# Fork/Join Parallelism and Its Analysis

CSE 332 Spring 2020

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# Announcements

- Quiz 3 released, due Friday

- Lecture questions: pollev.com/cse332

# Learning Objectives

- Articulate the high-level differences between "raw" threads and the fork/join-style parallelism
- Write embarrassingly-parallel code using primitives from the ForkJoin library
- Recognize when an algorithm can use maps and reductions
  - … and use maps and reductions to describe (parallel) algorithms
- Understand how to use work, span, speedup, and parallelism to calculate asymptotically optimal runtimes

# Lecture Outline

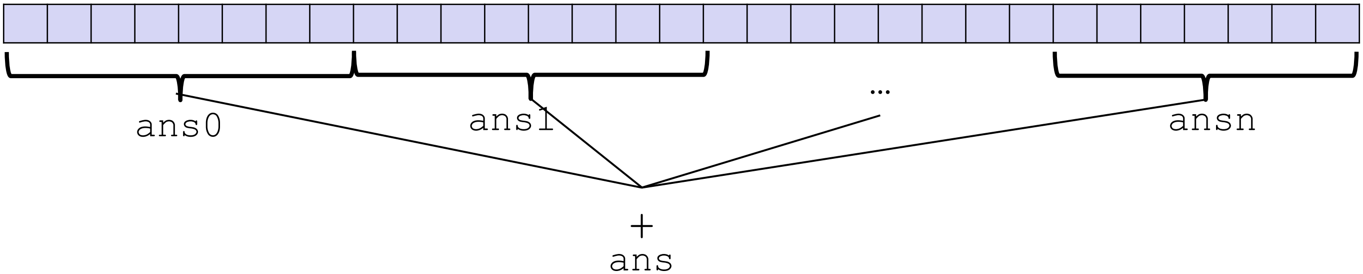
- Concurrency Frameworks in Java
  - Improving java.lang.Thread
    - **Asymptotically**
    - Constants
  - ForkJoin Library

- More examples of parallel programs
  - Common patterns: reduce and map
  - Non-array inputs

- Asymptotic Analysis for Fork/Join-style Parallelism

# Review: Many Small Chunks

- The solution: *cut up our problem into many small chunks*
  - We want far more chunks than the number of processors!
  - … but this will require changing our algorithm




1. *Portable?* Yes! (Substantially) more chunks than processors
2. *Adapts to Available Processors?* Yes! Hand out chunks as you go
3. *Load Balanced?* Yes(ish)! Variation is smaller if chunks are small

# Review: Abandoning java.lang.Thread

- For this specific problem (and for p3), the constants for Java's built-in Thread implementation are not great
  - Plus, there's complexity in Java's Thread that confuse rather than illuminate

- Also, the parallelism model is harder to reason about asymptotically

# Naïve Thread Creation/Joining Algorithm

- Suppose we create 1 thread to process every 1000 elements

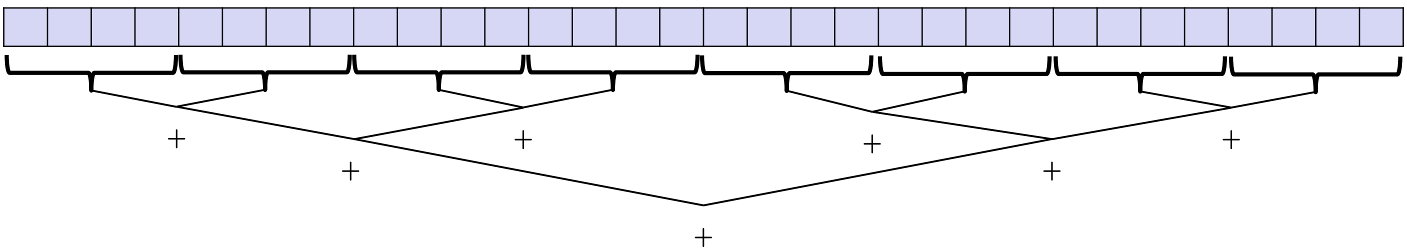
```
int sum(int[] arr){
  …
  int numThreads = arr.length / 1000;
  SumThread[] ts = new SumThread[numThreads];
  …
}
```

- "Combine results" part has `arr.length/1000` additions
  - Θ(N) to combine!
  - Previously, we had only 4 pieces (Θ(1) to combine)

- Will a Θ(N) algorithm to create threads/combine results be a bottleneck?

