Tries; Asymptotic Analysis 1

CSE 332 Spring 2020

Instructor: Richard Jiang

Teaching Assistants:

Hamsa Shankar Kristin Li Winston Jodjana
Maggie Jiang Hans Zhang Michael Duan
Jeffery Tian Annie Mao

Lecture Q&A: pollev.com/332summer

Lecture clarifications: tinyurl.com/332-06-24A

Announcements

- Due dates:
  - Exercise 1 is due this Friday
  - Quiz 1 will be available on Gradescope 12:00am Thursday and due 11:59 Friday
  - P1 Ckpt 1 is next Tuesday

- Before section tomorrow:
  - Try gitlab and IntelliJ, so TAs can help debug any issues during section
  - Please verify your section's Zoom links are in Canvas

Lecture Outline

- **Review: List, Stack, Queue, Dictionary and Set ADTs**
- The Trie data structure
- How do we Compare Algorithms?
- Analyzing Code
- How to Analyze Large Inputs - Asymptotics
- Big-Oh Definitions

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ADTs So Far (1 of 2)

List ADT. A collection storing an ordered sequence of elements.

- Each element is accessible by a zero-based index
- A list has a size defined as the number of elements in the list
- Elements can be added to the front, back, *or any index in the list*
- Optionally, elements can be removed from the front, back, *or any index in the list*

- Data structures that implement the List ADT include LinkedList and ArrayList
- When we restrict List's functionality, we end up with the 2 other ADTs we've seen so far

ADTs So Far (2 of 2)

Stack ADT. A collection storing an ordered sequence of elements.

- A stack has a size defined as the number of elements in the stack
- Elements can only be added and removed from the top (“LIFO”)

Queue ADT. A collection storing an ordered sequence of elements.

- A queue has a size defined as the number of elements in the queue
- Elements can only be added to one end and removed from the other (“FIFO”)

- Data structures that implement these ADTs are variants of LinkedList and ArrayList

Dictionary ADT

Dictionary ADT. A collection of keys, each associated with a value.

- A dictionary has a size defined as the number of elements in the dictionary
- You can add and remove (key, value) pairs , but the keys are unique
- Each value is accessible by its key via a "find" or "contains" operation

Terminology: a dictionary maps *keys* to *values*; an *item* or *data* refers to the (key, value) pair

- Also known as "**Map ADT**"
  - add(k, v)
  - contains(k)
  - find(k)
  - remove(k)

Set ADT

Set ADT. A collection of keys.

- A set has a size defined as the number of elements in the set
- You can add and remove keys, but the contained values are unique
- Each key is accessible via a "contains" operation

- Operations:
  - add(v)
  - contains(v)
  - remove(v)

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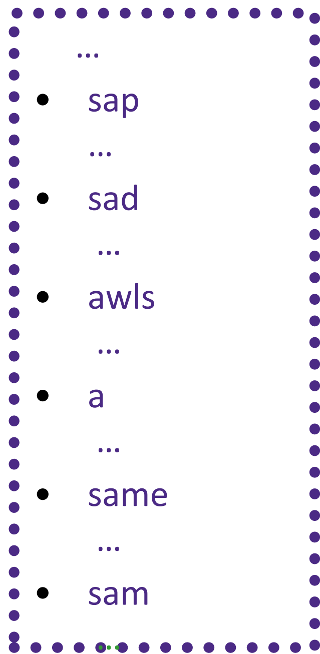
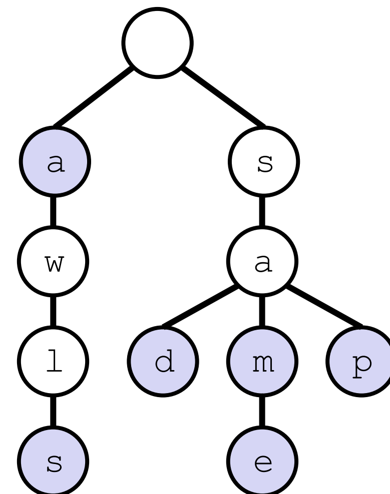
The Trie: A Specialized Data Structure

Tries view keys as a **sequence of characters**, keys must be **alphabetic**



Abstract Set ADT



Trie

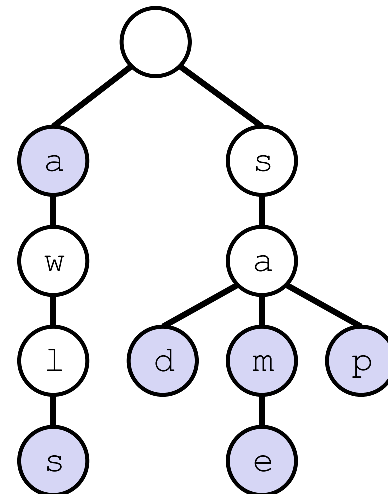
An Abstract Trie

Each level of the tree represents an index, and the children represent possible characters at that index.

