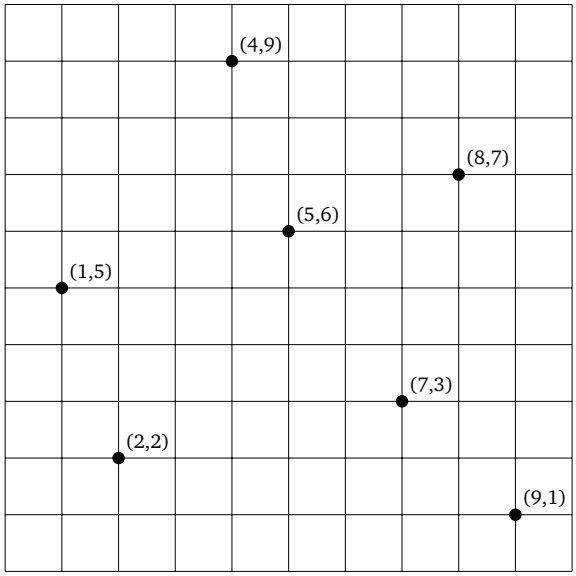
Section 05: k-d Tree, BFS DFS

k-d Tree

(a) Given the points shown in the grid to the right, draw a perfectly balanced k-d tree in the box below. For this tree, first split on the x dimension. The resulting tree should be complete with height 2. Then, draw the corresponding splitting planes on the grid to the right.

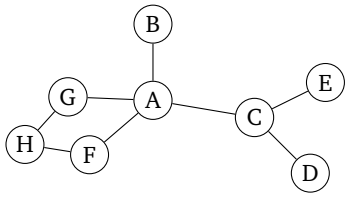


(b) Insert the point (6, 2) into the k-d tree you drew below. Then, add that point to the grid and draw the corresponding splitting plane.

(c) Find the nearest point to (3, 6) in your k-d tree. Mark each branch that is not visited (pruned in execution of `nearest`) with an `X` through the branch.

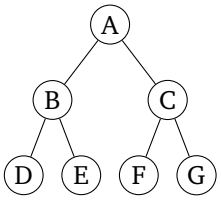
BFS DFS

(a) Consider the following graph. Suppose we want to traverse it, starting at node A.



If we traverse this using *breadth-first search*, what are *two* possible orderings of the nodes we visit? What if we use *depth-first search*?

(b) Same question, but on this graph:



(c) **Creative Question**
Given a maze in the form of the binary rectangular matrix, find length of the shortest path in a maze from given source to given destination.

The path can only be constrcuted out of cells having value 1 and at any given moment, we can only move one step in one of the four directions. The valid moves are:

Go Up: (x, y) -> (x - 1, y)
Go Left: (x, y) -> (x, y - 1)
Go Down: (x, y) -> (x + 1, y)
Go Right: (x, y) -> (x, y + 1)

For example, considering below binary matrix. If source = (0, 0) and destintion = (7, 5), the shortest path from source to destination has length 12.

[	**1**	**1**	**1**	1	1	0	0	1	1	1]
[	0	1	**1**	1	1	1	0	1	0	1]
[	0	0	**1**	0	1	1	1	0	0	1]
[	1	0	**1**	**1**	1	0	1	1	0	1]
[	0	0	0	**1**	0	0	0	1	0	1]
[	1	0	1	**1**	**1**	0	0	1	1	0]
[	0	0	0	0	**1**	0	0	1	0	1]
[	0	1	1	1	**1**	**1**	1	1	0	0]
[	1	1	1	1	1	0	0	1	1	1]
[	0	0	1	0	0	1	1	0	0	1]

Creative Graph Problems

(a) Given a digraph, design an algorithm to find a directed cycle with the minimum number of edges (or report that the graph is acyclic). The running time of your algorithm should be proportional to E V in the worst case.

Application: give a set of patients in need of kidney transplants, where each patient has a family member willing to donate a kidney, but of the wrong type. Willing to donate to another person provided their family member gets a kidney. Then hospital performs a "domino surgery" where all transplants are done simultaneously.

(b) Design a linear-time algorithm to determine whether a DAG has a vertex that is reachable from every other vertex.

(c) Consider the 2-D world on the right, we want to find the **number of islands** in this world. Black cell represents land and white cell represents water. Two black cells are considered to be in the same land if they are adjacent in one of these eight directions: NorthWest, North, NorthEast, East, SouthEast, South, SouthWest, West. Write pseudocode to solve this problem using BFS or DFS. There is no specific starting point. Then, breifly justify the runtime in terms of W, the width of the grid, H, the height of the grid, and k, the number of islands. (Feel free to write a helper function; for your information, the grid below contains 5 islands.)