CSE 332: Data Structures & Parallelism

Lecture 15: Analysis of Fork-Join Parallel Programs

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Autumn 2020

Outline

Done:

- How to use **`fork`** and **`join`** to write a parallel algorithm
- Why using divide-and-conquer with lots of small tasks is best
  - Combines results in parallel

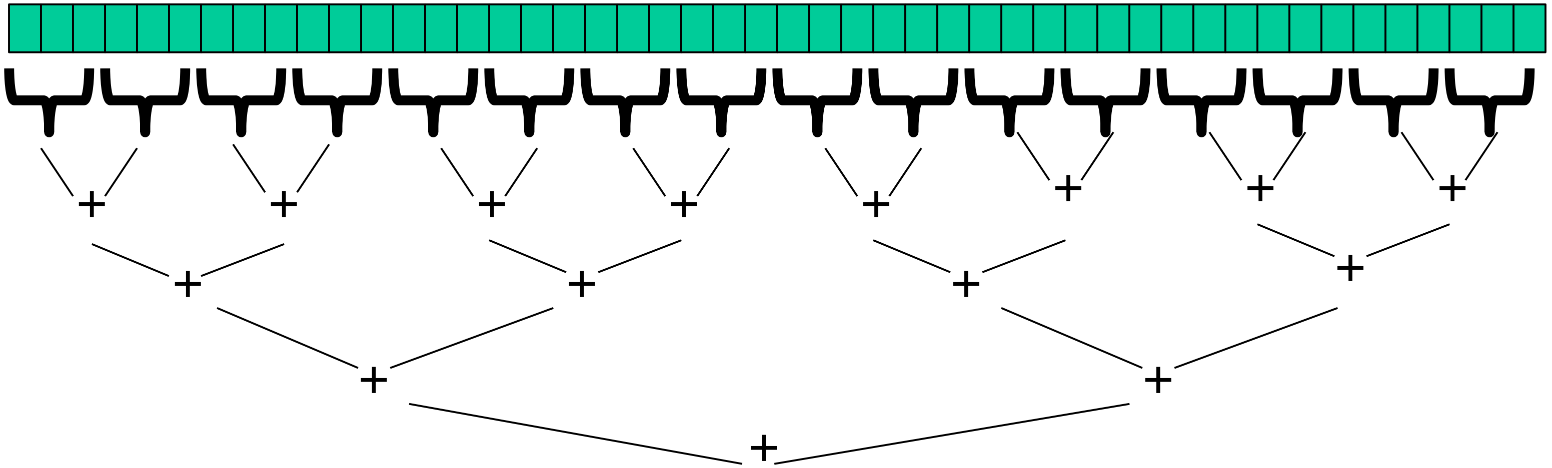
Now:

- More examples of simple parallel programs
- Arrays & balanced trees support parallelism better than linked lists
- Asymptotic analysis for fork-join parallelism
- Amdahl's Law

What else looks like this?

Saw summing an array went from $O(n)$ sequential to $O(\texttt{log}\ n)$ parallel (*assuming* ***a lot*** *of processors and very large n*)

