Section 09: Solutions

1. Parallel QuickSort

(a) Fill out the table below with each respective recurrence for the best case span and runtime.

Solution:

Quick Sort	Sequential Sort	Parallel Sort
Sequential Partition	$2T(N/2) + O(N)$	$T(N/2) + O(N)$
Parallel Partition	$2T(N/2) + O(logN)$	$T(N/2) + O(logN)$

(b) Which part of the recurrence is related to the Sequential Sort and explain why it is such?
Solution:

$2T(\frac{N}{2})$
Quicksort partitions the input into two equal sized pieces $\frac{N}{2}$, so there are 2 Quicksort calls of $T(\frac{N}{2})$. Since the two calls to Quicksort happen sequentially, the coefficient of the recurrence is 2.

(c) Which part of the recurrence is related to the Parallel Sort and explain why it is such?

Solution:

$T(\frac{N}{2})$
Quicksort partitions the input into two equal sized pieces of $\frac{N}{2}$, so there are 2 Quicksort calls of $T(\frac{N}{2})$. Since the two calls to Quicksort happen in parallel, the coefficient of the recurrence is 1.

(d) Which part of the recurrence is related to the Sequential Partition and explain why it is such?

Solution:

$O(N)$
The partition step requires all N elements to be examined to be properly partitioned using the pivot.

(e) Which part of the recurrence is related to the Parallel Partition and explain why it is such?

Solution:

$O(logN)$ The partition step uses two parallel packs (parallel map + parallel prefix + parallel map) to partition the N elements in parallel. The first pack is to partition the elements less than the pivot and the second pack is to partition the elements greater than the pivot. Both packs are $logN$span.