CSE 332: Data Structures and Parallelism

Summations

Gauss' Summation

Let $S = \sum_{i=0}^{n} i$.

$$\begin{array}{ccccccccccc} & S & = & 1 & + & 2 & + & \cdots & + & (n-1) & + & n \\ + & S & = & n & + & (n-1) & + & \cdots & + & 2 & + & 1 \\ \hline & 2S & = & (n+1) & + & (n+1) & + & \cdots & + & (n+1) & + & (n+1) \end{array}$$

So, $S = \frac{n(n+1)}{2}$.

Infinite Geometric Series

Let $S = \sum_{i=0}^{\infty} x^i$.

$$\begin{array}{rcl} S & = & 1 + x + x^2 + \cdots + x^{n-1} + x^n + x^{n+1} + \cdots \\ -xS & = & \quad -x + -x^2 + \cdots + -x^{n-1} + -x^n + -x^{n+1} + \cdots \\ S - xS & = & 1 \end{array}$$

So, $S = \frac{1}{1-x}$.

Finite Geometric Series

Let $S = \sum_{i=0}^{n} x^i$.

We know, from the above, that $\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$. Multiplying both sides by x^{n+1}, we get the equality:

$$x^{n+1} \sum_{i=0}^{\infty} x^i = \frac{x^{n+1}}{1-x}$$

Subtracting the second equality from the first gives us:

$$\begin{aligned} \left(\frac{1}{1-x}\right) - \left(\frac{x^{n+1}}{1-x}\right) &= \left(\sum_{i=0}^{\infty} x^i\right) - \left(x^{n+1} \sum_{i=0}^{\infty} x^i\right) \\ &= \left(\sum_{i=0}^{\infty} x^i\right) - \left(\sum_{i=0}^{\infty} x^{i+n+1}\right) \\ &= \left(\sum_{i=0}^{\infty} x^i\right) - \left(\sum_{i=n+1}^{\infty} x^i\right) \\ &= \left(\sum_{i=0}^{n} x^i\right) \end{aligned}$$

So, $\sum_{i=0}^{n} x^i = \left(\frac{1}{1-x}\right) - \left(\frac{x^{n+1}}{1-x}\right) = \frac{1-x^{n+1}}{1-x}$.

A few more useful formulas, more can be found on the bottom of slides from lecture 2

logs

$$x^{log_x n} = n$$

$$a^{log_b c} = c^{log_b a}$$

$$log_b a = \frac{log_d a}{log_d b}$$

summations

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=0}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$