DAG Shortest Paths

Given a weighted DAG (possibly negative edge weights), find the single-source shortest paths tree from s to every other vertex in the graph.
Your algorithm should be faster than Dijkstra's algorithm.

Positive Integer-Weighted Shortest Paths

Given a weighted, directed graph (possibly cyclic) with positive integer edge weights, find the single-source shortest paths tree from s to every other vertex in the graph.
Your algorithm should be faster than Dijkstra's algorithm on cyclic graphs.
Given a weighted DAG (possibly negative edge weights), find the single-source **longest** paths tree from s to every other vertex in the graph.

Give the runtime of your algorithm.

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How would you model vertex weights for shortest paths problems?