CSE 332: Data Structures and Parallelism

Exercises (Asymptotics)

Directions: *Submit your solutions on **Gradescope**. You must use a pdf file.*

EX05. Hell-$\mathcal{O}$! (20 points)

(a) [6 Points] The following Big-Oh proofs are incorrect and do not sufficiently prove that the given $f(n)$ is in the specified $\mathcal{O}(g(n))$. For each proof, explain why it is incorrect. *Note that the claims these proofs are trying to prove are true. Only the proofs themselves are incorrect.*

- I Let $f(n) = n^3 + n$. We want to prove that $f(n) \in \mathcal{O}(n^3)$. By the definition of big-Oh, $f(n) \in \mathcal{O}(n^3)$ iff $f(n) \leq c \cdot n^3$ for some $c > 0$ and all $n \geq n_0$. Using the definition of $f(n)$, $n^3 + n \leq c \cdot n^3$. Let $c = 2$ and $n_0 = 1$. By substituting in, we have $n^3 + n \leq 2n^3$. So, subtracting an n^3 from both sides, we get $n \leq n^3$. Dividing by n for both sides, we get $1 \leq n^2$, which is true for all $n \geq n_0$. So, $f(n) \in \mathcal{O}(n^3)$.
- II We want to prove that $4n^3$ is in $\mathcal{O}(n^3)$. Let $c \geq 4$ and $n_0 = 1$. Then we know $4n^3 \leq 4n^3$, so, $4 \cdot n^3 \leq c \cdot n^3$ for any value of $n \geq n_0$. By definition of $\mathcal{O}(n^3)$, $4n^3 \in \mathcal{O}(n^3)$.
- III We want to disprove that n^4 is in $\mathcal{O}(n^3)$. For this to be true, it must be the case that $n^4 \leq c \cdot n^3$ for some $c \geq 1$. But $n^4 \leq n^3$ can never be true because the 4 exponent causes n^4 to always be larger than n^3, thus $n^4 \notin \mathcal{O}(n^3)$.

(b) [14 Points] Use the formal definitions of Big-Oh, Big-Omega, and Big-Theta to *prove or disprove* each of the following statements. You should assume that the domain and co-domain of all functions in this exercise are the natural numbers. If you wish to disprove the claim, negate the quantified statement and prove the negation. We expect an English proof, thus something more formal that just picking a c and n0 value. You may not use Calculus (e.g., limits, differentiation, integrals) for these questions.

- I $2^{n+3} \in \Theta(2^n)$
- II $(2^n)^{1/3} \in \Theta(2^n)$ (Note, this problem is significantly harder than the others.)