Q Overall Asymptotic Runtime Bound for dup1

$$R_{\text{best}}(N) = 2$$

$$R_{\text{worst}}(N) = \frac{N^2 + 3N + 2}{2}$$

Give an **overall** asymptotic runtime bound for R as a combination of **Θ**, **O**, and/or **Ω** notation. Take into account both the best and the worst case runtimes (R_{best} and R_{worst}).

Then, give a few other valid runtime bounds for R_{best} , R_{worst} , and R using asymptotic notation.

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Q1: Give an overall asymptotic runtime bound for R as a combination of **Θ**, **O**, and/or **Ω** notation. Take into account both the best and the worst case runtimes (R_{best} and R_{worst}).

Q2: Then, give a few other valid runtime bounds for R_{best} , R_{worst} , and R using asymptotic notation.

Q Mystery

Give a tight asymptotic runtime bound for mystery as a function of N, the length of the array, in the best case, worst case, and overall.

```
boolean mystery(int[] a, int target) {
    int N = a.length;
    for (int i = 0; i < N; i += 1)
        if (a[i] == target)
            return true;
    return false;
}
```

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Q1: Give a tight asymptotic runtime bound for mystery as a function of N, the length of the array, in the best case, worst case, and overall.

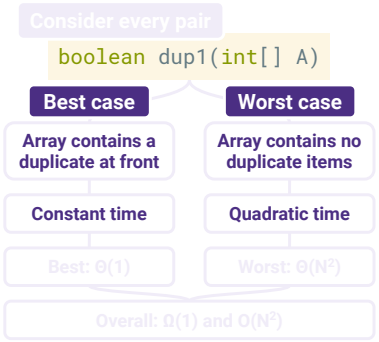
Runtime Analysis Process

Comprehending. Understanding the implementation details of a program.

Modeling. Counting the number of steps in terms of N, the size of the input.

- **Case Analysis**. How certain conditions affect the program execution.
- **Asymptotic Analysis**. Describing what happens for very large N, as N→∞.

Formalizing. Summarizing the final result in precise English or math notation.



Repeat After Me…

There is no magic shortcut for these problems (except in a few well-behaved cases).

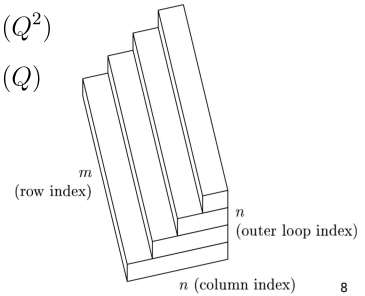
Know these two summations since they're common patterns.

$$1 + 2 + 3 + 4 + \cdots + Q = \frac{Q(Q+1)}{2} \quad \in \Theta(Q^2)$$

$$1 + 2 + 4 + 8 + \cdots + Q = 2Q - 1 \quad \in \Theta(Q)$$

Strategies.

1. Find the exact count of steps.
2. Write out examples.
3. Use a geometric argument–visualizations!



Numerical Linear Algebra (Lloyed N. Trefethen, David Bau, III/SIAM)

Real world programs are often messy and difficult to model.

?: What's different between these two summations?

?: How did we apply these strategies to analyze printParty?

Q Runtime: f0

```
static void f0(int N) {
    if (N < 10000)
        for (int i = 0; i < N * N; i += 1)
            System.out.println("hello");
    else
        System.out.println(N * N * N);
}
```

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?: What happens when N is less than 10000? What happens when N is greater than 10000?

?: What is the asymptotic variable in this problem?

Q1: Give the order of growth of the runtime in Θ notation as a function of N. Your answer should be simple with no unnecessary leading constants or summations.

Q Runtime: f1

```
public static void f1(int N) {
    for (int i = 1; i < N; i *= i)
        System.out.println("hello");
    for (int i = 1; i < N; i *= 2)
        System.out.println("hiya");
    for (int i = 1; i < N; i += 1)
        System.out.println("hi");
}
```

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Q1: Give the order of growth of the runtime in Θ notation as a function of N. Your answer should be simple with no unnecessary leading constants or summations.

Q Runtime: f2

```
static void f2(int N) {
    for (int i = 0; i < N; i += 1) {
        // Math.pow takes constant time
        int big = Math.pow(2, i + 1) - 1;
        for (int j = 0; j < big; j += 1)
            System.out.println("hello");
    }
}
```

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Q1: Give the order of growth of the runtime in Θ notation as a function of N. Your answer should be simple with no unnecessary leading constants or summations.

Q Runtime: f3

```
static void f3(int N) {
    for (int x = 0; x < N; x += 1) {
        int i = N / 2;
        while (i != x)
            if (i > x)
                i -= 1;
            else
                i += 1;
    } }
```

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Q1: Give the order of growth of the runtime in Θ notation as a function of N. Your answer should be simple with no unnecessary leading constants or summations.