# Smarter Thread Creation/Joining: Divide and Conquer!

- Divide and Conquer:
  - "Grows" the number of threads to fit the problem
  - Uses parallelism for the recursive calls
  - This style of parallel programming is called "fork/join"




- Fork/Join Phases:

1. Divide the problem
   - Start with full problem at root
   - Make two new threads, halving the problem, until size is at cutoff
2. Combine answers as we return from recursion

# Fork/Join-style Parallelism (1 of 2)

```
class SumThread extends java.lang.Thread {
  // … member fields and constructors elided …
  public void run() {  // override: implement "main"
    if(hi - lo < SEQUENTIAL_CUTOFF) {
      // Just do the calculation in this thread
      for (int i=lo; i < hi; i++)
        ans += arr[i];
    }
    else {
      // Create two new threads to calculate the left and right sums
      SumThread left = new SumThread(arr, lo, (hi+lo)/2);
      SumThread right= new SumThread(arr, (hi+lo)/2, hi);
      left.start();
      right.start();

      // Combine their results
      left.join();  // don't move this up a line (why?)
      right.join();
      ans = left.ans + right.ans;
    }
  }
}
```

# Fork/Join-style Parallelism (2 of 2)

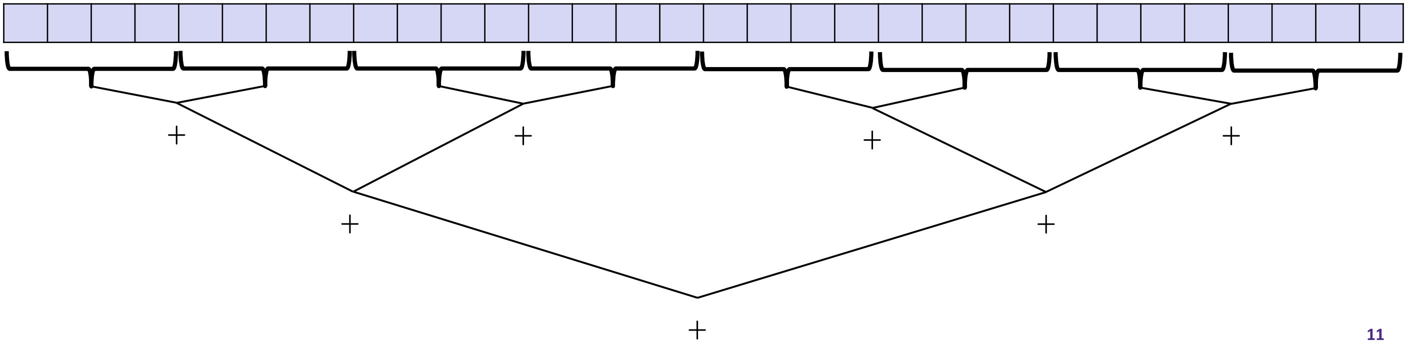
```
class SumThread extends java.lang.Thread {
  int lo, int hi, int[] arr; // input: arguments
  int ans = 0;               // output: result
  SumThread(int[] a, int l, int h) { … }
  public void run(){ … }     // override: implement "main"
}
```

```
int sum(int[] arr) {
  SumThread t = new SumThread(arr, 0, arr.length);// just 1 obj since
  t.run();                                        // we don't need
  return t.ans;                                   // parallelism to
}                                                 // start recursion
```

- The computation and the result-combining are both in parallel
  - Using recursive divide-and-conquer makes this natural
  - Easier to write *and* more efficient asymptotically!

# Fork/Join-style Parallelism Really Works!

- The key is in parallelizing both the executor-creation and the result-combining phases
  - If enough processors, runtime is **height of the tree:** $O(\texttt{log}\ n)$
    - Optimal and exponentially faster than sequential $O(n)$
  - Relies on operations being associative (like +) $(a+b)+c = a+(b+c)$
- We'll write all our parallel algorithms in this style
  - But using a special library engineered for this style



# Being Pragmatic #1: Performance Tuning

- Wait until computer has more processors ;)
  - Communication overhead may still dominate at 4 processors, but this configuration is rare for servers (circa 2020)
    - attu6 has 4 CPUs with 14 cores each = 56 "processors"

See /proc/cpuinfo on the attus

- Beware memory-hierarchy issues!
  - Won't focus on this, but crucial for parallel performance

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    - Asymptotically
    - **Constants**
  - ForkJoin Library
- More examples of parallel programs
  - Common patterns: reduce and map
  - Non-array inputs
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# Poll Everywhere