This trie stores the set of strings:

`awls, a, sad,`

`same, sap, sam`



How to deal with `a` and `awls`?

- Mark which nodes *complete* strings (shown in purple)

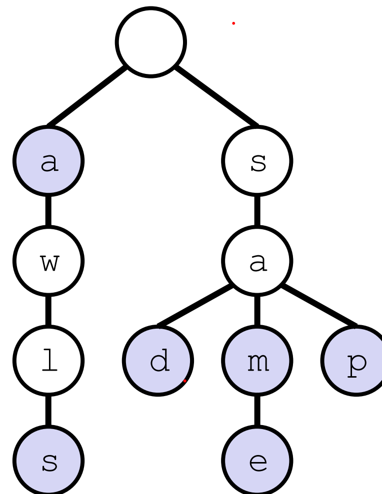
Searching in Tries

contains("sam"): true, purple. **hit**.

contains("sa"): false, white. **miss**.

contains("a"): true, purple. **hit**.

contains("saq"): false, fell off. **miss**.



Two ways to have a **search miss**.

1. If the final node isn't purple (not a key).
2. If we fall off the tree.

Keys as “a sequence of tokens” (1 of 2)

- Most dictionaries treat their keys as an “atomic blob”: you can’t disassemble the key into smaller components

- Tries take the opposite view: keys are a **sequence of tokens**
  - `String`s are made of `Character`s

- But “tokens” don’t have to come from the Latin alphabet
  - `Character` includes most Unicode codepoints (eg, 蛋糕)
  - `List<E>`
  - `byte[]`

Keys as “a sequence of tokens” (2 of 2)

- But “tokens” don’t have to come from the Latin alphabet
  - `Character` includes most Unicode codepoints (eg 蛋糕)
  - `List<E>`
  - `byte[]`

- Tries are defined by 3 types instead of 2:
  - An “alphabet”: the domain of the characters/tokens
  - A “key”: a sequence of “characters” from the alphabet
  - A “value”: the usual Dictionary value

Poll Everywhere

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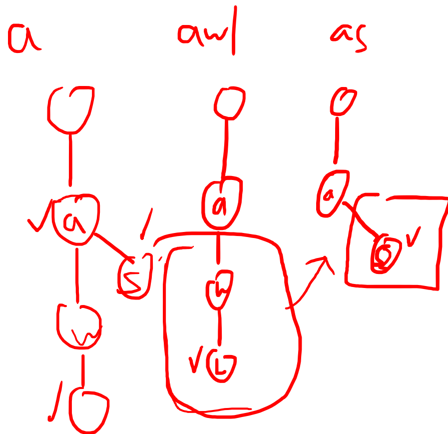
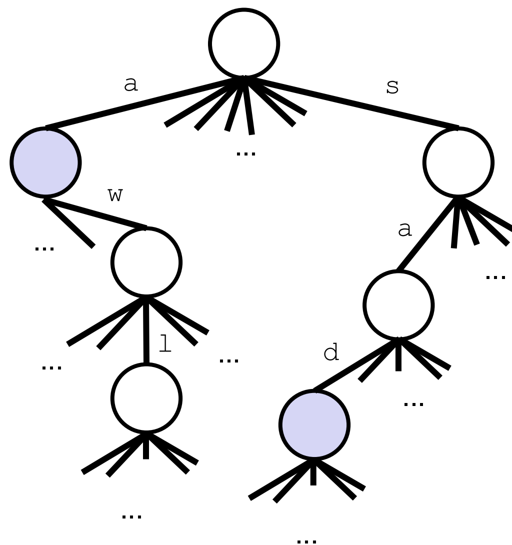
- Does the structure of a trie depend on the order in which strings are inserted?

A. Yes

B. No

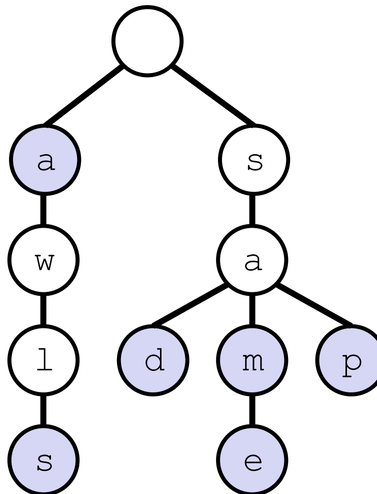
C. I'm not sure





Trie Deletion

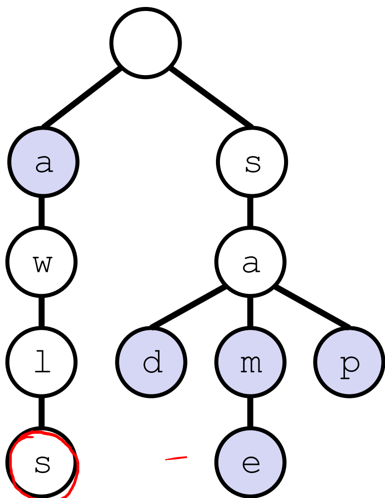
What do we do when a user calls `delete("awls")`?

