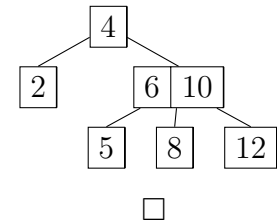
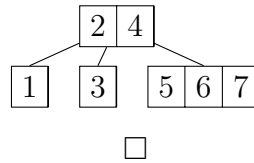
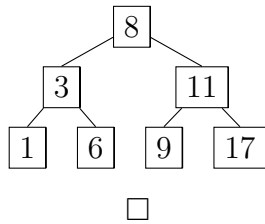
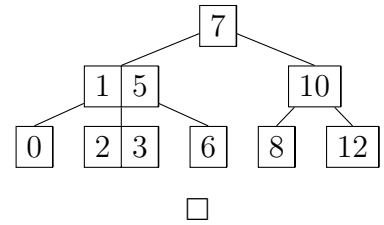
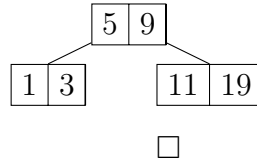
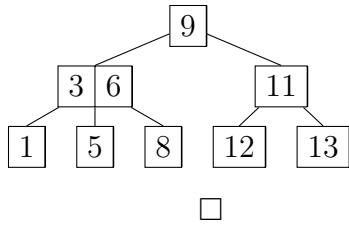


1 B-Tree

(a) Select all **VALID** 2-3 Tree structures.

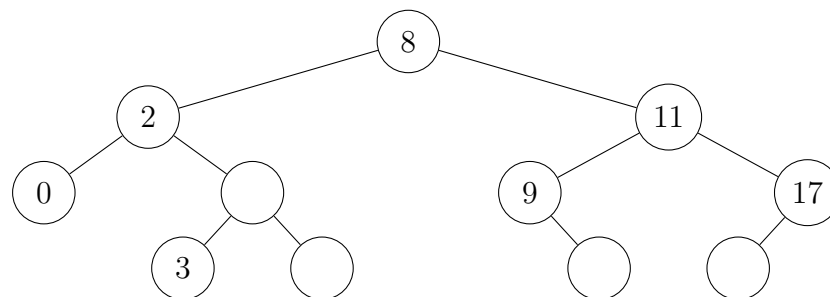


(b) Draw the 2-3 tree that results from **inserting** these items in order: **9,2,6,4,5,11,12,3** (hint: try to keep the invariants.)

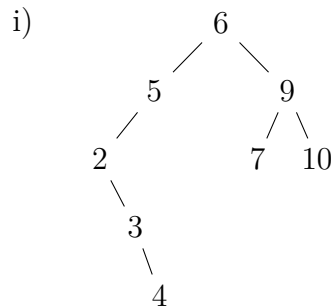


2 Binary Search Tree

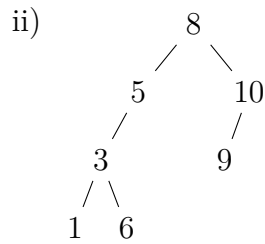
(a) Fill-in the blank nodes of the following binary search tree with valid **integer values**.



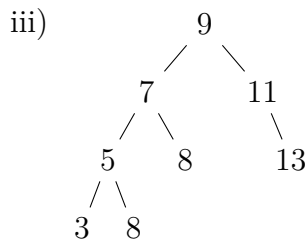
- (b) Given the following binary trees, determine if each is a binary search tree, and whether the height of the tree is the same as the height of the optimal binary search tree containing the given elements.



Binary Search Tree: Valid Invalid
Height: Balanced Unbalanced



Binary Search Tree: Valid Invalid
Height: Balanced Unbalanced



Binary Search Tree: Valid Invalid
Height: Balanced Unbalanced

- (c) (Optional) Given any insertion order of distinct values x_1, x_2, \dots, x_n to insert into BST, we want to build a **balanced BST**. Assume that N is the size of the insertion list.

- `findMedian(list)` runs in $f(N)$ time,
- `insertIntoTree(x)` runs in $g(N)$ time and
- `partition(list, x)` which partitions `list` into two lists, one containing values less than `x` and another containing values more than `x`, runs in $h(N)$ time

Give a recurrence relation modeling the worst-case runtime to build this balanced BST.

$$T(N) = \underline{\hspace{10em}} \quad \text{for } N > 1$$