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- Assume that thread creation and joining are expensive. Which of the following optimizations might improve our constants?
  1. Use a cutoff, after which computation proceeds sequentially
  2. Somehow create fewer threads
  3. Somehow reuse threads when they're done

A. Cutoff only

B. Cutoff + Fewer Threads

C. Cutoff + Thread Reuse

D. Cutoff + Fewer Threads + Thread Reuse

E. I'm not sure …

# Being Pragmatic #2: Constants Matter

- *In theory*, can divide down to single elements, do all the result-combining in parallel, and get optimal speedup
  - Total time: O(n / numProcessors + log n)

- *In practice*, thread creation/joins eat into the savings, so:
  1. Use a cutoff, after which computation proceeds sequentially
     - Cutoff value depends on type of computation; 500-1000 is a good start
     - Eliminates *almost all* the recursive thread creation (bottom levels of tree)
     - *Exactly* like QuickSort switching to InsertionSort, but more important here
  2. Do not create *two* recursive threads; create one thread and do the other piece of work “yourself”
     - Halves the number of threads created (?!?!)

# Halving the Created Threads: Code

- If the *language* had built-in support for fork/join-style parallelism, this hand-optimization would be unnecessary
- But the *library* we're using expects you to do it yourself
  - … and the difference is surprisingly substantial
- Again: no difference in theory, "only" the constants

run() is a nomal function call! Execution won't proceed until it completes

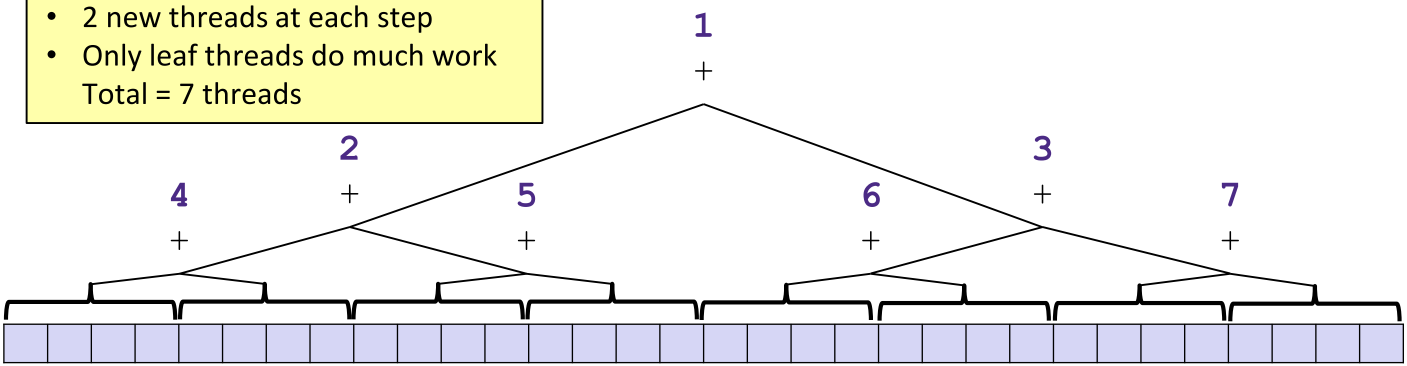
```
// Don't do this:
SumThread left  = …
SumThread right = …

left.start();
right.start();

left.join();
right.join();
ans = left.ans + right.ans;
```

```
// Do this instead:
SumThread left  = …
SumThread right = …

left.start();
right.run();

left.join();
// no right.join() needed
ans = left.ans + right.ans;
```

# Halving the Created Threads: Pictorially

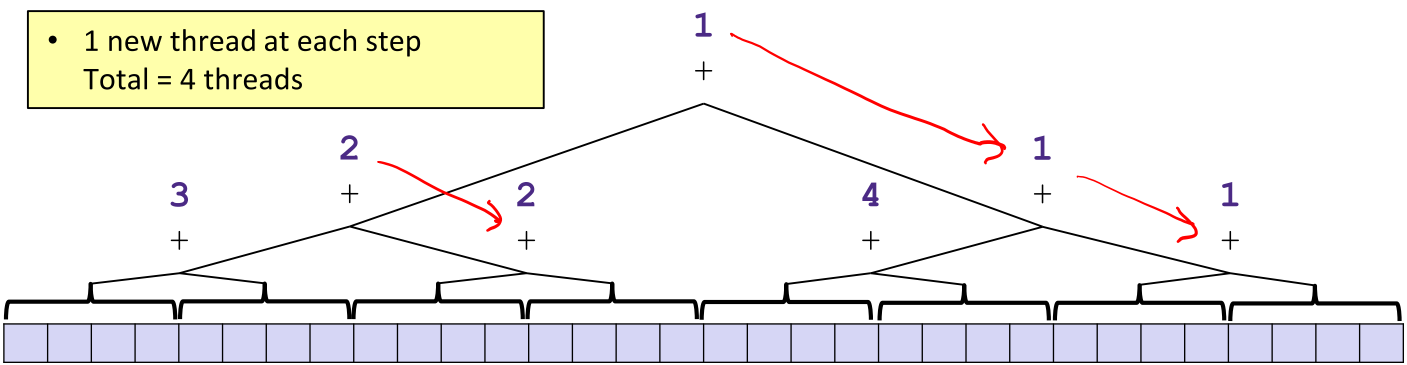
- 2 new threads at each step
- Only leaf threads do much work