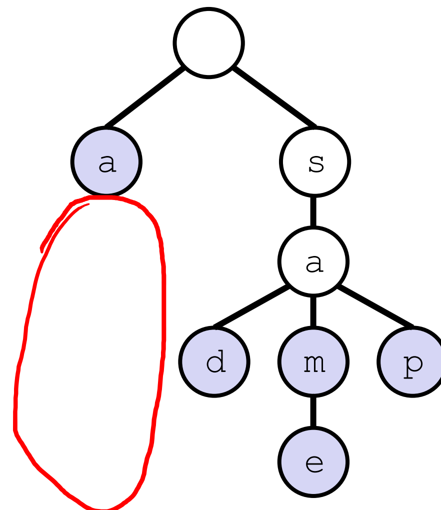


The Trie: A Specialized Data Structure

What are the tradeoffs of each deletion algorithm?

`delete("awls")`





Lazy Deletion

Normal Deletion

The Trie: A Specialized Data Structure

What are the tradeoffs of each deletion algorithm?

`delete("awls")`

Lazy Deletion

Better if..

- You want a faster delete
- Insertion is common
- Reinserting deleted *keys* is expected
- Easier to code (yes, this counts)

Normal Deletion

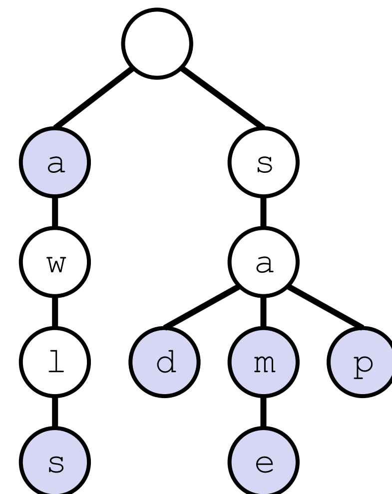
Better if..

- Find/prefix operations are common
- Better space management

String-Specific Operations

- The main appeal of tries is their efficient prefix matching!

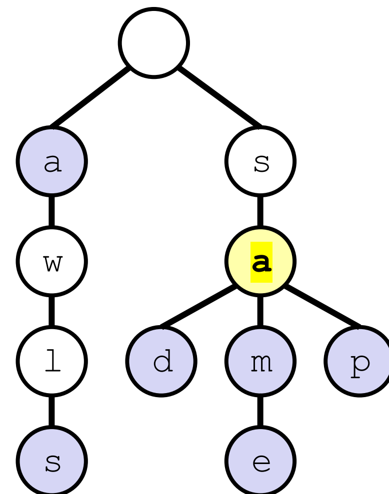
- **Prefix match**
  - `findPrefix("sa")`
- **Longest prefix**
  - `longestPrefixOf("sample")`



Prefix Operations with Tries

Describe in English an algorithm for `findPrefix`.

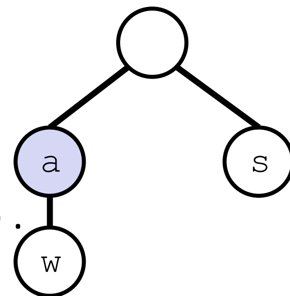
```
findPrefix("sa"):
    ["sad","sam","same","sap"]
```



Collecting Trie Keys

Describe in English an algorithm to collect all the keys in a trie.

`collect(): ["a","awls","sad","sam","same","sap"]`



1. Create an empty list of results `x`.
2. For character `c` in `root.next.keys()`:
   Call `colHelp(r + c, x, root.next.get(c))`.
3. Return `x`.

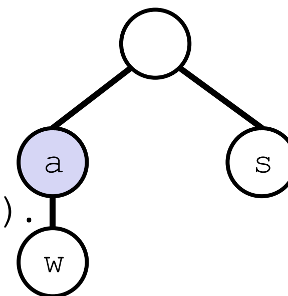
`colHelp(String s, List<String> x, Node n)`

1. ???

Collecting Trie Keys

Describe in English an algorithm to collect all the keys in a trie.

`collect(): ["a","awls","sad","sam","same","sap"]`



1. Create an empty list of results `x`.
2. For character `c` in `root.next.keys()`:
   Call `colHelp(r + c, x, root.next.get(c))`.
3. Return `x`.

`colHelp(String s, List<String> x, Node n)`

1. If `n.isKey`, then `x.add(s)`.
2. For character `c` in `n.next.keys()`:
   Call `colHelp(s + c, x, n.next.get(c))`.

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What do we care about?

- Correctness:
  - Does the algorithm do what is intended.
- Performance:
  - Speed **time complexity**
  - Memory **space complexity**
- Why analyze?
  - To make good design decisions
  - Enable you to look at an algorithm (or code) and identify the bottlenecks, etc.

Q: How should we compare two algorithms?

A: How should we compare two algorithms?

