Hash Tables (cont.);

CSE 332 Summer 2020

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Lecture clarifications: tinyurl.com/332-07-17A

Announcements

- No announcements! We're just busy grading

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Lecture Outline

- Hash Tables
  - Collision Resolution: Open Addressing
    - **Quadratic Probing**
    - Double Hashing
  - Collision Avoidance: Rehashing
  - Java-specific Hash Table Concerns
  - Conclusion

- Comparison Sorting
  - Simple Algorithms: InsertionSort and SelectionSort
  - Fancier Algorithms: HeapSort

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Hash Table Components

```
HashTable h;
h.add("cat", 100);
h.add("bee", 50);
```

0	-
1	-
2	100
3	50
4	-

- Implementing a hash table requires the following components:


```
hashFunction("cat") == 2;
2 % 5 == 2
hashFunction("bee") == 2525393088;
2525393088 % 5 == 3
```

Linear Probing: Add Example

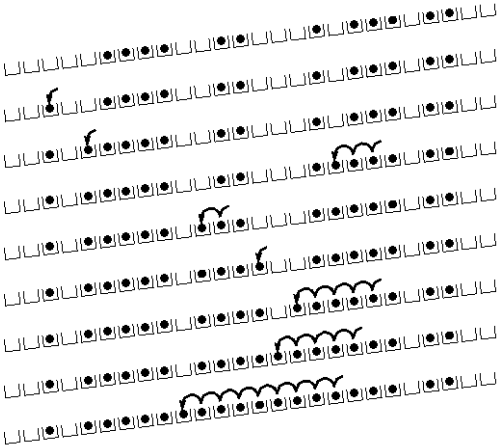
- Example: add 89, 18, 49, 58, 79
  - Let hashFunction(x) = x
  - Let `TableSize` = 10

0	49
1	58
2	79
3	-
4	-
5	-
6	-
7	-
8	18
9	89

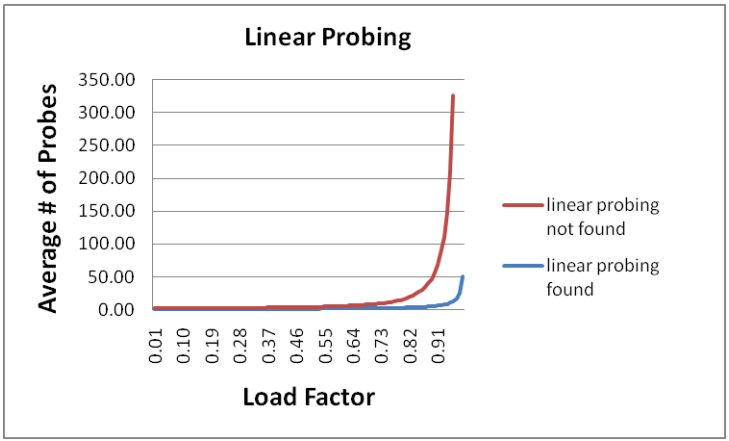
Linear Probing: Primary Clustering

- Linear probing uses the following probe function:
  - `i`th probe: `(h(key) + i) % TableSize`

- Which tends to produce long probe sequences due to **primary clustering**



[R. Sedgewick]



Quadratic Probing

- Avoid primary clustering by changing the probe function:
  - **`i`**th probe: **`(h(key) + i^2) % TableSize`**
  - Probe sequence becomes:
    - 0th probe: **`h(key) % TableSize`**
    - 1st probe: **`(h(key) + 1) % TableSize`**
    - 2nd probe: **`(h(key) + 4) % TableSize`**
    - 3rd probe: **`(h(key) + 9) % TableSize`**
- Intuition: Probes quickly "leave the neighborhood"

Reminder: a dictionary maps *keys* to *values*; an *item* or *data* refers to the (key, value) pair

Quadratic Probing: Add Example

- Example: add 89, 18, 49, 58, 79
  - Let hashFunction(x) = x
  - Let `TableSize` = 10

0	49
1	-
2	58
3	79
4	-
5	-
6	-
7	-
8	18
9	89

Quadratic Probing: Another Add Example

- Example: add 76, 40, 48, 5, 55, 47
  - Let hashFunction(x) = x
  - Let `TableSize` = 7

76%7 = 6
40%7 = 5
48%7 = 6
5%7 = 5
55%7 = 6
47%7 = 5

0	48
1	-
2	5
3	55
4	-
5	40
6	76

Quadratic Probing: Another Add Example

- Example: add ~~76, 40, 48, 5, 55,~~ 47
  - Let hashFunction(x) = x
  - Let `TableSize` = 7
  - (47 + 1) % 7 = 6 collision!
  - (47 + 4) % 7 = 2 collision!
  - (47 + 9) % 7 = 0 collision!
  - (47 + 16) % 7 = 0 collision!
  - (47 + 25) % 7 = 2 collision!