  Total = 7 threads




- 1 new thread at each step

  Total = 4 threads



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# Finally! The ForkJoin Library

- Even using fork/join-style code, java.lang.Thread is still too "heavyweight"
  - Constant factors, especially space overhead
  - Creating 20,000 Java threads just a bad idea ☹
- So use the **ForkJoin Library** instead
  - Introduced in Java 8 (2014)
  - Similar libraries available for other languages
    - C/C++: Cilk (inventors), Intel's Thread Building Blocks
    - C#: Task Parallel Library
    - …
  - Its implementation is a fascinating but advanced topic

# Thread -> ForkJoin: Terminology

| Java Built-in Threads | ForkJoin Library |
|---|---|
| Subclass `Thread` | Subclass `RecursiveTask<V>` |
| Override `run()` | Override `compute()` |
| Call `start()` to begin parallel computation | Call `fork()` to begin parallel computation |
| Return results via member fields (eg, `ans`) | Return results via return value (ie, an instance of `V`) |
| Call `join()`, then check its "returned" member field | Call `join()`, then check its return value |
| Halve created threads by calling `run()` directly | Halve created threads by calling `compute()` directly |
| Begin recursion with top-level call to `run()` (instead of `start()`) | Begin recursion by creating a `ForkJoinPool` and calling its `invoke()` |

# Fork/Join-style Parallelism with ForkJoin (1 of 2)

```
class SumTask extends RecursiveTask<Integer> {
  int lo; int hi; int[] arr;     // just the "input" arguments!
```

No output params! Returns directly!

```
  protected Integer compute() {  // override: implement "main"
    if(hi - lo < SEQUENTIAL_CUTOFF) {
      // Just do the calculation in this thread
      int ans = 0;    // local variable instead of a member field
      for (int i=lo; i < hi; i++)
        ans += arr[i];
      return ans;     // direct return of answer
    } else {
      // Create ONE new thread to calculate the left sum
      SumTask left  = new SumTask(arr, lo, (hi+lo)/2);
      SumTask right = new SumTask(arr, (hi+lo)/2, hi);
      left.fork();    // create a thread and call its compute()
      int rightAns = right.compute(); // call compute() directly

      // Combine results
      int leftAns  = left.join();
      return leftAns + rightAns;
    }
  }
}
```

# Fork/Join-style Parallelism with ForkJoin (2 of 2)

```
class SumTask extends RecursiveTask<Integer> {
  int lo; int hi; int[] arr;          // input: arguments
  SumTask(int[] a, int l, int h) { … }
  protected Integer compute() { … } // override: implement “main”
}
```

```
static final ForkJoinPool POOL = new ForkJoinPool();
```

Required setup: instantiate a pool

```
int sum(int[] arr) {
  SumTask task = new SumTask(arr, 0, arr.length);

  // invoke() returns the value which is returned by the
  // top-level compute()
  return POOL.invoke(task);
}
```

# Being Pragmatic #2: Performance Tuning the Library

- Sequential threshold
  - Library documentation recommends doing approximately 100-5000 basic operations in each "piece" of your algorithm
- ForkJoin library needs to "warm up"
  - May see slow results before JVM re-optimizes the library internals
  - Put computations in a loop to see the "long-term benefit"

# Summary: Parallelism

- **Parallelism**: increasing efficiency/decreasing total runtime
- **Concurrency**: correctly accessing shared resources
  - They intersect when *parallel computations access shared resources*

- Model: shared memory with explicit threads:
  - **Threads** are the minimum fields necessary to represent "computation": a program counter and a stack
  - Everything else is **shared** (eg, static variables, heap)