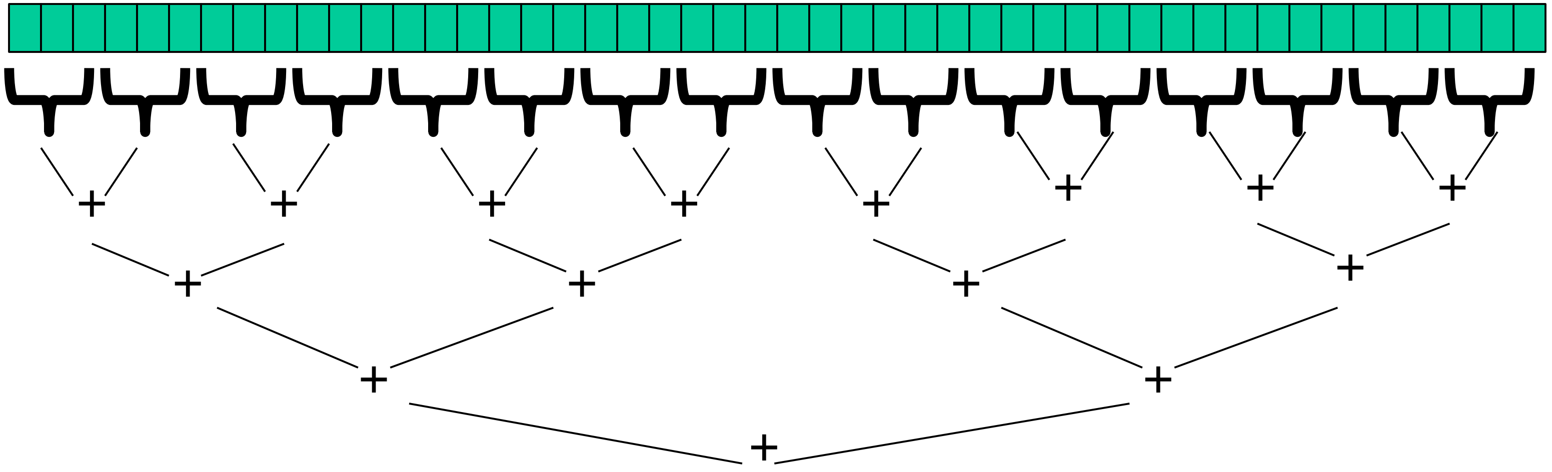
– Exponential speed-up in theory (n / $\texttt{log}\ n$ grows exponentially)



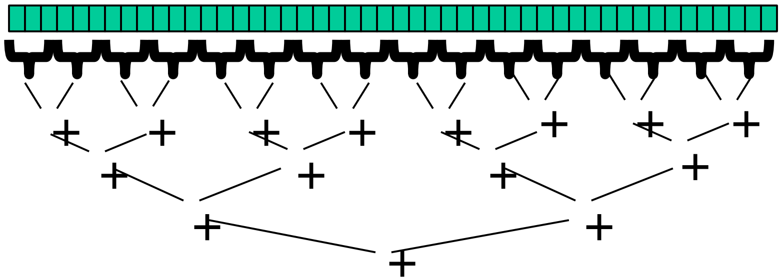
- Anything that can use results from two halves and merge them in $O(1)$ time has the same property…

Extending Parallel Sum

- We can tweak the ‘parallel sum’ algorithm to do all kinds of things; just specify 2 parts (usually)
  - Describe how to compute the result at the ‘cut-off’ (Sum: Iterate through sequentially and add them up)
  - Describe how to merge results (Sum: Just add ‘left’ and ‘right’ results)



Examples



- Parallelization (for some algorithms)
  - Describe how to compute result at the ‘cut-off’
  - Describe how to merge results
- How would we do the following (assuming data is given as an array)?
  1. Maximum or minimum element
  2. Is there an element satisfying some property (e.g., is there a 17)?
  3. Left-most element satisfying some property (e.g., first 17)
  4. Smallest rectangle encompassing a number of points
  5. Counts; for example, number of strings that start with a vowel
  6. Are these elements in sorted order?

Reductions

- This class of computations are called reductions
  - We ‘reduce’ a large array of data to a single item
  - Produce single answer from collection via an associative operator
  - Examples: max, count, leftmost, rightmost, sum, product, …
- Note: Recursive results don’t have to be single numbers or strings. They can be arrays or objects with multiple fields.
  - Example: create a Histogram of test results from a much larger array of actual test results
- While many can be parallelized due to nice properties like associativity of addition, some things are inherently sequential
  - How we process **`arr[i]`** may depend entirely on the result of processing **`arr[i-1]`**

Even easier: Maps (Data Parallelism)

- A map operates on each element of a collection independently to create a new collection of the same size
  - No combining results
  - For arrays, this is so trivial some hardware has direct support
- Canonical example: Vector addition

```
int[] vector_add(int[] arr1, int[] arr2){
  assert (arr1.length == arr2.length);
  result = new int[arr1.length];
  FORALL(i=0; i < arr1.length; i++) {
    result[i] = arr1[i] + arr2[i];
  }
  return result;
}
```