- Uh, why NOT just run the program and time it??
  - Too much ***variability***, not reliable or ***portable***:
    - Hardware: processor(s), memory, etc.
    - OS, Java version, libraries, drivers
    - Other programs running
    - Implementation dependent
  - Choice of input
    - Testing (inexhaustive) may ***miss*** worst-case input
    - Timing does not ***explain*** relative timing among inputs (what happens when *n* doubles in size)

- Often want to evaluate an ***algorithm***, not an implementation
  - Even ***before*** creating the implementation ("coding it up")

Comparing Algorithms (1 of 2)

- When is one ***algorithm*** (not ***implementation***) better than another?
  - Various possible answers (clarity, security, …)
  - But a big one is ***performance***: for sufficiently large inputs, runs in less time (our focus) or less space than the other
- Large inputs (n) because probably any algorithm is "plenty good" for small inputs (if *n* is 10, probably anything is fast enough)
- Want a model that's:
  - ***Independent*** of CPU speed, programming language, coding tricks, etc.
  - General and rigorous, ***complementary*** to "coding it up and timing it on some test cases": can do analysis before coding!

Comparing Algorithms (2 of 2)

- Want a model that's:
  - ***Independent*** of CPU speed, programming language, coding tricks, etc.
  - General and rigorous, ***complementary*** to "coding it up and timing it on some test cases": can do analysis before coding!
  - ***Descriptive of large inputs***; most algorithms are "good enough" for small inputs
    - If *n* is 10, probably anything is fast enough
    - But what consider to be "large"?

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Analyzing Code (1 of 2)

- We abstract away the computer by counting "operations" (time complexity) and "elements" (space complexity)
  - Remember: "Independent of CPU speed, programming language, coding tricks, etc."

- Basic ***elements*** take "some amount of" ***constant space***
  - Integers in an array
  - Nodes in a linked list
  - Etc.
  - (This is an *approximation of reality*: a very useful "lie".)

Analyzing Code (2 of 2)

- Basic ***operations*** take "some amount of" ***constant time***
  - Arithmetic
  - Assignment
  - Access one Java field **or array index**
  - Etc.
  - (Again, this is an *approximation of reality*)

Consecutive statements	Sum of time of each statement
Loops	Num iterations * time for loop body
Recurrence	Solve recurrence equation
Function Calls	Time of function's body
Conditionals	Time of condition + time of {slower/faster} branch

Which Branch To Analyze?

- Case Analysis != Asymptotic Analysis
- We'll start by focusing on two cases:
  - ***Worst-case complexity***: max # steps algorithm takes on "most challenging" input of size N
  - ***Best-case complexity*****:** min # steps algorithm takes on "easiest" input of size N
- Unless otherwise stated, we usually refer to the ***worst case***
  - But there are uses for other types of analyses
  - So for now we'll analyze the ***slower branch***

Examples

```
b = b + 5
c = b / a
b = c + 100
```

2
2
2
→ 6

```
for (i = 0; i < n; i++) {
  sum++;
}
```

n · 5
→ 5n+1

```
if (j < 5) {
  sum++;
} else {
  for (i = 0; i < n; i++) {
    sum++;
  }
}
```

3
5n+1
5n+2

Another Example

```
int coolFunction(int n, int sum) {
  int i, j;
  for (i = 0; i < n; i++) {
    for (j = 0; j < n; j++) {
      sum++;
    }
  }
  print "This program is great!";
  for (i = 0; i < n; i++) {
    sum++;
  }
  return sum
}
```

Analyzing Loops, Formally

- We use summations to quantify the runtime

```
for (i = 0; i < n; i++) {
   sum++;
}
```

5

$$\sum_{i=0}^{n-1} 5 + 1$$

Example Problem

Find an integer in a *sorted* array

2	3	5	16	37	50	73	75	126

```
// requires array is sorted
// returns whether k is in array
boolean find(int[] arr, int k) {
   ???
}
```

Example Solution: Linear Search

Find an integer in a *sorted* array

2	3	5	16	37	50	73	75	126

127

```
// requires array is sorted
// returns whether k is in array
boolean find(int[] arr, int k) {
 for(int i=0; i < arr.length; ++i)
    if(arr[i] == k)
      return true;
 return false;
}
```

Best case: 6 → O(1)

Worst case: n·5 → O(n)

Example Solution: Linear Search Runtimes

Find an integer in a *sorted* array

2	3	5	16	37	50	73	75	126

```
// requires array is sorted
// returns whether k is in array
boolean find(int[] arr, int k) {
 for(int i=0; i < arr.length; ++i)
    if(arr[i] == k)
      return true;
 return false;
}
```

Best case: 6 “ish” steps = $O(1)$

Worst case: 5 “ish” * (arr.length) + 1
= O(arr.length)

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Ignoring Constant Factors (1 of 2)