- Threading:
  - `run()`/`compute()` are "regular" function calls, but `start()`/`fork()` create a new thread and then call `run()`/`compute()`
  - Parallelizing many small chunks of work is portable, adaptable, and load-balancable

# Summary: Fork/Join Parallelism

- **Fork/Join Parallelism** is a *model* that grows the parallelism to fit the problem size using recursion

- The **ForkJoin library** that cleanly enables this model
  - You still need to manually specify the sequential cutoff
  - Halving the created threads also requires manual intervention

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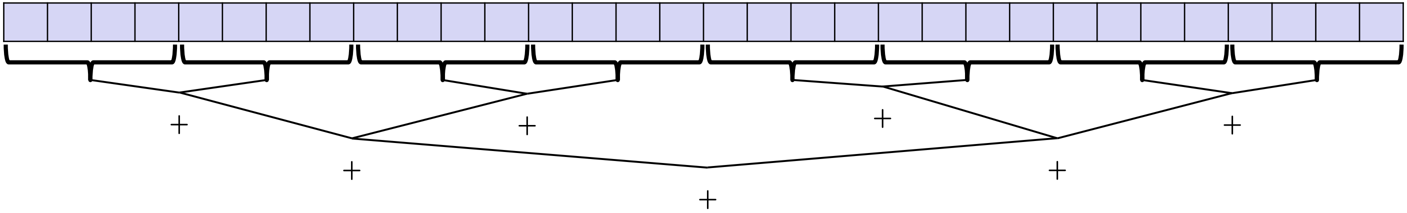
- Concurrency Frameworks in Java
  - Improving java.lang.Thread
    - Asymptotically
    - Constants
  - ForkJoin Library
- More examples of parallel programs
  - **Common patterns: reduce and map**
  - Non-array inputs
- Asymptotic Analysis for Fork/Join-style Parallelism

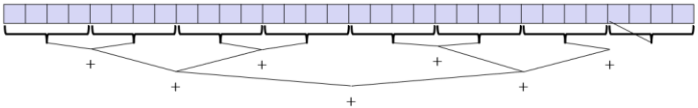
# A Common Pattern

- Summing went from $O(n)$ sequential to $O(\texttt{log}\ n)$ parallel
  - Assuming **a lot** of processors and very large n
  - Exponential speed-up in theory: $n$ / `log` $n$ grows exponentially)




- Any solution which can merge two subsolutions in $O(1)$ time has this property! Usually just need to "plug in" 2 parts:
  - How to compute the result at the cut-off
    *(Parallel-Sum: Iterate through sequentially and add up)*
  - How to merge results
    *(Parallel-Sum: Just <u>add</u> 'left' and 'right' results)*

# Examples



- Parallelization Pattern #1:
  - How to compute result at the ‘cut-off’
  - How to merge results

- Assume the input is an array; how would we do the following?
  1. Maximum or minimum element
  2. Is there an element satisfying some property (e.g., is there a 17)?
  3. Left-most element satisfying some property (e.g., first 17)
  4. Smallest rectangle encompassing a number of points
  5. Counts; for example, number of strings that start with a vowel
  6. Are these elements in sorted order?

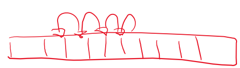
# A Common Pattern: Reductions

- This class of computations are called **reductions**
  - We ‘reduce’ a large array of data to a single final result
  - Intermediate results must be combined with an **associative operator**
  - *Examples*: max, count, leftmost, rightmost, sum, product, …

- Intermediate and final results can be “aggregates”: arrays or multi-field objects
  - *Example*: histogram from a much larger array of test results

- Some things are inherently sequential
  - *Example*: How we process **`arr[i]`** depends entirely on the result of processing **`arr[i-1]`**

for i from 1 to n
arr[i] = arr[i-1] + f()



DAG looks like a linked list!

# Another Common Pattern: Maps

- A **map** transforms each element of a collection independently, creating a new-but-same-sized collection of modified elements
  - No combining results

- *Example*: Vector addition

```
int[] vectorAdd(int[] arr1, int[] arr2) {
  assert(arr1.length == arr2.length);

  result = new int[arr1.length];
  FORALL (i=0; i < arr1.length; i++) {
    result[i] = arr1[i] + arr2[i];
  }
  return result;
}
```

- Using a map? Only need to "plug in" one part:
  - How to map element E to transformed E'
  - *(Vector-add: generate result[i] from arr1[i])*