Maps in ForkJoin Framework

```
class VecAdd extends RecursiveAction {
  int lo; int hi; int[] res; int[] arr1; int[] arr2;
  VecAdd(int l,int h,int[] r,int[] a1,int[] a2){ … }
  protected void compute(){
    if(hi – lo < SEQUENTIAL_CUTOFF) {
      for(int i=lo; i < hi; i++)
        res[i] = arr1[i] + arr2[i];
    } else {
      int mid = (hi+lo)/2;
      VecAdd left = new VecAdd(lo,mid,res,arr1,arr2);
      VecAdd right= new VecAdd(mid,hi,res,arr1,arr2);
      left.fork();
      right.compute();
      left.join();
    }
  }
}
static final ForkJoinPool POOL = new ForkJoinPool();
int[] add(int[] arr1, int[] arr2){
  assert (arr1.length == arr2.length);
  int[] ans = new int[arr1.length];
  POOL.invoke(new VecAdd(0,arr.length,ans,arr1,arr2);
  return ans;
}
```

Maps and reductions

Maps and reductions: the “workhorses” of parallel programming

- By far the two most important and common patterns
  - Two more-advanced patterns in next lecture
- Learn to recognize when an algorithm can be written in terms of maps and reductions
- Use maps and reductions to describe (parallel) algorithms
- Programming them becomes “trivial” with a little practice
  - Exactly like sequential for-loops seem second-nature

Map vs reduce in ForkJoin framework

- In our examples:
- Reduce:
  - Parallel-sum extended RecursiveTask
  - Result was returned from compute()
- Map:
  - Class extended was RecursiveAction
  - Nothing returned from compute()
  - In the above code, the ‘answer’ array was passed in as a parameter
- Doesn’t *have* to be this way
  - Map can use RecursiveTask to, say, return an array
  - Reduce could use RecursiveAction; depending on what you’re passing back via RecursiveTask, could store it as a class variable and access it via ‘left’ or ‘right’ when done

Digression: MapReduce on clusters

- You may have heard of Google's "map/reduce"
  - Or the open-source version Hadoop
- Idea: Perform maps/reduces on data using many machines
  - The system takes care of distributing the data and managing fault tolerance
  - You just write code to map one element and reduce elements to a combined result
- Separates how to do recursive divide-and-conquer from what computation to perform
  - Old idea in higher-order functional programming transferred to large-scale distributed computing
  - Complementary approach to declarative queries for databases

Trees

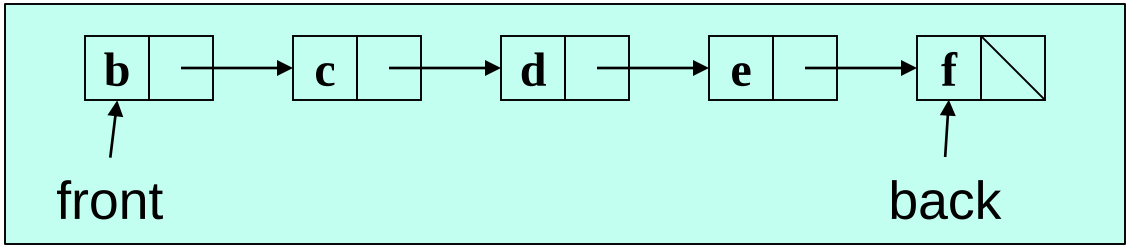
- Maps and reductions work just fine on balanced trees
  - Divide-and-conquer each child rather than array sub-ranges
  - Correct for unbalanced trees, but won't get much speed-up

- Example: minimum element in an *unsorted* but balanced binary tree in $O(\texttt{log}\ n)$ time given enough processors

- How to do the sequential cut-off?
  - Store number-of-descendants at each node (easy to maintain)
  - Or could approximate it with, e.g., AVL-tree height

Linked lists

- Can you parallelize maps or reduces over linked lists?
  - Example: Increment all elements of a linked list
  - Example: Sum all elements of a linked list
  - Parallelism still beneficial for expensive per-element operations



- Once again, data structures matter!