0	48
1	-
2	5
3	55
4	-
5	40
6	76

- **Will we ever get a 1 or 4?!?**

Quadratic Probing: Another Add Example

- Example: add 76, 40, 48, 5, 55, 47

0	48
1	-
2	5
3	55
4	-
5	40
6	76

- Will we ever get a 1 or 4?!?
  - add(47) will *always* fail here. Why?
  - For all i, $(5 + i^2) \% 7$ is 0, 2, 5, or 6
  - Proof uses induction and
    - $(5 + i^2) \% 7 = (5 + (i - 7)^2) \% 7$

- In fact, for all c and k,
  - $(c + i^2) \% k = (c + (i - k)^2) \% k$

Quadratic Probing: Bad News / Good News

- Bad News:
  - After `TableSize` probes, we cycle through the same indices
- Good News:
  - If **`TableSize` is prime and $\lambda < ½$,** then quadratic probing will find an empty slot in at most **`TableSize/2`** probes
  - So: If you keep $\lambda < ½$ and **`TableSize`** is prime, no need to detect cycles
  - Textbook has proof

Quadratic Probing: Success Guarantee (1 of 2)

If `TableSize` is prime and $\lambda < ½$, then quadratic probing will find an empty bucket in `TableSize/2` probes or fewer

- Intuition: if the table is less than half full, then probing `TableSize/2` distinct buckets must find an empty one
  - Therefore, prove the first `TableSize/2` probes are distinct

Any i^{th} and any j^{th} probe results in a distinct bucket

- Theorem: for all **`0 ≤ i,j ≤ TableSize/2`**, and **`i ≠ j`**

`(h(x) + i²) % TableSize ≠ (h(x) + j²) % TableSize`

Quadratic Probing: Success Guarantee (2 of 2)

- Proof, by contradiction: suppose that for some i ≠ j:

  `(h(x) + i^2) % TableSize = (h(x) + j^2) % TableSize`

  ⇒ `i^2 % TableSize = j^2 % TableSize`

  ⇒ `(i^2 - j^2) % TableSize = 0`

  ⇒ `[(i + j)(i - j)] % TableSize = 0`

  ⇒ `[(i + j)(i - j)] = k*TableSize` for some k ≥1

  or

  `[(i + j)(i - j)] = 0`

  } CONTRADICTION!

- How can `i+j = 0` or `i+j = k*TableSize` when:

  `0 ≤ i,j` and `i≠j` and `i,j ≤ TableSize/2`?

- How can `i-j = 0` or `i-j = k*TableSize` when

  `i ≠ j` and `i,j ≤ TableSize/2` ?

Quadratic Probing: Secondary Clustering

- Quadratic probing does not suffer from primary clustering!
  - We don't grow "big blobs" by adding to the end of a cluster

- Quadratic probing does not resolve collisions between different keys that hash *to the same* ***index***
  - These keys **have the same series of moves** looking for an empty spot
  - Called secondary clustering ☹

- Since the problem occurs when we have the different keys hashing to the same initial index, can we avoid secondary clustering with *a probe function that also incorporates the key*?
  - Known as double hashing

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Double Hashing

- Double hashing:
  - **`i`**th probe: **`(h(key) + i*g(key)) % TableSize`**
  - Probe sequence becomes:
    - 0th probe: **`h(key) % TableSize`**
    - 1st probe: **`(h(key) + g(key)) % TableSize`**
    - 2nd probe: **`(h(key) + 2*g(key)) % TableSize`**
    - …
- Idea:
  - **`g(key)`** lets us "go different places from initial collisions"
    - It is very unlikely that for some key, **`h(key) == g(key)`**
    - (assuming good hash functions h and g)
  - **`i*g(key)`** lets us "leave the neighborhood"
- Detail: Ensure **`g(key)`** can't generate 0