- Binary search is $O(\texttt{log}\ n)$ and linear is $O(n)$
  - But which is actually ***faster***?
  - Depending on ***constant factors*** and ***size of n***, in a particular situation, linear search could be faster!
- Depends on constant factors:
  - How *many* assignments, additions, etc. for each *n*
- Depends on size of *n:*
  - Remember: "Descriptive of large inputs; most algorithms are 'good enough' for small inputs"
  - Each data structure's and algorithm's behavior can vary for every finite N
  - So we pick $N \rightarrow \infty$ as our definition of "large"

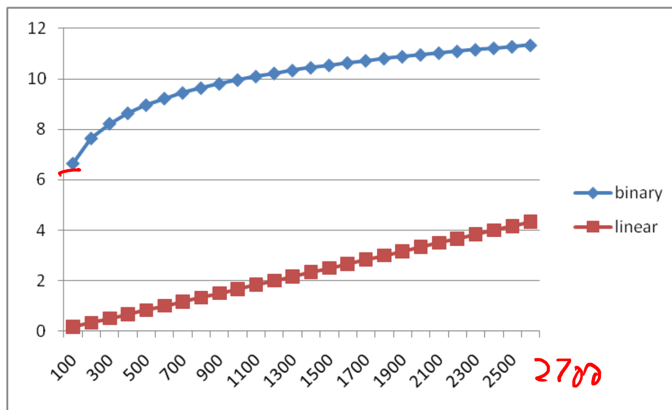
Ignoring Constant Factors (2 of 2)

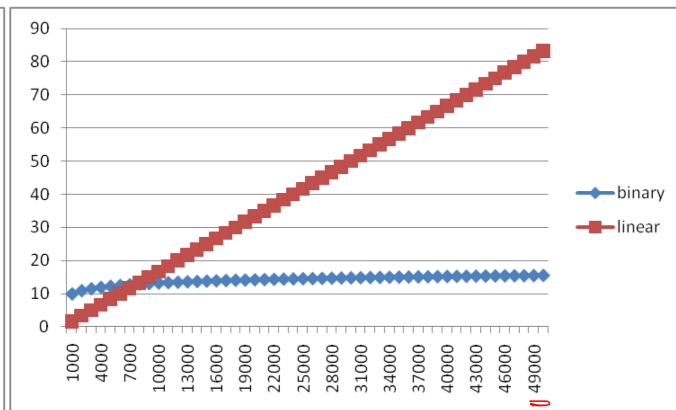
- How formalize the intuitive idea of how an algorithm behaves as N→∞?
  - There exists some n_0 such that for all $n > n_0$ ***binary search "wins"***

- Let's play with a couple plots to get some intuition…

Example: Binary Search vs Linear Search

- Let's "help" linear search "win"
  - Run it on a computer 100x as fast (say 2018 model vs. 1990)
  - Use a new compiler/language that is 3x as fast
  - Be a clever programmer to eliminate half the work
  - Each iteration is 600x as fast as in binary search





When we're dealing with infinity, constants and lower-order terms don't meaningfully add to the final result

Intuitive Simplifications

- When we're dealing with infinity, constants and lower-order terms don't meaningfully add to the final result

- (1) Eliminate lower-order terms
  - $6 + \frac{1}{2}N^2 + \frac{3}{2}N + 1 + \frac{1}{2}N^2 + \frac{1}{2}N + \frac{1}{2}N^2 - \frac{1}{2}N + N^2 + N$
  - $6 + \frac{1}{2}N^2 + \frac{3}{2}N + 1 + \frac{1}{2}N^2 + \frac{1}{2}N + \frac{1}{2}N^2 - \frac{1}{2}N + N^2 + N$
  - $\frac{5}{2}N^2$

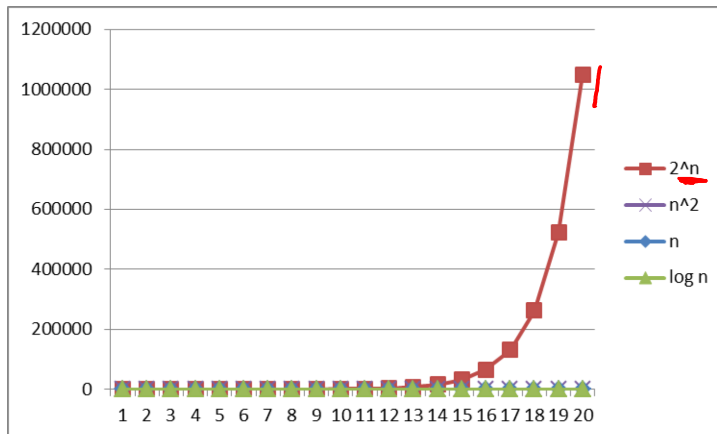
- (2) Ignore multiplicative constants
  - $\frac{5}{2}N^2$
  - N^2

$O(n^2)$

Logarithms and Exponents

- Definition: $\log_2 x = y$ if $x = 2^y$
  - Note: since so much is binary in CS, `log` almost always means $\log_2$

- Just as exponents grow *very* quickly, logarithms grow *very* slowly
  - So, $\log_2$ 1,000,000 = "a little under 20"