- For parallelism, balanced trees generally better than lists so that we can get to all the data exponentially faster $O(\mathtt{log}\ n)$ vs. $O(n)$
  - Trees have the same flexibility as lists compared to arrays (in terms of say inserting an item in the middle of the list)

Analyzing algorithms

- How to measure efficiency?
  - Want asymptotic bounds
  - Want to analyze the algorithm without regard to a specific number of processors
  - The key “magic” of the ForkJoin Framework is getting expected run-time performance asymptotically optimal for the available number of processors
    - So we can analyze algorithms assuming this guarantee

Work and Span

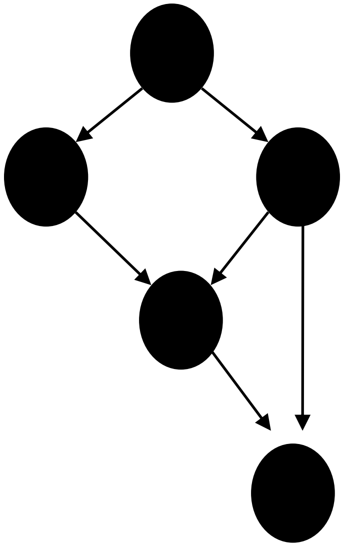
Let $\mathbf{T_P}$ be the running time if there are **P** processors available

Two key measures of run-time:

- Work: How long it would take 1 processor = $\mathbf{T_1}$
  - Just "sequentialize" the recursive forking
  - Cumulative work that all processors must complete
- Span: How long it would take infinity processors = $\mathbf{T_\infty}$
  - The hypothetical ideal for parallelization
  - This is the longest "dependence chain" in the computation
  - Example: $O($`log` $n)$ for summing an array
    - Notice in this example having > $n/2$ processors is no additional help
  - Also called "critical path length" or "computational depth"

The DAG (Directed Acyclic Graph)

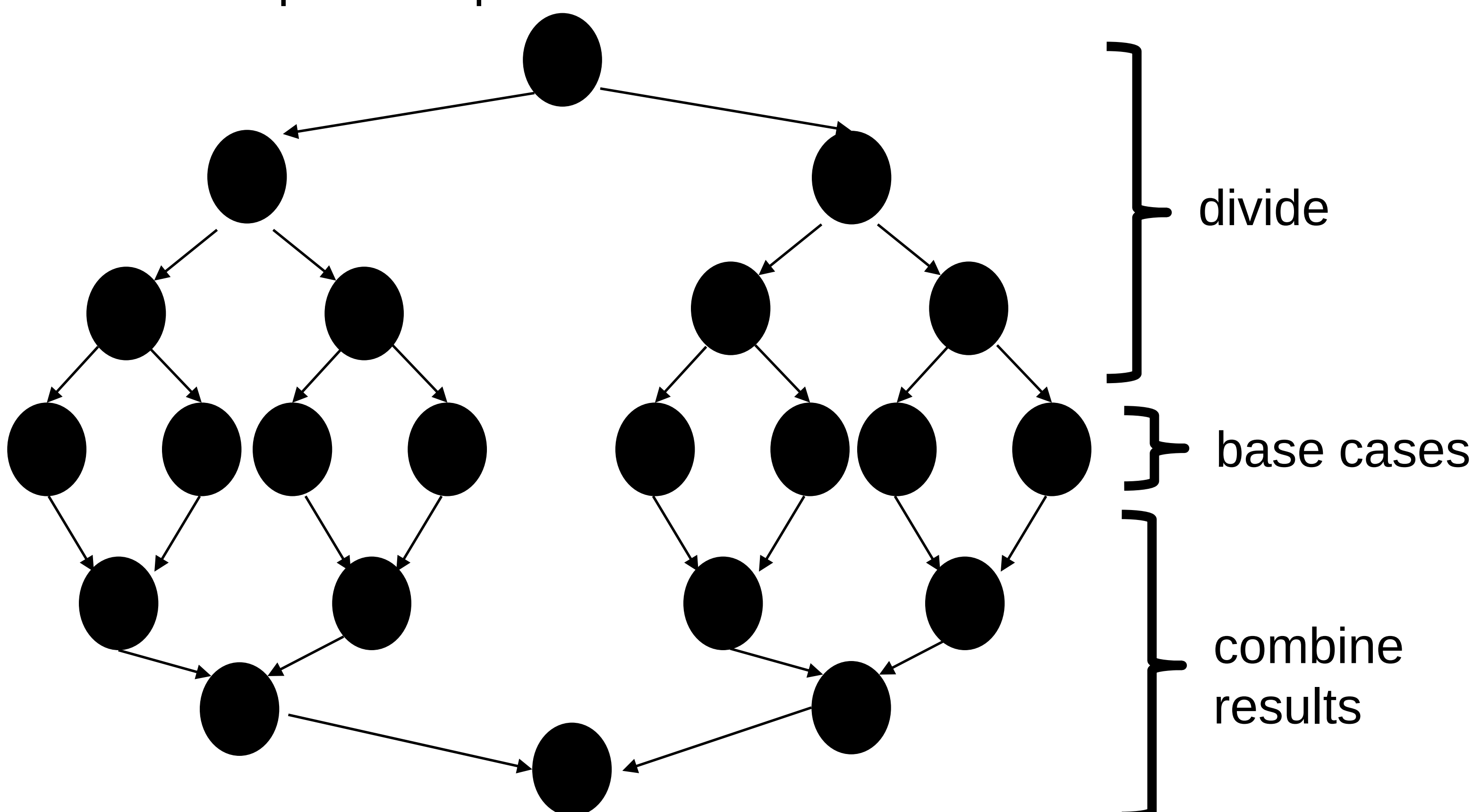
- A program execution using **`fork`** and **`join`** can be seen as a DAG
- [A DAG is a graph that is <u>directed</u> (edges have direction (arrows)), and those arrows do not create a <u>cycle</u> (ability to trace a path that starts and ends at the same node).]
  - **Nodes**: Pieces of work
  - **Edges**: Source must finish before destination starts