Double Hashing: Add Example

- Example: add 13, 28, 33, 147, 43
  - Let hashFunction(x) = x%`TableSize`
  - Let g(x) = 1 + ((x/`TableSize`) % (`TableSize`-1))
  - Let `TableSize` = 10

0	-
1	-
2	-
3	13
4	-
5	-
6	-
7	33
8	28
9	147

Double Hashing: Add Example

- Example: add ~~13, 28, 33, 147,~~ 43
  - Let hashFunction(x) = x%`TableSize`
  - Let g(x) = 1 + ((x/`TableSize`) % (`TableSize`-1))
  - Let `TableSize` = 10
  - h(43) = 3
  - g(43) = 1 + (4%9) = 5
  - 3 + 0*5 = 3 collision!
  - 3 + 1*5 = 8 collision!
  - 3 + 2*5 = 13 collision
  - **Will we ever get anything else?!?**

0	-
1	-
2	-
3	13
4	-
5	-
6	-
7	33
8	28
9	147

Double Hashing: Add Example

- Example: add ~~13, 28, 33, 147,~~ 43
  - Let hashFunction(x) = x%`TableSize`
  - Let g(x) = 1 + ((x/`TableSize`) % (`TableSize`-1))
  - Let `TableSize` = 10
  - Will we ever get anything else?!?
    - add(43) will always fail here. Why?

0	-
1	-
2	-
3	13
4	-
5	-
6	-
7	33
8	28
9	147

Double Hashing: Considerations (1 of 2)

- Our example implies the possibility of infinite probe sequences ☹
  - But we can be avoid infinite probes if our functions are:
    - h(key) = key % p
    - g(key) = q – (key % q)
  - And p and q are primes, with 2 < q < p

Double Hashing: Considerations (2 of 2)

- Double hashing:
  - **`i`**th probe: **`(h(key) + i*g(key)) % TableSize`**

- Assume **`g(key)`** divides `TableSize`
  - That is, there exists some integer `x` such that `x*g(key)=TableSize`
  - Therefore: after x probes, we'll "loop through" the same indices as before
  - Example:
    - `TableSize=50`
    - `g(key)=25`
    - Probe sequence:
      - i=0: h(key)
      - i=1: h(key)+25
      - i=2: h(key)+50 = h(key)
      - i=3: h(key)+75 = h(key)+25
      - …

- Bottom line: don't let **`g(key)`** divide `TableSize`
  - That is, choose a prime `TableSize` when using double hashing

Double Hashing: Performance

- Assume g() distributes its keys uniformly over its range
  - That is: probability of `g(key1)%p == g(key2)%p` is `1/p`
- We won't prove the following:
  - Average # of probes (in the limit as TableSize $\to\infty$), **unsuccessful** find:

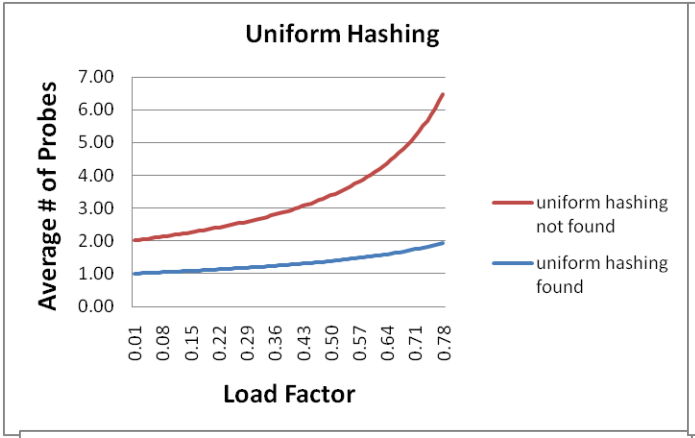
$$\frac{1}{1-\lambda}$$

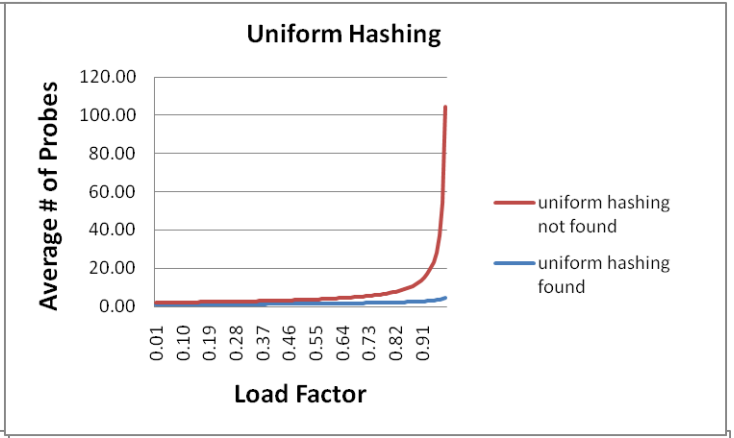
  - Average # of probes (in the limit as TableSize $\to\infty$), **successful** find:

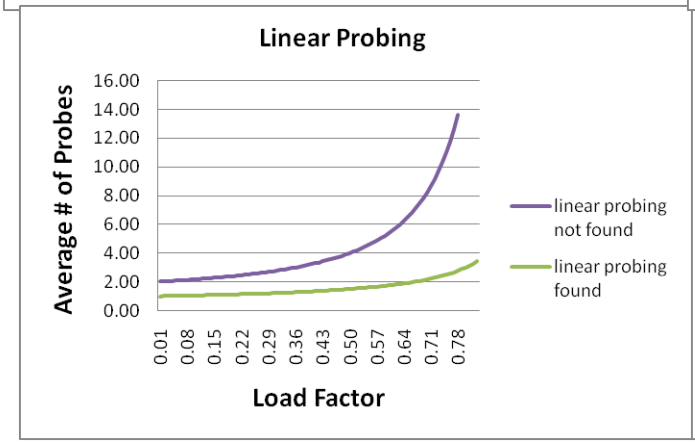
$$\frac{1}{\lambda}\log_e\left(\frac{1}{1-\lambda}\right)$$

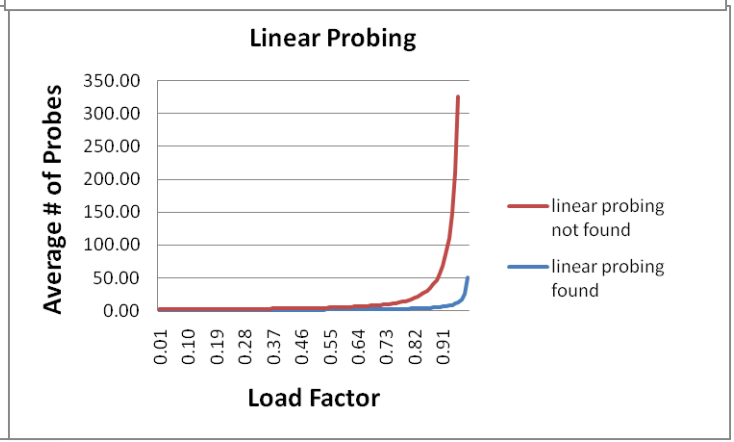
- Bottom line:
  - Performance of unsuccessful finds degrades with λ (but not as quickly as linear probing degrades)
  - Performance of successful finds degrades not nearly as quickly

Double Hashing vs Linear Probing Performance









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Separate Chaining vs Open Addressing

- Separate Chaining
  - `find, add, remove` proportional to λ if using unsorted LL
  - If using another data structure for buckets (e.g. AVL tree) , runtime is proportional to runtime for that structure

- Open addressing: has clustering issues as table fills ($\lambda \geq 1/2$)
  - Why use it:
    - Some runtime for allocating nodes; open addressing could be faster?
    - Less memory usage for nodes
    - Easier data representation?