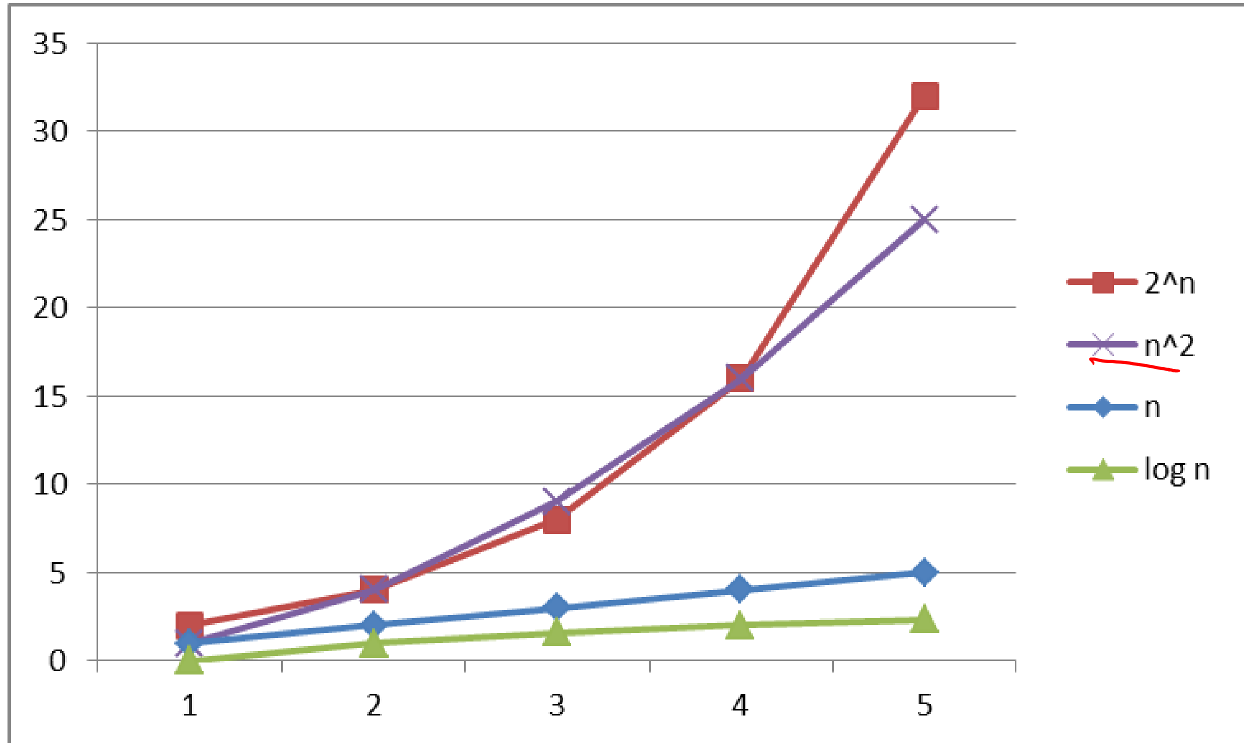
Log base doesn't matter (much)

- "Any base *B* log is equivalent to base 2 log within a constant factor"
  - ***And we are about to prove constant factors don't matter!***
  - In particular, $\log_2 x = 3.22 \log_{10} x$
- Why a constant multiplier ?
  - $\log_B x = (\log_A x) / (\log_A B)$