- A **`fork`** "ends a node" and makes two outgoing edges
  - New thread
  - Continuation of current thread
- A **`join`** "ends a node" and makes a node with two incoming edges
  - Node just ended
  - Last node of thread joined on

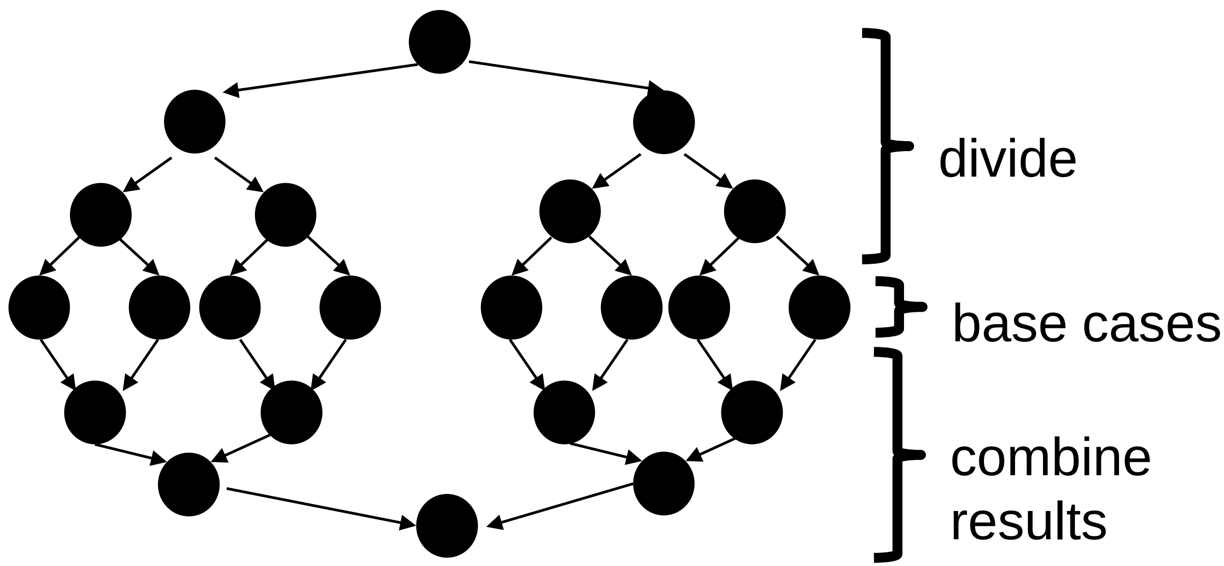
Our simple examples

- **`fork`** and **`join`** are very flexible, but divide-and-conquer maps and reductions use them in a very basic way:
  – A tree on top of an upside-down tree