# Maps in the ForkJoin Library (1 of 2)

- Many small tasks still helps with load balancing
  - Maybe not for vector-add, but definitely for compute-intensive maps
  - The forking is O(log n); theoretically other approaches are O(1)

```
class VectorAdd extends RecursiveAction {
  // input: arguments
  int lo; int hi; int[] res; int[] v1; int[] v2;

  protected void compute(){
    if(hi - lo < SEQUENTIAL_CUTOFF) {
      for(int i=lo; i < hi; i++)
        res[i] = v1[i] + v2[i];
    } else {
      int mid = (hi+lo)/2;
      VectorAdd left = new VectorAdd(lo, mid, res, v1, v2);
      VectorAdd right= new VectorAdd(mid, hi, res, v1, v2);
      left.fork();
      right.compute();
      left.join();
    }
  }
}
```

# Maps in the ForkJoin Library (2 of 2)

```
class VectorAdd extends RecursiveAction {
  // input: arguments
  int lo; int hi; int[] res; int[] v1; int[] v2;

  protected void compute() { … } // override: implement "main"
}
```

```
static final ForkJoinPool POOL = new ForkJoinPool();

int[] add(int[] arr1, int[] arr2){
  assert (arr1.length == arr2.length);

  // Use ans as an "output argument" instead of looking at the
  // top-level compute()'s return value (which is void).
  int[] ans = new int[arr1.length];

  POOL.invoke(new VectorAdd(0, arr.length, ans, arr1, arr2);
  return ans;
}
```

# Map and Reduce in the ForkJoin Library

- Map (vector-add)
  - `VectorAdd` extended `RecursiveAction`
  - Result was an output parameter; nothing returned from `compute()`
- Reduce (parallel-sum):
  - `SumTask` extended `RecursiveTask`
  - Result directly returned from `compute()`
- … but it doesn't *have* to be this way
  - Map could've used `RecursiveTask` to return an array
  - Reduce could've used `RecursiveAction` and returned result as an output parameter

# Maps and Reductions, Generally

- Maps and reductions are the "workhorses" of parallel programming
  - By far, the two most important and common patterns
    - Two more-advanced patterns in next lecture
- Remember the learning objectives!
  - Recognize when an algorithm can use maps and reductions
  - Use maps and reductions to describe (parallel) algorithms
- Goal: programming them becomes "trivial"
  - Exactly like sequential for-loops seem second-nature nowadays

# Digression: MapReduce on clusters

- You may have heard of Google’s “map/reduce”
  - Or the open-source version Hadoop
- Performs maps and reduces using many machines
  - System takes distributes input data and manages fault tolerance
  - You just write code to map one element and reduce elements to a combined result
- Separates how the recursive divide-and-conquer “frame” from the computation to perform
  - An old idea in higher-order functional programming, transferred to large-scale distributed computing
  - Complementary approach to declarative queries for databases

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# Parallelized Computation on Trees

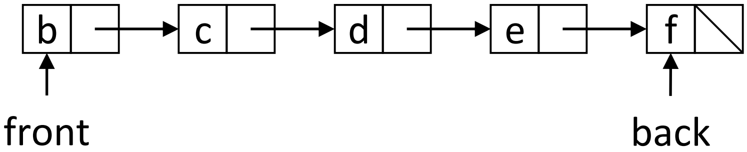
- Maps and reductions work on trees
  - Divide-and-conquer each child rather than array sub-ranges
  - Correct for unbalanced trees, but won't get much speed-up unless tree is balanced

- *Example*: minimum in an *unsorted-*but-balanced binary tree
  - $O($`log` $n)$ time given enough processors

- How to do the sequential cut-off?
  - Store number-of-descendants at each node (easy to maintain)
  - Or could approximate it with, e.g., AVL-tree height

# Parallelized Computation on Linked Lists

- Can you parallelize maps or reduces over linked lists?
  - *Example*: Increment all elements of a linked list
  - *Example*: Sum all elements of a linked list




- Parallelism still helps with expensive per-element operations

- Once again, data structures matter!
  - Balanced trees allow faster access to all the data: O(log n) vs. O(n)
  - Trees and lists have the same flexibility compared to arrays (eg, inserting an item in the middle of the list)

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# Analyzing Parallel Algorithms

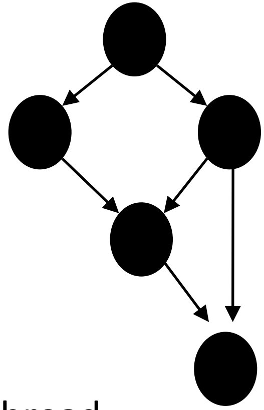
- How to measure efficiency?
  - Want asymptotic bounds
  - Want an analysis that's independent of a specific number of processors
- Fork/Join parallelism gets *asymptotically optimal* runtime for the available number of processors
  - So we can analyze algorithms assuming this guarantee

