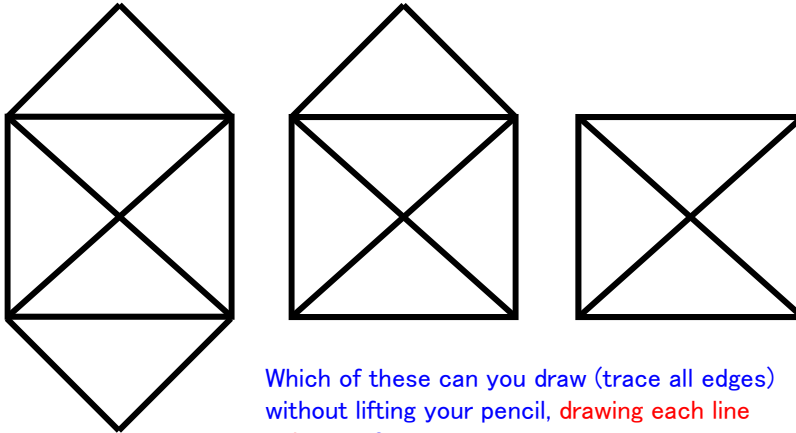


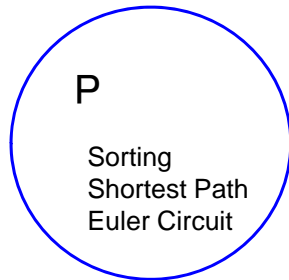
Try it!



Which of these can you draw (trace all edges) without lifting your pencil, drawing each line only once?
Can you start and end at the same point?

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Hamiltonian Circuit
Satisfiability (SAT)
Vertex Cover
Travelling Salesman

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Euler Circuits

- **Euler circuit**: a path through a graph that *visits each edge exactly once and starts and ends at the same vertex*
- Named after Leonhard Euler (1707-1783), who cracked this problem and founded graph theory in 1736
- An **Euler circuit** exists *iff*
 - the graph is connected and
 - each vertex has **even** degree (= # of edges on the vertex)

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The Road Inspector: Finding Euler Circuits

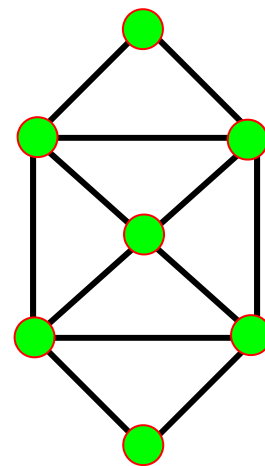
Given a connected, undirected graph $G = (V, E)$, find an Euler circuit in G

Can check if one exists:

- Check if all vertices have even degree

Basic Euler Circuit Algorithm:

1. Do an edge walk from a start vertex until you are back to the start vertex.
 - You never get stuck because of the even degree property.
2. “Remove” the walk, leaving several components each with the even degree property.
 - Recursively find Euler circuits for these.
3. Splice all these circuits into an Euler circuit

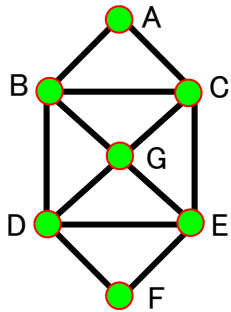


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Running time?

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Euler Circuit Example



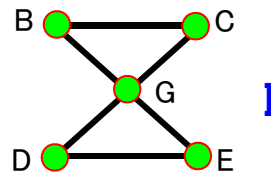
Euler(A) :

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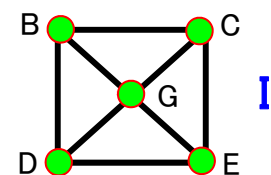
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Hamiltonian Circuits

- **Euler circuit:** A cycle that goes through each *edge* exactly once
- **Hamiltonian circuit:** A cycle that goes through each *vertex* exactly once
- Does graph I have:
 - An Euler circuit?
 - A Hamiltonian circuit?
- Does graph II have:
 - An Euler circuit?
 - A Hamiltonian circuit?
- Which problem sounds harder?



I



II

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Polynomial vs. Exponential Time

- All of the algorithms we have discussed in this class have been **polynomial time** algorithms:
 - Examples: $O(\log N)$, $O(N)$, $O(N \log N)$, $O(N^2)$
 - Algorithms whose running time is $O(N^k)$ for some $k > 0$
- **Exponential time** b^N is asymptotically worse than any polynomial function N^k for any k

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The Complexity Class P

- **P** is the set of all problems that can be solved in *polynomial worst case time*
 - All *problems* that have some *algorithm* whose running time is $O(N^k)$ for some k
- **Examples of problems in P:**
sorting, shortest path, Euler circuit, *etc.*

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