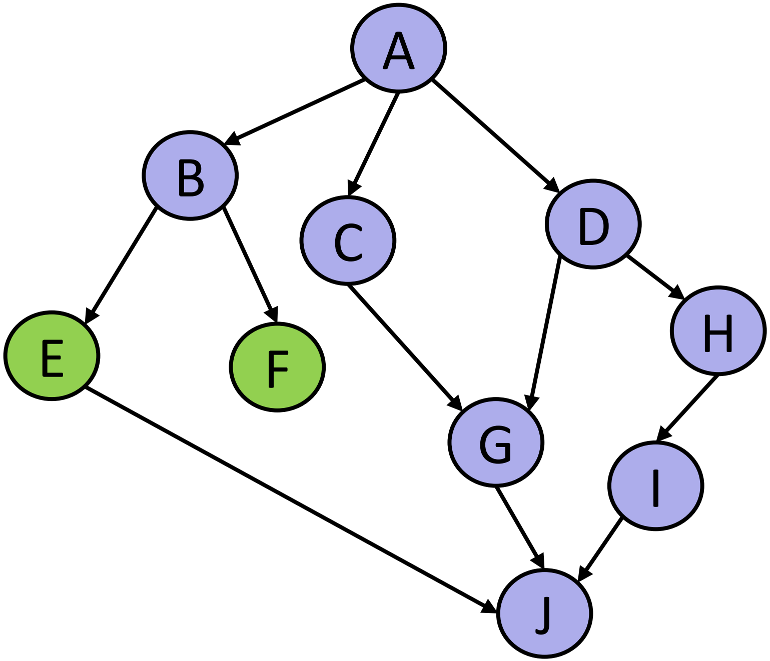
Stack:
E, F

Marked:
A, B, C, D, G, H, I, J, E, F

Order Processed:
A, D, H, I, J, G, C, B

```
DFSIterative(Node start) {
  s.push(start)
  mark start as visited

  while (!s.empty())
    next = s.pop()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.push(u)
}
```



Depth-First Search on a Graph

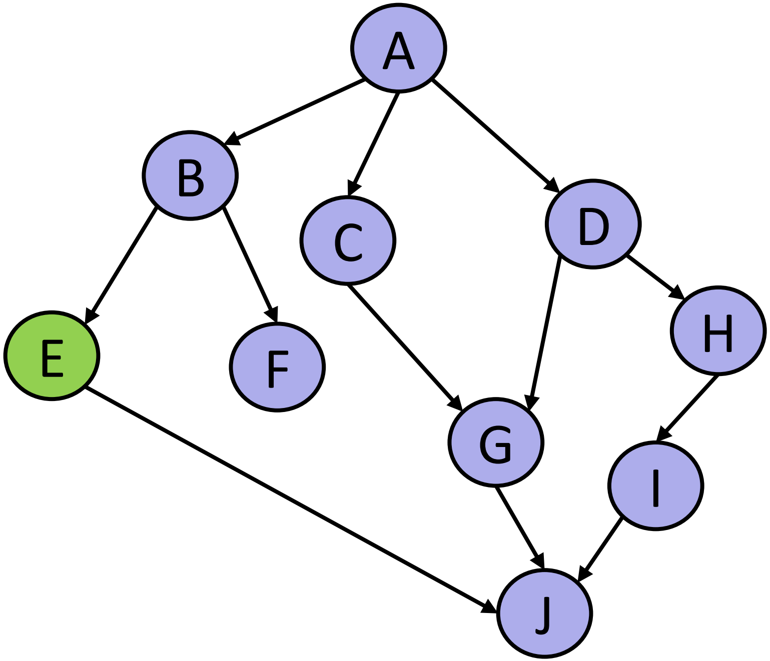
Stack:
E

Marked:
A, B, C, D, G, H, I, J, E, F

Order Processed:
A, D, H, I, J, G, C, B, F

```
DFSIterative(Node start) {
  s.push(start)
  mark start as visited

  while (!s.empty())
    next = s.pop()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.push(u)
}
```



Depth-First Search on a Graph

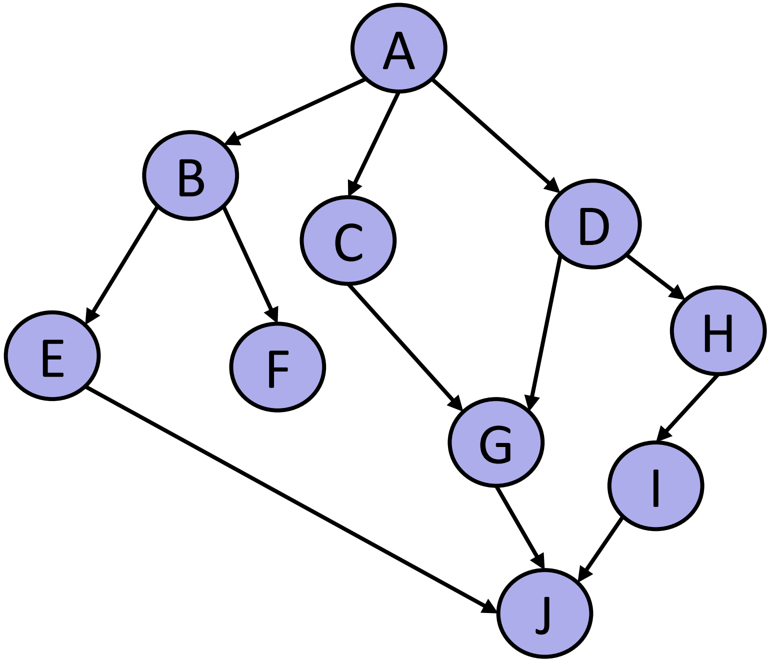
Stack:

Marked:
A, B, C, D, G, H, I, J, E, F

Order Processed:
A, D, H, I, J, G, C, B, F

```
DFSIterative(Node start) {
  s.push(start)
  mark start as visited

  while (!s.empty())
    next = s.pop()
    process(next)
    foreach u adjacent to next
      if (!u.marked)
        mark u
        q.push(u)
}
```



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gradescope.com/courses/275833

- Were the Pre/In/Post-Order Traversals from 143 examples of BFS or DFS?