# Modelling Fork/Join Parallelism with DAGs

- A program execution using can be modeled as a DAG
  - Nodes: Pieces of work
  - Edges: Source must finish before destination can start

A directed acyclic graph (DAG) is:
- A graph that is directed (edges have direction/arrows)
- And whose edges do not create a cycle (ability to trace a path that starts and ends at the same node)

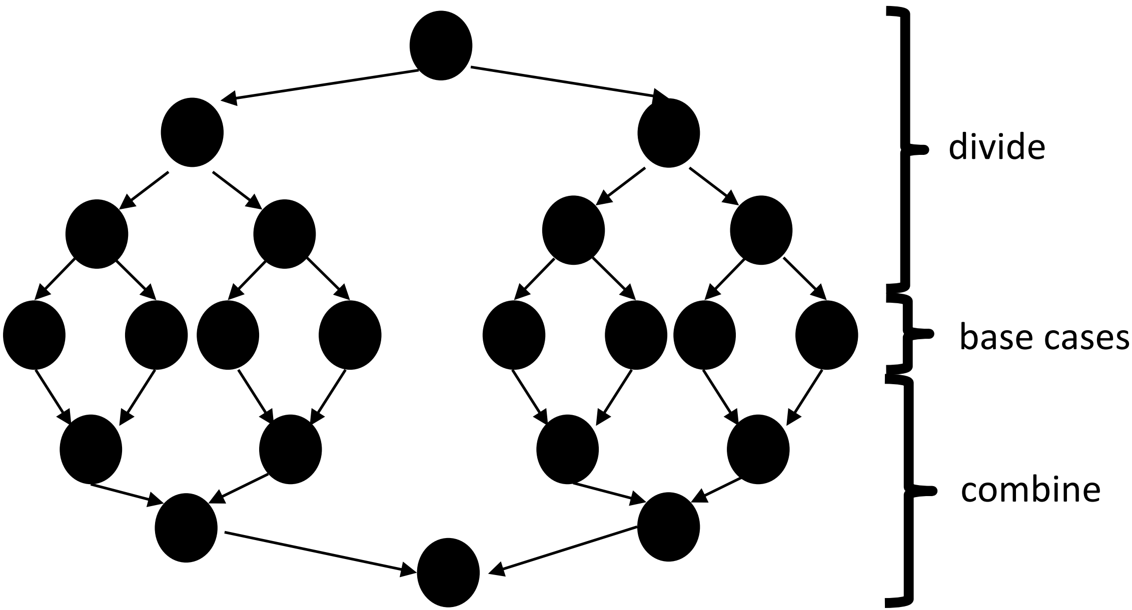
- A `fork` makes two outgoing edges:
  - New thread
  - Continuation of current thread



- A `join` takes two incoming edges
  - The final node of the joined thread
  - The computation that just finished in the current thread

# Our Simple Examples

- `fork` and `join` are very flexible, but maps and reductions use them in a very basic way
  - A (perfect) tree, on top of an upside-down (perfect) tree



# Aside: More Interesting DAGs?

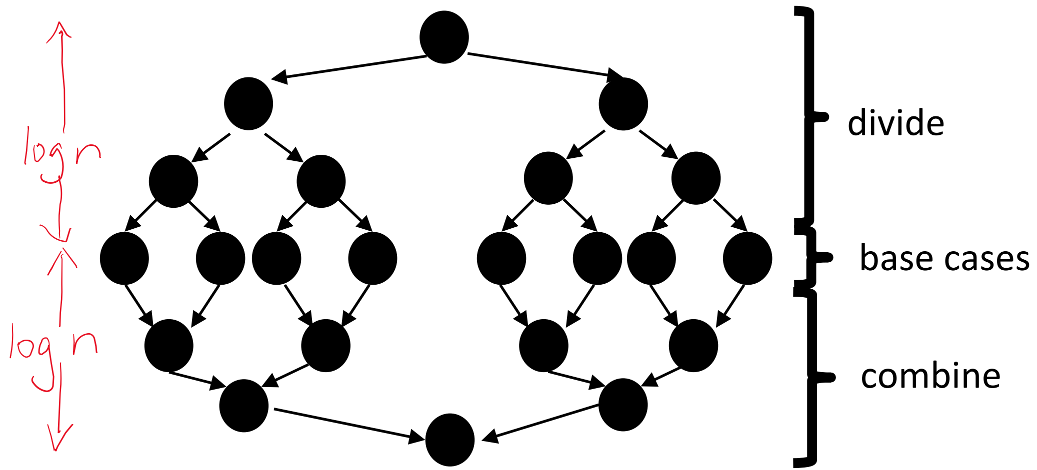
- The execution DAGs are not always this simple
  - *Example*: combining results might so expensive that we parallelize it. Then each node in the *inverted* tree would expand into another set of nodes for that parallel computation