Rehashing (1 of 3)

- As with array-based stacks/queues/lists, if table gets too full, create a bigger table and "copy" everything over
- With separate chaining, we decide what "too full" means
  - Keep load factor reasonable (e.g., < 1)?
  - Consider average or max size of non-empty chains?
- For open addressing, half-full is a good rule of thumb

Rehashing (2 of 3)

- Can't actually copy to the same indices in the new table
  - We'd calculated the index based on **`TableSize`**
- For each key/value in old table, must `add` into new table
  - Iterate over old table: O(n)
  - n calls to the hash function: n · O(1) = O(n)
- Can we avoid all those hash function calls?
  - Space/time tradeoff: Could store `h(key)` with each item
  - Iterating over the table is still O(n); saving `h(key)` only helps by a constant factor

Rehashing (3 of 3)

- New table size
  - Twice-as-big is a good idea, except … ummm … that won't be prime!
  - So go *about* twice-as-big
    - Hard-coded list of primes (you probably won't grow more than 20-30 times)
    - Calculate primes after that

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Hashing and Equality Testing

- Our examples use an `int` key, which overlooks a critical detail:
  - We hash **`K`** to get a table index
  - While chaining or probing, we need to test whether the current **`K'`** is equal to the **`K`** we're looking for
- So a Java hash table needs a hash *and* an equality function
  - Fortunately, in Java every object defines an **`equals`** and a **`hashCode`** method

```
class Object {
  boolean equals(Object o) {…}
  int hashCode() {…}
  …
}
```

Overriding equals()? Override hashCode() too

- The Java library (and your project's hash table) make a very important assumption that *all* clients must satisfy:
  - Object-oriented way of saying it:

    If `a.equals(b)`, then `a.hashCode() == b.hashCode()`
  - Functor way of saying it:

    If `c.compare(a,b) == 0`, then

    `h.hashCode(a) == h.hashCode(b)`

- In other words, if you ever override equals:
  - You must also override hashCode() in a consistent way
  - See Core Java book, Ch. 5, for other "gotchas" with equals()

compareTo() rules

- Java also makes assumptions about `compareTo()` that affect:
  - All our dictionaries
  - Sorting (next major topic)

- Comparison must impose a consistent, total ordering:
  - For all **`a`**, **`b`**, and **`c`**,
    - If **`a.compareTo(b) < 0`**, then **`b.compareTo(a) > 0`**
    - If **`a.compareTo(b) == 0`**, then **`b.compareTo(a) == 0`**
    - If **`a.compareTo(b) < 0`** and **`b.compareTo(c) < 0`**, then **`a.compareTo(c) < 0`**

A Generally-Good hashCode()


```
int result = 17;  // start at a prime

foreach field f
  int fieldHashcode =
    boolean: (f ? 1: 0)
    byte, char, short, int: (int) f
    long: (int) (f ^ (f >>> 32))
    float: Float.floatToIntBits(f)
    double: Double.doubleToLongBits(f),
        then above conversion to int
    Object: object.hashCode()
  result = 31 * result + fieldHashcode;

return result;
```

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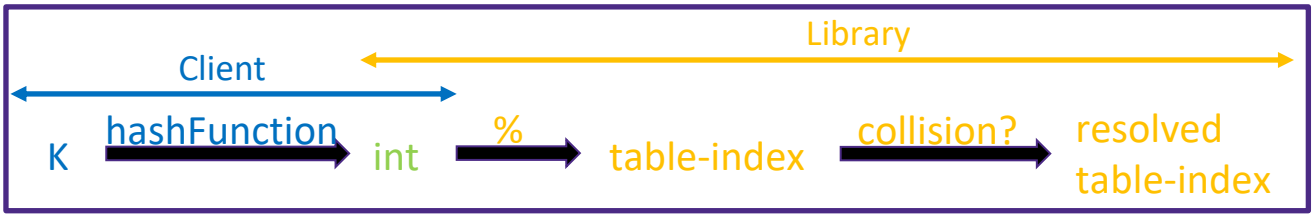
Who Hashes What?

- When used as a library, hash tables generally have two roles: client vs library



- We learned both, but you'll spend more time as clients
  - Both roles must contribute to minimizing collisions
  - Client should aim for different ints for expected keys
    - Avoid "wasting" any part of K or the int's bits
  - Library should aim for putting "similar" ints in different indices
    - Conversion to index is almost always "mod table-size"
    - Using prime numbers for table-size is common

Summary: Hash Tables vs. Balanced Trees

- In terms of a Dictionary ADT for just **`add`**, **`find`**, **`remove`**, hash tables and balanced trees are just different data structures
  - Hash tables *O*(1) on average (assuming few collisions)
  - Balanced trees *O*(**`log`** *n*) worst-case