Our simple examples, in more detail

Our **`fork`** and **`join`** frequently look like this:



In this context, the span ($\mathbf{T}_\infty$) is:

- The longest dependence-chain; longest ‘branch’ in parallel ‘tree’
- Example: $O(\log n)$ for summing an array; we halve the data down to our cut-off, then add back together; $O(\log n)$ steps, O(1) time for each
- Also called “critical path length” or “computational depth”

More interesting DAGs?

- The DAGs are not always this simple
- Example:
  - Suppose combining two results might be expensive enough that we want to parallelize each one
  - Then each node in the inverted tree on the previous slide would itself expand into another set of nodes for that parallel computation

Connecting to performance

- Recall: $\mathbf{T_P}$ = running time if there are **P** processors available
- Work = $\mathbf{T_1}$ = sum of run-time of all nodes in the DAG
  - That lonely processor does everything
  - Any topological sort is a legal execution
  - $O(n)$ for simple maps and reductions
- Span = $\mathbf{T_\infty}$ = sum of run-time of all nodes on the most-expensive path in the DAG
  - Note: costs are on the nodes not the edges
  - Our infinite army can do everything that is ready to be done, but still has to wait for earlier results
  - $O(\log n)$ for simple maps and reductions

Definitions

A couple more terms:

- Speed-up on **P** processors: $\mathbf{T_1 / T_P}$
- If speed-up is **P** as we vary **P**, we call it perfect linear speed-up
  - Perfect linear speed-up means doubling **P** halves running time
  - Usually our goal; hard to get in practice
- Parallelism is the maximum possible speed-up: $\mathbf{T_1 / T_\infty}$
  - At some point, adding processors won't help
  - What that point is depends on the span

Parallel algorithms is about decreasing span without increasing work too much

Optimal T_P: Thanks ForkJoin library!

- So we know $\mathbf{T_1}$ and $\mathbf{T_\infty}$ but we want $\mathbf{T_P}$ (e.g., **P**=4)
- Ignoring memory-hierarchy issues (caching), $\mathbf{T_P}$ can't beat
  - $\mathbf{T_1 / P}$ why not?
  - $\mathbf{T_\infty}$ why not?
- So an *asymptotically* optimal execution would be:

$$\mathbf{T_P = O((T_1 / P) + T_\infty)}$$

  - First term dominates for small **P**, second for large **P**
- The ForkJoin Framework gives an *expected-time guarantee* of asymptotically optimal!
  - Expected time because it flips coins when *scheduling*
  - How? For an advanced course (few need to know)
  - Guarantee requires a few assumptions about your code…

Division of responsibility

- Our job as ForkJoin Framework users:
  - Pick a good algorithm, write a program
  - When run, program creates a DAG of things to do
  - *Make all the nodes a small-ish and approximately equal amount of work*

- The framework-writer's job:
  - Assign work to available processors to avoid idling
    - Let framework-user ignore all scheduling issues
  - Keep constant factors low
  - Give the expected-time optimal guarantee assuming framework-user did his/her job

$$\mathbf{T_P = \mathit{O}((T_1 / P) + T_\infty)}$$

Examples

$$\mathbf{T_P = \mathit{O}((T_1 / P) + T_\infty)}$$

- In the algorithms seen so far (e.g., sum an array):
  - $\mathbf{T_1} = O(n)$
  - $\mathbf{T_\infty} = O(\mathtt{log}\ n)$
  - So expect (ignoring overheads): $\mathbf{T_P = \mathit{O}(\mathit{n}/P + \mathtt{log}\ \mathit{n})}$
- Suppose instead:
  - $\mathbf{T_1} = O(n^2)$
  - $\mathbf{T_\infty} = O(n)$
  - So expect (ignoring overheads): $\mathbf{T_P = \mathit{O}(\mathit{n}^2/P + \mathit{n})}$