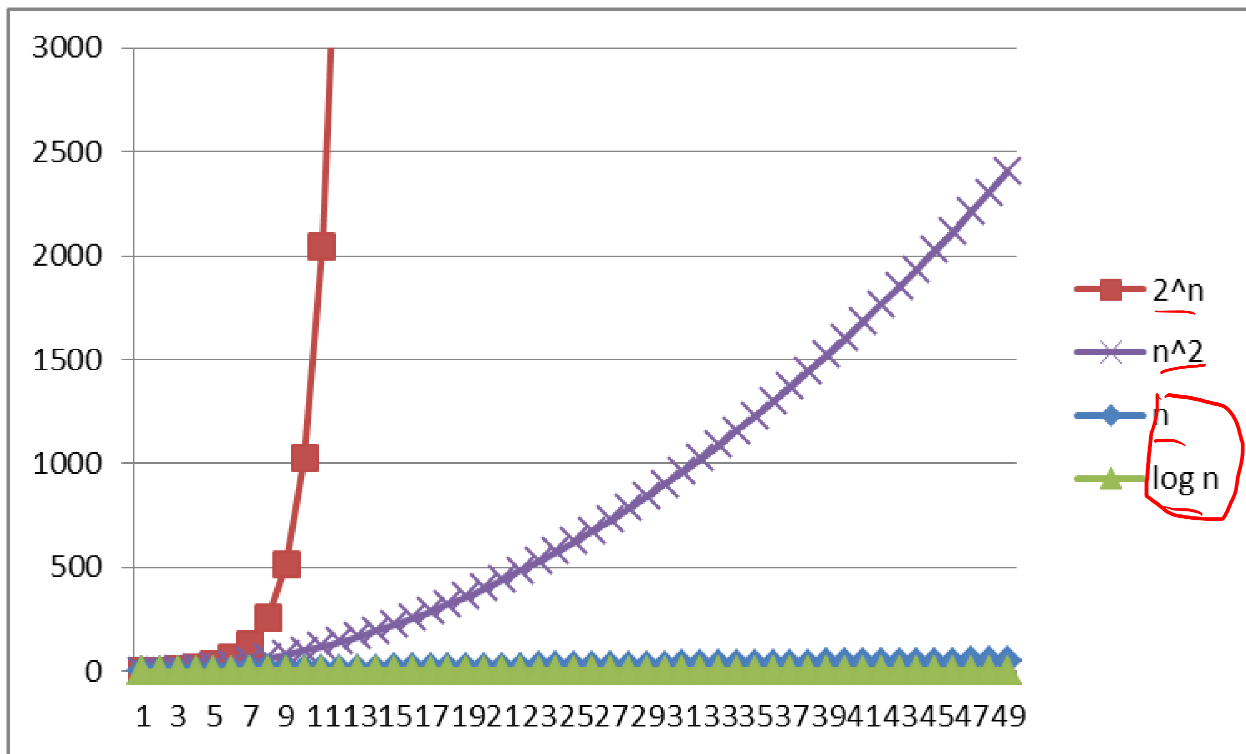
Review: Properties of logarithms

- `log(A*B) = log A + log B`
  - So `log(N`^{k}`)= k log N`
- `log(A/B) = log A – log B`
- `x =` $\log_2 2^x$
- `log(log x)` is written `log log x`
  - Grows as slowly as 2^{2^y} grows fast
  - Ex: $\log_2 \log_2 4billion \sim \log_2 \log_2 2^{32} = \log_2 32 = 5$
- `(log x)(log x)` is written `log`^{2}`x`
  - It is greater than `log x` for all `x > 2`

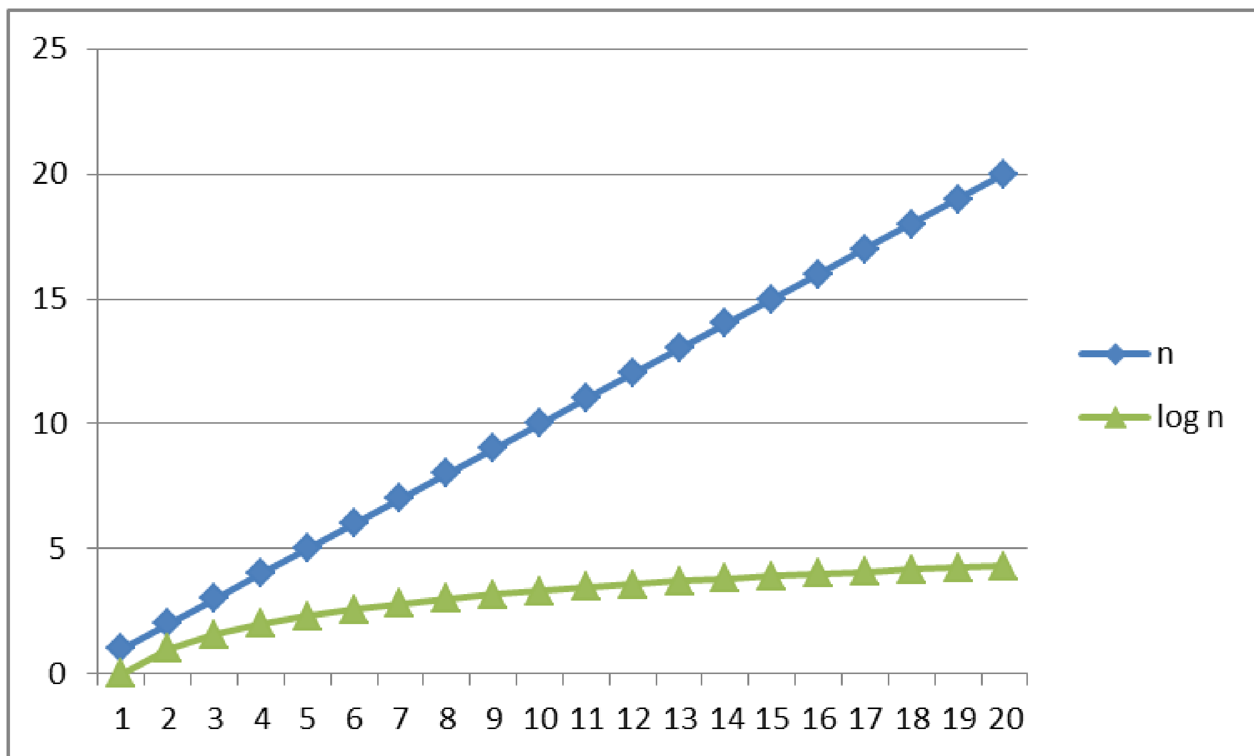
Logarithms and Exponents



Logarithms and Exponents



Logarithms and Exponents



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Introduction: Asymptotic Notation