- Constant-time is better, right?
  - Yes, but you need "hashing to behave" (must avoid collisions)
  - Yes, but what if we want to **`findMin`**, **`findMax`**, **`predecessor`**, and **`successor`**, **`printSorted?`**
    - Hash tables are not designed to efficiently implement these operations
    - Your textbook considers hash tables to be a different ADT
    - Not so important to argue over the definitions

Summary: Hash Tables

- The hash table is one of the most important data structures
  - Useful in many, many real-world applications
  - Popular topic for job interview questions
- **Crucial** to use a good hash function
  - Hash tables rely on good distribution
  - Not overly expensive to calculate (bit shifts good!)
- Important to keep hash table at a good size
  - Prime table size
  - "Preferred λ" depends on type of table
- What we skipped:
  - Perfect hashing, universal hash functions, hopscotch hashing, cuckoo hashing

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Introduction to Sorting (1 of 2)

- Stacks, queues, priority queues, and dictionaries/sets all provide one element at a time

- But often we want "all the items" in some order
  - Alphabetical list of people
  - Population list of countries
  - Search engine results by relevance

- Different sorting algorithms have different asymptotic and constant-factor trade-offs
  - Knowing one way to sort just isn't enough; no single "best sort"
  - **Sorting is an excellent case-study in making trade-offs!**

Introduction to Sorting (2 of 2)

- *Preprocessing* (e.g. sorting) data to make subsequent operations faster is a general technique in computing!
  - Example: Sort the items so that you can:
    - Find the $\mathbf{k}^{th}$ largest in constant time for any $\mathbf{k}$
    - Perform binary search to find an item in logarithmic time
  - Whether preprocessing is beneficial depends on
    - How often the items will change
    - How many items there are

- Preprocessing's benefits depend on
  - How often the items will change and how many items there are
  - **Sorting is an excellent case-study in making trade-offs!**

Comparison Sorting: Definition

- Problem: We have *n* comparable items in an array, and we want to rearrange them to be in increasing order

- Input:
  - An array A of (key, value) pairs
  - A comparison function (consistent and total)
    - Given keys a & b, what is their relative ordering? <, =, >?
    - Ex: keys that implement Comparable or have a Comparator

- Output/Side-Effect:
  - Reorganize the elements of A such that for any index i and j,

    if $i < j$ then $A[i] \leq A[j]$
  - [Usually unspoken] A must have all the same items it started with
  - Could also sort in reverse order, of course

Comparison Sort: Variations (1 of 2)

1. Maybe elements are in a linked list
   - Could convert to array and back in linear time, but some algorithms can still "work" on linked lists

2. Maybe if there are ties we should preserve the original ordering
   - Sorts that do this naturally are called **stable sorts**

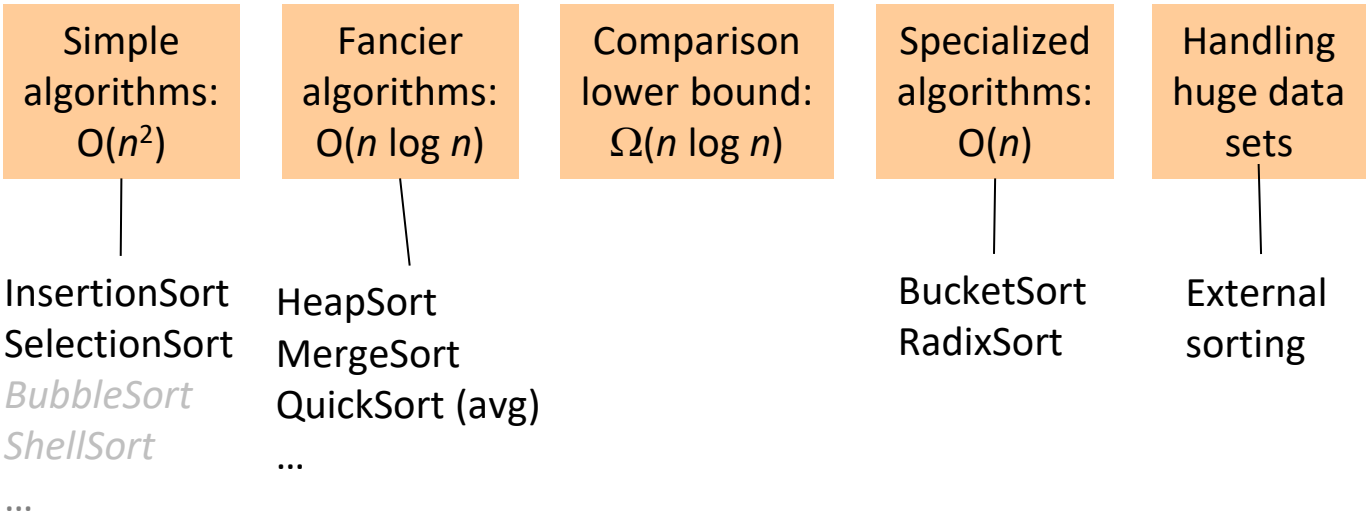
3. Maybe we must not use more than O(1) "auxiliary space"
   - These are called **in-place sorts**
   - Not allowed to allocate memory proportional to input (i.e., O(n)), but can allocate O(1) # of variables
   - Work is done by swapping around in the array

