Valid Partitions

Given the array \([4, 5, 1, 3, 7, 2, 6]\), which arrays are valid partitions on the pivot 4?

- \([4, 5, 1, 3, 7, 2, 6]\)
- \([1, 2, 3, 4, 5, 6, 7]\)
- \([1, 7, 5, 3, 4, 2, 6]\)
- \([1, 3, 4, 2, 5, 7, 6]\)
- \([1, 3, 4, 2, 5, 6, 7]\)
- \([2, 1, 3, 4, 5, 7, 6]\)
- \([3, 2, 1, 4, 7, 5, 7]\)

Hoare Partitioning

Given the array \([4, 5, 1, 3, 7, 2, 6]\) and pivot 6, give the array after the first swap. Then, give the array after the entire partition operation is complete.

- Q1: Given the array \([4, 5, 1, 3, 7, 2, 6]\) and pivot 6, give the array after the first swap.
- Q2: Then, give the array after the entire partition operation is complete.
- Q3: Is Hoare partitioning stable?
### Selection Algorithms

**Selection.** Given an array of \( N \) items, find item of rank \( K \).

For finding the median, choose \( K = N / 2 \).

Characterize the difficulty of this problem.

- Why is the time complexity of selection in \( \Omega(N) \)?
- Describe an \( O(N \log N) \) runtime algorithm for selection for any \( K \).
- Describe an \( O(N) \) runtime algorithm for selection for \( K = 0, 1, 2 \).

### Partition Sort

Consider a generalization of quicksort: PartitionSort(input, lo, hi, subsort).

1. If \( hi - lo \leq 1 \), return.
2. Partition around input[lo], the leftmost item of the current subproblem.
3. Call subsort on the left and right subproblems.

Give the worst-case runtime for each call below.

- PartitionSort(input, 0, N, InsertionSort)
- PartitionSort(input, 0, N, MergeSort)
- PartitionSort(input, 0, N, PartitionSort)

### Questions

**Q1:** Why is the time complexity of selection in \( \Omega(N) \)?

**Q2:** Describe an \( O(N \log N) \) runtime algorithm for selection for any \( K \).

**Q3:** Describe an \( O(N) \) runtime algorithm for selection for \( K = 0, 1, 2 \).