- About to show formal definition, which amounts to our earlier intuitive simplifications:
  - Eliminate lower-order terms
  - Ignore multiplicative constants

- Examples:
  - $4n + 5$ — $O(n)$
  - $0.5n \log n + 2n + 7$ — $O(n \log n)$
  - $n^3 + 2^n + 3n$ — $O(2^n)$
  - $n \log (10n^2)$ — $O(n \log n)$

Big-Oh relates functions

- We use O on a function f(n) (for example n^2) to mean *the set of functions with asymptotic behavior* ***less than or equal to*** f(n)

- So ($3n^2+17$) **is in** $O(n^2)$
  - $3n^2+17$ and n^2 have the same **asymptotic behavior**

- Confusingly, we also say/write:
  - ($3n^2+17$) **is** $O(n^2)$
  - ($3n^2+17$) $\in$ $O(n^2)$
  - ($3n^2+17$) **=** $O(n^2)$ ← *least ideal*

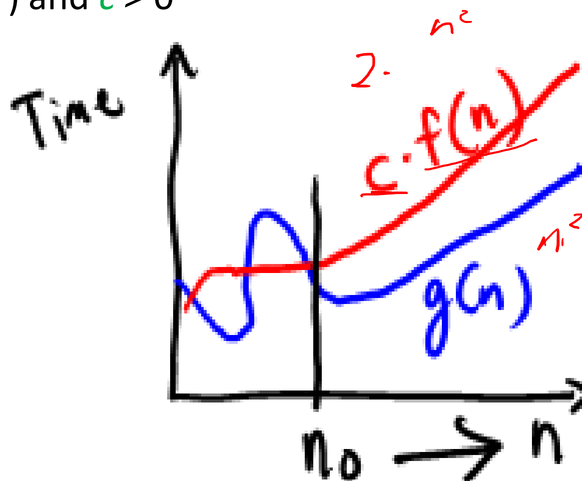
- But we would never say $O(n^2)$ = ($3n^2+17$)

Big-Oh, Formally (1 of 3)

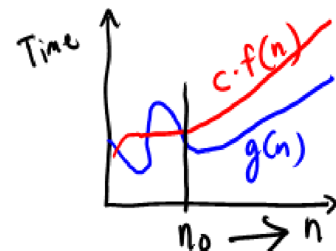
Definition: $g(n)$ is in $O(\ f(n)\ )$ iff there exist positive constants c and n_0 such that

$g(n) \leq c\ f(n)$ for all $n \geq n_0$

Note: $n_0 \geq 1$ (and a natural number) and $c > 0$



Big-Oh, Formally (2 of 3)

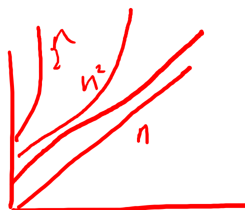


> Definition: **g(*n*)** is in O(**f(*n*)**) iff there exist positive constants ***c*** and n_0 such that
>
> **g(*n*)** ≤ ***c*** **f(*n*)** for all ***n*** ≥ n_0

Note: $n_0 \geq 1$ (and a natural number) and *c* > 0

To show **g(*n*)** is in O(**f(*n*)**), pick a ***c*** large enough to "cover the constant factors" and n_0 large enough to "cover the lower-order terms"

O(?)



$n \in O(n)$

$n \in O(n^2)$

$n \in O(2^n)$

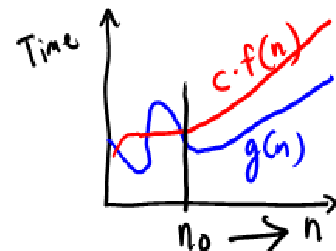
$n + 4$ $O(n)$

$n_0 = 5$

$c = 2$

$n + 4 \leq 2n$

Big-Oh, Formally (3 of 3)



Definition: g(n) is in O(f(n)) iff there exist positive constants c and n_0 such that

$g(n) \leq c\, f(n)$ for all $n \geq n_0$

Note: $n_0 \geq 1$ (and a natural number) and $c > 0$

Example: Let g(n) = 3n + 4 and f(n) = n

c = 4 and n_0 = 5 is one possibility

$3n+4 \leq 4n \quad \forall n \geq 5$

$\therefore 3n+4 \in O(n)$

Example: Let g(n) = 3n + 4 and f(n) = n^5

c = 3 and n_0 = 2 is one possibility

$3n+4 \leq 3n^5 \quad \forall n \geq 2$

$\therefore 3n+4 \in O(n^5)$

Example: Let g(n) = 3n + 4 and f(n) = 2^n

c = 100000000 and n_0 = 1 is one possibility

$3n+4 \leq 100000000 \cdot 2^n \quad \forall n \geq 1$

$\therefore 3n+4 \in O(2^n)$