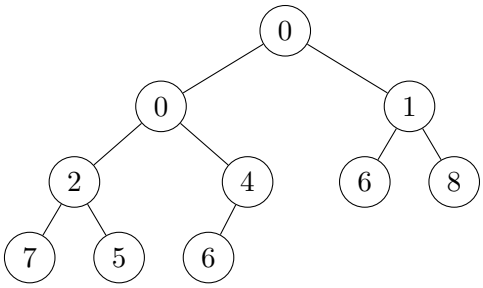
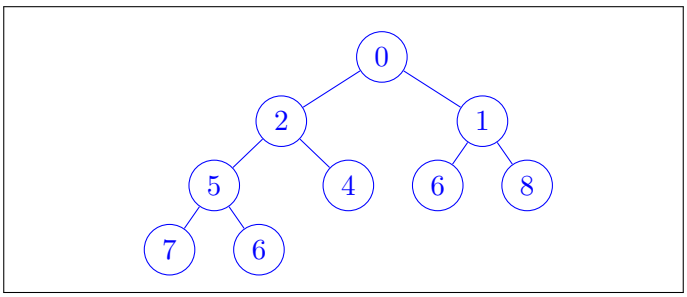
1 Heap

(a) Suppose we have the **min heap** below, with array representation as shown. Show the heap and array representation after the smallest value is removed, using the procedure described in class.





	0	1	2	3	4	5	6	7	8	9	10
Initial:	–	0	0	1	2	4	6	8	7	5	6
Your Answer:	–	0	2	1	5	4	6	8	7	6	–

We first swap the root and the last leave (`arr[1]` and `arr[10]`). Then, we can safely remove the last leave at `arr[10]`, which now stores 0 (the smallest value), while holding the tree completeness. Then, we sink 6 from the root to its proper place, as shown above.

2 Left-Leaning Red-Black Tree

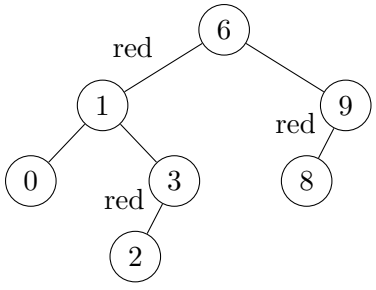
LLRB Tree Invariants:

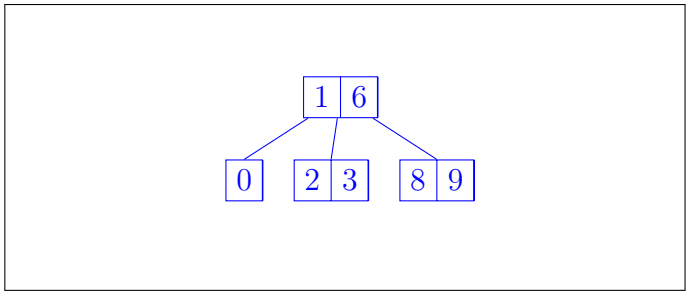
- Every root-to-leaf path has the same number of black edges.
- Red edges lean left.
- No node has two red edges connected to it, either above/below or left/right.

LLRB Tree Problems and Solutions:

- Red link to right child and no red link to left child?: Rotate left.
- Two left reds in a row?: Rotate right.
- Red links to both children?: Flip colors.

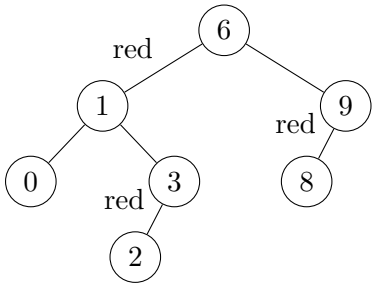
(a) Draw the **2-3 tree** corresponding to the following left-leaning red-black tree.

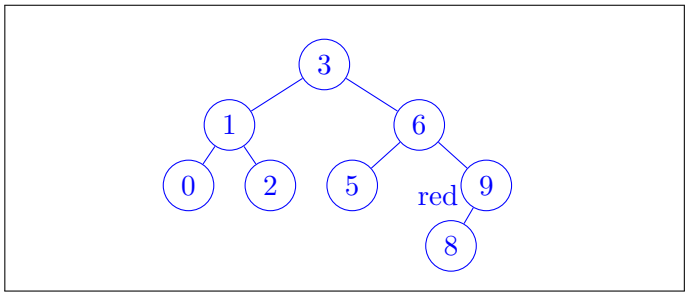


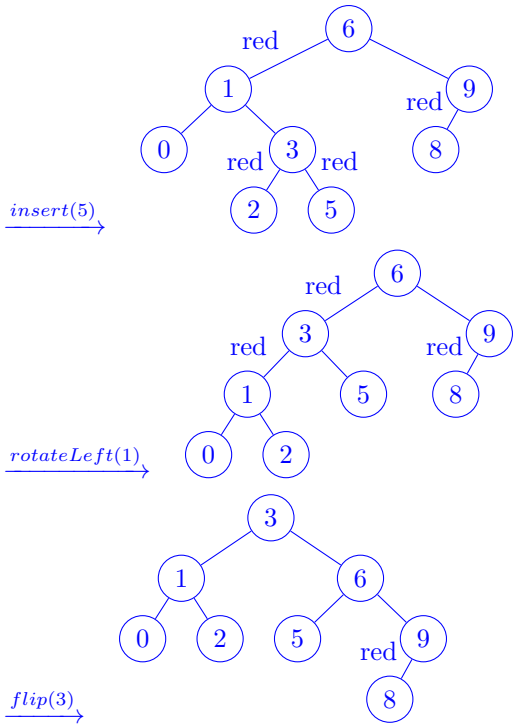


Will you want to pick up your worksheet later? Circle one: Yes / No
How confident do you feel with the material this week? Circle one: 1 / 2 / 3 / 4