# Definitions: Work and Span

- Let $\mathbf{T_P}$ be the *running time* if there are **P** *processors* available
- Two important definitions:
  - **Work**: How long it would take with 1 processor (ie, $\mathbf{T_1}$)
    - Just "sequentialize" the recursive forking
    - Cumulative work that all processors must complete
  - **Span**: How long it would take with infinitely many processors (ie, $\mathbf{T_\infty}$)
    - The hypothetical ideal; aka "critical path length" or "computational depth"
    - This is the longest "dependence chain" in the computation
    - *Example*: $O($`log` $n)$ for summing an array
      - Notice how having $>n/2$ processors doesn't reduce the span

# Our Simple Examples + Our Definitions

- In this context, the span ($T_\infty$) is:
  - The longest dependence-chain; i.e., longest ‘branch’ in parallel ‘tree’
  - *Example*: O(log n) for summing an array
    - We halve the data down to our sequential cut-off, then add back together
    - 2 * log n steps, O(1) time for each: O(log n)



# Work and Span in Fork/Join-style DAGs

- **Span** ($T_\infty$) = sum of runtime of all nodes in the DAG's *most-expensive path*
  - Note: costs are on the nodes not the edges
  - O(log n) for simple maps and reductions

- **Work** ($T_1$) = sum of runtime of all nodes in the DAG
  - Any topological sort is a legal execution
  - O(n) for simple maps and reductions

# More Definitions: Speed-up and Parallelism

$T_1 = 100s$
$T_4 = 50s$ } speed-up = 2

- **Speed-up**, using **P** processors: $\mathbf{T_1 / T_P}$
- If speed-up is **P** as we vary **P**, we call it **perfect linear speed-up**
  - Perfect linear speed-up means doubling **P** halves running time
  - Usually our goal, but hard to get in practice

If $T_4 = 25s$ in prev. example, would've had perfect linear speed-up

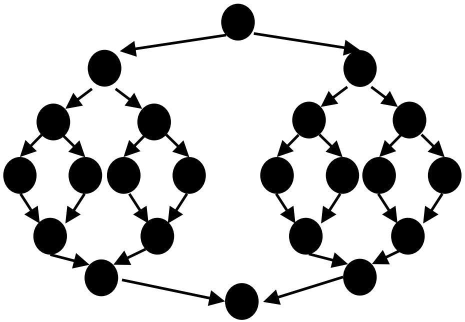
- **Parallelism**: $\mathbf{T_1 / T_\infty}$
  - Parallelism is the maximum possible speed-up; the point at which adding processors doesn't help
  - That point depends on the span

If $T_\infty = 5s$, parallelism = 20

*Parallel algorithms attempt to decrease span without increasing work too much*

# Obtaining Optimality for $T_P$

- What is the asymptotically optimal $\mathbf{T_P}$, for any value of **P**?
  - (as usual, we ignore memory-hierarchy issues; i.e. caching)



- We know $\mathbf{T_P}$ is greater than or equal to:
  - $\mathbf{T_1 / P}$ *(why?)*
  - $\mathbf{T_\infty}$ *(why?)*

- So an *asymptotically optimal* execution must be:

$$O(\,(T_1/P) + T_\infty\,)$$

  - First term dominates for small **P**, second for large **P**

# Optimal $T_P$: Thanks, ForkJoin library!

- The ForkJoin library gives an *expected-time guarantee* of asymptotically optimal!
  - "Expected time" because it flips coins when scheduling

- To obtain this guarantee, our job as ForkJoin library users is to make all the nodes in our execution DAG *small-ish* and *approximately equal*

- In exchange, the library-writers:
  - Assign work to avoid **idling**; we can ignore **scheduling** issues
  - Keep constant factors low
  - Give the **expected-time optimal guarantee** (assuming the library user did their job): $\mathbf{T_P = O((T_1 / P) + T_\infty)}$