And now for the bad news…

- So far: talked about a parallel program in terms of work and span
- In practice, it's common that your program has:

  a) parts that **parallelize well:**
  - Such as maps/reduces over arrays and trees

  b) …and parts that **don't parallelize at all:**
  - Such as reading a linked list, getting input, or just doing computations where each step needs the results of previous step

- These **unparallelized** parts can turn out to be a big bottleneck, which brings us to Amdahl's Law …

Amdahl's Law (mostly bad news)

Let the ***work*** (time to run on 1 processor) be 1 unit time

Let **S** be the portion of the execution that can't be parallelized

Then: $\mathbf{T_1 = S + (1\text{-}S) = 1}$

Suppose we get perfect linear speedup *on the parallel portion*

Then: $\mathbf{T_P = S + (1\text{-}S)/P}$

So the overall speedup with **P** processors is (Amdahl's Law):

$$\mathbf{T_1 / T_P = 1 / (S + (1\text{-}S)/P)}$$

And the parallelism (infinite processors) is:

$$\mathbf{T_1 / T_\infty = 1 / S}$$

Amdahl's Law Example

Suppose: $\mathbf{T_1 = S + (1\text{-}S) = 1}$ (aka total program execution time)

$\mathbf{T_1 = 1/3 + 2/3 = 1}$

$\mathbf{T_1 = 33\ sec + 67\ sec = 100\ sec}$

Time on P processors: $\mathbf{T_P = S + (1\text{-}S)/P}$

So: $\mathbf{T_P = 33\ sec + (67\ sec)/P}$

$\mathbf{T_3 = 33\ sec + (67\ sec)/3 =}$

Why such bad news?

$T_1 / T_P = 1 / (S + (1-S)/P)$ $T_1 / T_\infty = 1 / S$

- Suppose 33% of a program is sequential
  - Then a billion processors won't give a speedup over 3!!!
- No matter how many processors you use, your speedup is bounded by the sequential portion of the program.

The future and Amdahl's Law

Speedup: $T_1 / T_P = 1 / (S + (1-S)/P)$

Max Parallelism: $T_1 / T_\infty = 1 / S$

- Suppose you miss the good old days (1980-2005) where 12ish years was long enough to get 100x speedup
  - Now suppose in 12 years, clock speed is the same but you get 256 processors instead of 1
  - What portion of the program must be parallelizable to get 100x speedup?

All is not lost

Amdahl's Law is a bummer!

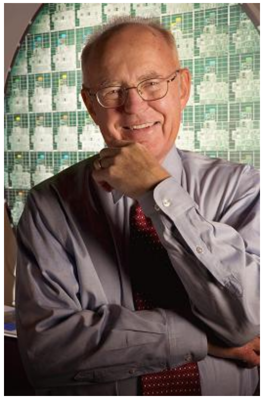
- Unparallelized parts become a bottleneck very quickly
- But it doesn't mean additional processors are worthless

- We can find new parallel algorithms
  - Some things that seem entirely sequential turn out to be parallelizable
  - Eg. How can we parallelize the following?
    - Take an array of numbers, return the 'running sum' array:

`input`	**6**	**4**	**16**	**10**	**16**	**14**	**2**	**8**
`output`	**6**	**10**	**26**	**36**	**52**	**66**	**68**	**76**

  - At a glance, not sure; we'll explore this shortly
- We can also change the problem we're solving or do new things
  - Example: Video games use tons of parallel processors
    - They are not rendering 10-year-old graphics faster
    - They are rendering richer environments and more beautiful (terrible?) monsters

Moore and Amdahl

- Moore's "Law" is an observation about the progress of the semiconductor industry
  - Transistor density doubles roughly every 18 months
- Amdahl's Law is a mathematical theorem
  - Diminishing returns of adding more processors
- Both are incredibly important in designing computer systems