(b) Draw the left-leaning red-black tree after **inserting 5**. Label red edges **red**.







Another approach is to insert 5 into the corresponding 2-3 tree (from 2(a)) and convert it back to LLRB tree.

Will you want to pick up your worksheet later? Circle one: Yes / No
How confident do you feel with the material this week? Circle one: 1 / 2 / 3 / 4

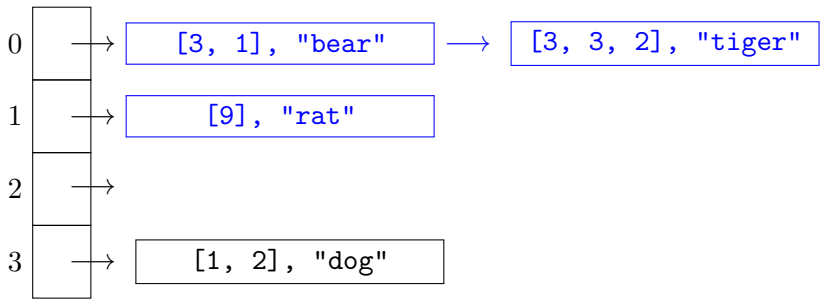
3 Hashing

For the following problems, assume that..

- `IntList` is a list of integers.
- The **hash code** of an `IntList` is the sum of the integers in the list.
- `IntList`s are considered equal only if they have the same size and the same values in the same order.
- `FourBucketHashMap` uses separate chaining and that new items are added to the **back** of each bucket.
- `FourBucketHashMap` always has **four** buckets and never resizes.

(a) Draw the hash table that is created by the following code. The result of the first `put` is provided for you.

```
1  FourBucketHashMap<IntList, String> fbhm = new FourBucketHashMap<>();
2  fbhm.put(IntList.of(1, 2), "dog");
3  fbhm.put(IntList.of(3, 1), "bear");
4  fbhm.put(IntList.of(9), "rat");
5  fbhm.put(IntList.of(3, 3, 2), "tiger");
```

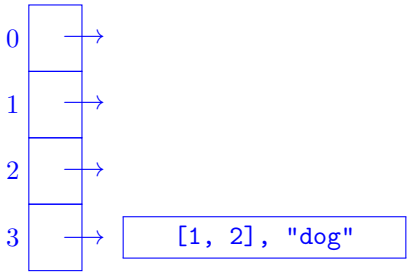


(b) Consider the following code:

```
1  FourBucketHashMap<IntList, String> fbhm = new FourBucketHashMap<>();
2  IntList list1 = IntList.of(1, 2);
3  fbhm.put(list1, "dog");
4  \\ Part i
5  list1.add(3);
6  \\ Part ii
```

i) At Part i (line 4), what will be returned from the following statement?

At line 4, our hash table looks like:

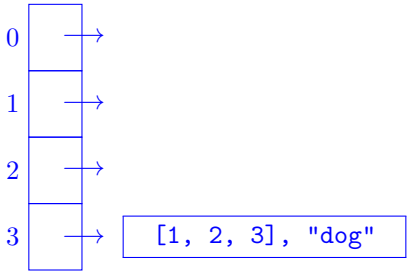


`fbhm.get(IntList.of(1, 2));` ● `"dog"` ○ `[1, 2]` ○ `null`

This will look up the bucket $(1+2) \bmod 4 = 3$. In the bucket 3, `IntList.of(1, 2)` is equivalent to `[1, 2]`, so `"dog"` which is the stored value is returned.

ii) At Part ii (line 6), what will be returned from the following statements?

At line 6, our hash table looks like:



`fbhm.get(IntList.of(1, 2));` ○ `"dog"` ○ `[1, 2]` ● `null`

This will look up the bucket $(1+2) \bmod 4 = 3$. In the bucket 3, `IntList.of(1, 2)` is NOT equivalent to `[1, 2, 3]`, so we cannot find the matched key. Hence, return `null`.

`fbhm.get(IntList.of(1, 2, 3));` ○ `"dog"` ○ `[1, 2, 3]` ● `null`

This will look up the bucket $(1+2+3) \bmod 4 = 2$. Since the bucket 2 is empty, we definitely cannot find the matched key. Hence, return `null`.

iii) Is there a problem with the code? If so, explain below.

Adding 3 into `list1` changes its hash code, causing `list1` to live in the wrong bucket.