Comparison Sort: Variations (2 of 2)

4. Maybe we can do more with elements than just compare
   - Comparison sorts assume a binary 'compare' operator
   - In special cases we can sometimes get faster algorithms

5. Maybe we have too many items to fit in memory
   - Use an **external sorting** algorithm

Sorting: The Big Picture



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InsertionSort

- Idea: At step **`k`**, insert the **`k`**th element in the correct position
  - Sort first two elements
  - Now insert 3rd element in order
  - …
- Loop invariant ("when loop index is **`i`**"):
  - First **`i`** elements are sorted
- Time:

  Best-case: _____ Worst-case: _____ "Average" case: _____
- Characteristics:

  Stable: _____ In-place: _____

Demo:
https://docs.google.com/presentation/d/10b9aRqpGJu8pUk8OpfqUIEEm8ou-zmmC7b_BE5wgNg0/present

SelectionSort

- Idea: At step **`k`**, select the smallest element, put it at **`k`**th position
  - Find smallest element, put it 1st
  - Find next smallest element, put it 2nd
  - …
- Loop invariant ("when loop index is **`i`**"):
  - First **`i`** elements are the **`i`** smallest elements in sorted order
- Time:

  Best-case: _____ Worst-case: _____ "Average" case: _____
- Characteristics:

  Stable: _____ In-place: _____

Demo:
https://docs.google.com/presentation/d/1p6g3r9BpwTARjUylA0V0yspP2temzHNJEJjCG41I4r0/edit

InsertionSort vs. SelectionSort

- Different algorithms, same problem

- Same worst-case and "average"-case asymptotic complexity
  - InsertionSort has better best-case complexity; preferable when input is "mostly sorted"

- Other algorithms are more efficient *for non-small arrays that are not already almost sorted*
  - InsertionSort may do well on small arrays (empirically: N < ~15)

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Naïve HeapSort

- Idea: Put everything into a **MIN** heap. At step **k**, remove the **k**th-min element
  - insert each `arr[i]` –OR– better yet, use `buildHeap`
  - ```
    for(i=0; i < arr.length; i++)
        arr[i] = deleteMin();
    ```

- Loop invariant ("when loop index is **i**"):
  - First **i** elements are the **i** smallest elements in sorted order

- Time:

  Best-case: _____ Worst-case: _____ "Average" case: _____

Demo: https://goo.gl/EZWwSJ
    ```

In-place HeapSort

- Idea: Put everything into a **MAX** heap. At step **k**, remove the **k**th-max element
  - insert each `arr[i]` –OR– better yet, use `buildHeap`
  - ```
    for(i=0; i < arr.length; i++)
        arr[arr.length - i] = deleteMax();
    ```
- Loop invariant ("when loop index is **i**"): same as naïve version
- Time:

  Best-case: _____ Worst-case: _____ "Average" case: _____
- Characteristics:

  Stable: _____ In-place: _____

Demo: https://docs.google.com/presentation/d/1SzcQC48OB9agStD0dFRgccU-tyjD6m3esrSC-GLxmNc/present
    ```

Aside: "AVL sort" and "data structure sorts"

- We can also use a balanced tree to:
  - `add` each element: total time $O(n \log n)$
  - Do an in-order traversal $O(n)$

- Cannot be made in-place and constants worse than HeapSort
  - Both are $O(n \log n)$ in worst, best, and average case; neither parallelizes well

- There are algorithms similar to using a HashTable that we will cover….