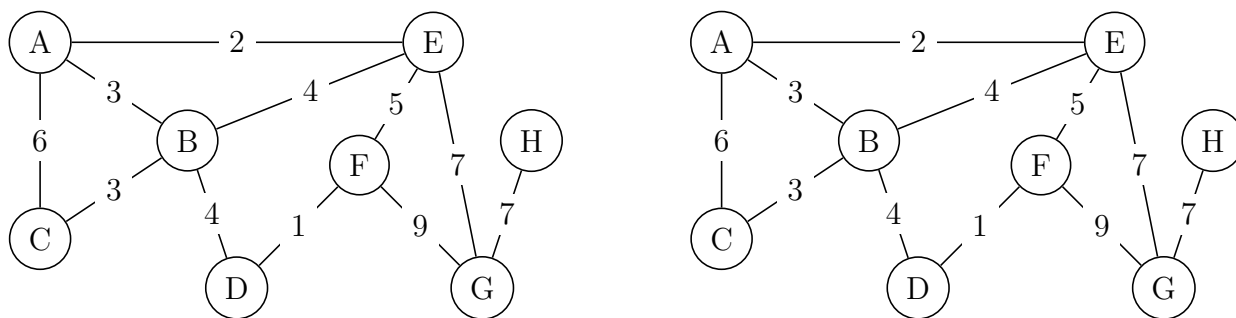


# 1 Minimum Spanning Tree

Consider this undirected graph,  $G$ , with 8 vertices and 11 weighted edges, for problems (a), (b), and (d). Denote each edge with alphabetical overbar notation  $\overline{AB}$ , which represents the edge from  $A$  to  $B$ . For your convenience, the graph is printed twice.



(a) What are the edges on the cut between ABCE and DFGH? You may not need all blanks.

\_\_\_\_\_

(b) What is the minimum edge on the cut between ABE and the rest of  $G$ ? \_\_\_\_\_

(c) **True / False:** Given any undirected graph,  $G$ ,  $G$  will always have a unique MST.

(d) List the edges in the order that they're added to the MST by each algorithm. Assume ties are broken in alphabetical order (i.e. the edge  $\overline{BD}$  would be considered before  $\overline{BE}$ ). You may not need all blanks.

Prim's algorithm from A: \_\_\_\_\_

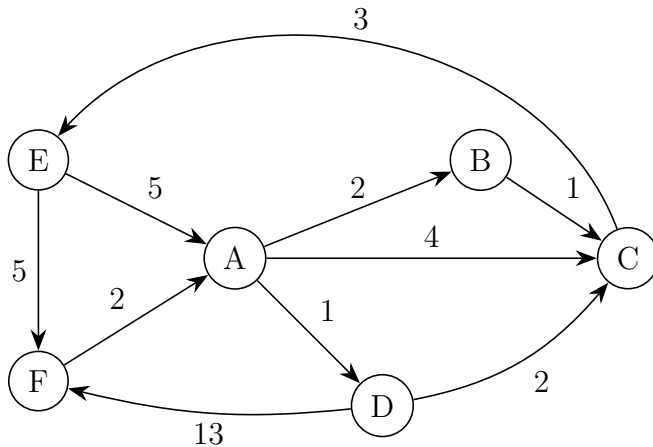
Kruskal's algorithm: \_\_\_\_\_

(e) We are trying to find the MST of a graph where the edge weights only range between 0 and 255. One TA suggests that it is possible to find the MST in a time faster than  $O(|E|\log|E|)$ . Is this true? If so, briefly explain how this can be done. If not, briefly explain why.

(f) **(Optional)** Give a scenario where Prim's algorithm is more preferred than Kruskal's algorithm.

## 2 Single-Source & Single-Pair Shortest Path

- (a) For the graph below, run Dijkstra’s algorithm to find **single-source** shortest paths from A to every other vertices. You must show your work at every step and give the vertex order visited by Dijkstra’s algorithm, assuming that we always visit alphabetically earlier vertex first if there are multiple valid choices. The first step is provided for you.



Fringe:

Vertex	Priority	Removed?
A	0	Yes
B	$\infty$ 2	
C	$\infty$ 4	
D	$\infty$ 1	
E	$\infty$	
F	$\infty$	

Visiting order:

A \_ \_ \_ \_ \_

Vertex	distTo	edgeToVertex
A	0	–
B	$\infty$ 2	AB
C	$\infty$ 4	AC
D	$\infty$ 1	AD
E	$\infty$	
F	$\infty$	

- (b) **(Optional)** Draw the shortest paths tree (SPT) resulting from the shortest paths found by Dijkstra’s algorithm in problem 2(a).

- (c) Using the graph from problem 2(a), suppose we are trying to find a **single-pair** shortest path from A to F. Specify your heuristic function for A\* Search by **circling any possible numbers** for  $h(E)$  that returns the **wrong shortest paths tree**.

$h(A) = 0$

$h(B) = 10$

$h(C) = 9$

$h(D) = 15$

$h(E) = -3 \quad 0 \quad 5 \quad 9 \quad 11 \quad 14$

$h(F) = 0$