Lecture Outline

- Graph Representations
  - Adjacency Matrix
  - Adjacency List
- Topological Sort
- Traversals
  - Breadth-first
  - Depth-first
  - **Conclusion**

Saving the Path

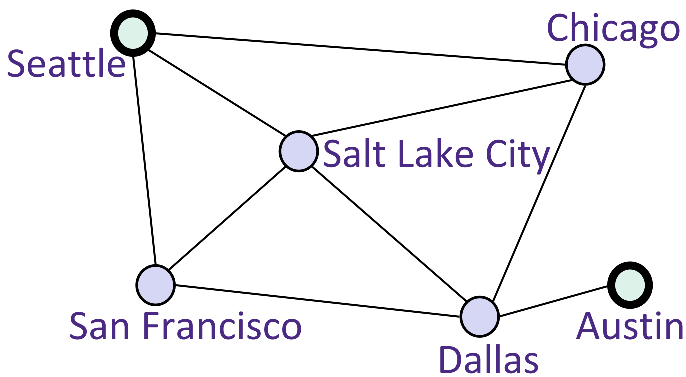
- These graph traversals can answer the "reachability question":
  - "*Is there* a path from node x to node y?"

- But what if we want to *output the actual path* or its length?
  - Eg, getting driving directions vs knowing it's possible to get there

- Modifications:
  - Instead of just "marking" a node, store the path's *previous node*
    - ie: when processing u, if we add v to the "remaining work" set `v.prev` to `u`
  - When you reach the goal, follow `prev` fields backwards to start
    - (don't forget to reverse the answer)
  - Path length:
    - Same idea, but also store integer distance at each node

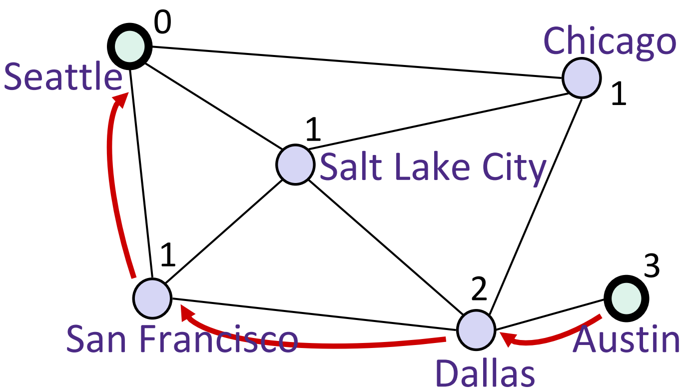
Saving the Path: Example using BFS (1 of 2)

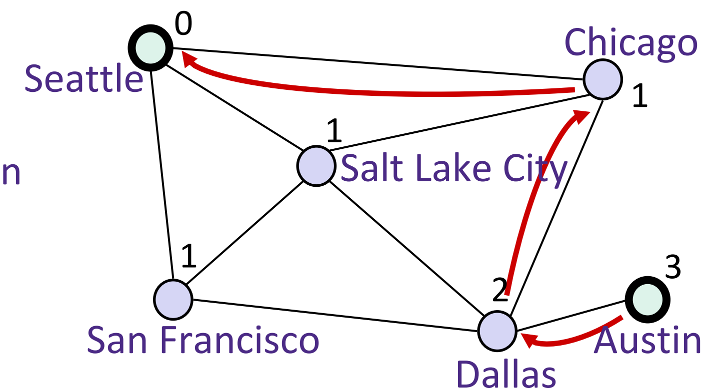
- Find the shortest path from Seattle to Austin
  - Remember marked nodes are not re-enqueued
  - Shortest paths may not be unique



Saving the Path: Example using BFS (2 of 2)

- Find the shortest path from Seattle to Austin
  - Remember marked nodes are not re-enqueued
  - Shortest paths may not be unique





DFS/BFS Comparison

- Breadth-first search:
  - Always finds shortest paths, i.e., finds "optimal solutions"
    - Better for "what is the shortest path from x to y?"
  - But queue may hold up to O(|V|) nodes
    - Eg, at the bottom level of perfect binary tree, queue contains |V|/2 nodes

- Depth-first search:
  - Can use less space when finding a path
    - If longest path in the graph is p and highest out-degree is d then stack never has more than d*p elements

It Doesn't Have to be Either/Or

- A third approach: Iterative deepening (IDDFS):
  - Try DFS, but don't allow recursion more than K levels deep
  - If fails to find a solution, increment K and start the entire search over

- Like BFS, finds shortest paths. Like DFS, less space

Summary

- Two very different "standard" graph representations
  - Must understand tradeoffs to choose between adj list and adj matrix
- TopoSort finds a total ordering in a DAG representing a partial ordering
  - Runtime for TopoSort was dependent on graph representation and a helper data structure!
- We can traverse both trees and graphs
  - Depth-first-style tree traversals have 3 flavors (named by when the processing happens)
  - Breadth-first-style tree traversals are called "level-order"
  - Graphs can have "pre-" and "post-" style traversals, but ordering is less important than in trees