CSE 332: Data Structures and Parallelism

Exercises (Graphs)

Directions: *Submit your solutions on **Gradescope**. You must submit a pdf file.*

EX15. Diijjkkstra? (20 points)

Please use **directed** graphs in your examples for this problem.

(a) [7 Points] If there is more than one minimum cost path from x to y, will Dijkstra's Algorithm always find the path with the fewest edges? If not, explain in a few sentences how to modify Dijkstra's algorithm so that if there is more than one minimum cost path from x to y, a path with the fewest edges is chosen. Assume no negative-cost edges. Please use **directed** graphs in your examples for this problem.

(b) [6 Points] Give a concrete example (show the DIRECTED graph and the starting vertex) where Dijkstra's Algorithm gives the wrong answer in the presence of a negative-cost edge but no negative-cost cycles. Explain briefly why Dijkstra's Algorithm fails on your example. The example need not be complex; it is possible to demonstrate the point using as few as 3 vertices.

(c) [7 Points] Suppose you are given a weighted directed graph that has at least one negative-cost edge but no negative-cost cycles. We want to find the shortest paths, where the cost of a path is the sum of the weights of edges along the path. Consider the following strategy to find shortest paths in this graph: Uniformly add a constant k to the cost of every edge, so that all costs become non-negative, then run Dijkstra's Algorithm and return that result with the edge costs reverted back to their original values (i.e., with k subtracted).

- Give a concrete example in the form of a directed graph and a starting vertex, where this technique fails (Dijkstra's would not find what is actually the shortest path) and explain why it fails.
- Also, give a general explanation as to why this technique does not work. Think about your example and why the original least cost path is no longer the least cost path after adding k.