Section 8: Minimax & Alpha Beta Pruning

University of Washington

November 15, 2017

Game algorithm: Minimax

Minimax

- suppose our opponent makes the best move every time!
- so we want to minimize the possible max gain our opponent will get by maximizing our own gain
- So called "minimax".

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Values of the game

In a chess game:

- My gain is my opponent's loss (and vice versa)
- If the position value is 50 for me, it should be -50 for my opponent.
- If I reach $+\infty$, I win; if my opponent reaches $-\infty$, he/she wins.
- So I want **MAX**, my opponent wants **MIN**
- A zero-sum game (Google it if you are interested).

Introducing the players!

For the following slides, assume:

- It's blue's turn!
- **MIN** wants to minimize the value
- **MAX** wants to maximize the value

Minimax, Code


```
int minimax(Position p, boolean is_max) {
  if (p is a leaf) {
    // always position value of MAX
    return p.evaluate();
  }
  if (is_max) { // MAX
    int bestValue = −∞
    for (move in p.getMoves()) {
      p.applyMove();
      int value = minimax(p, is_max);
      p.undoMove();
      if (value > bestValue) {
        bestValue = value;
      }
    }
  } else { // MIN
    int bestValue = ∞
    for (move in p.getMoves()) {
      p.applyMove();
      int value = minimax(p, is_max);
      p.undoMove();
      if (value < bestValue) {
        bestValue = value;
      }
    }
  }
}
```

The highlighted parts are the only differences!

How do we simplify Minimax?

A fact

$$\max(a, b) = -\min(-a, -b)$$

By maximizing the **negation** of the values, we found the **negation** of the min value.

Change Minimax Code

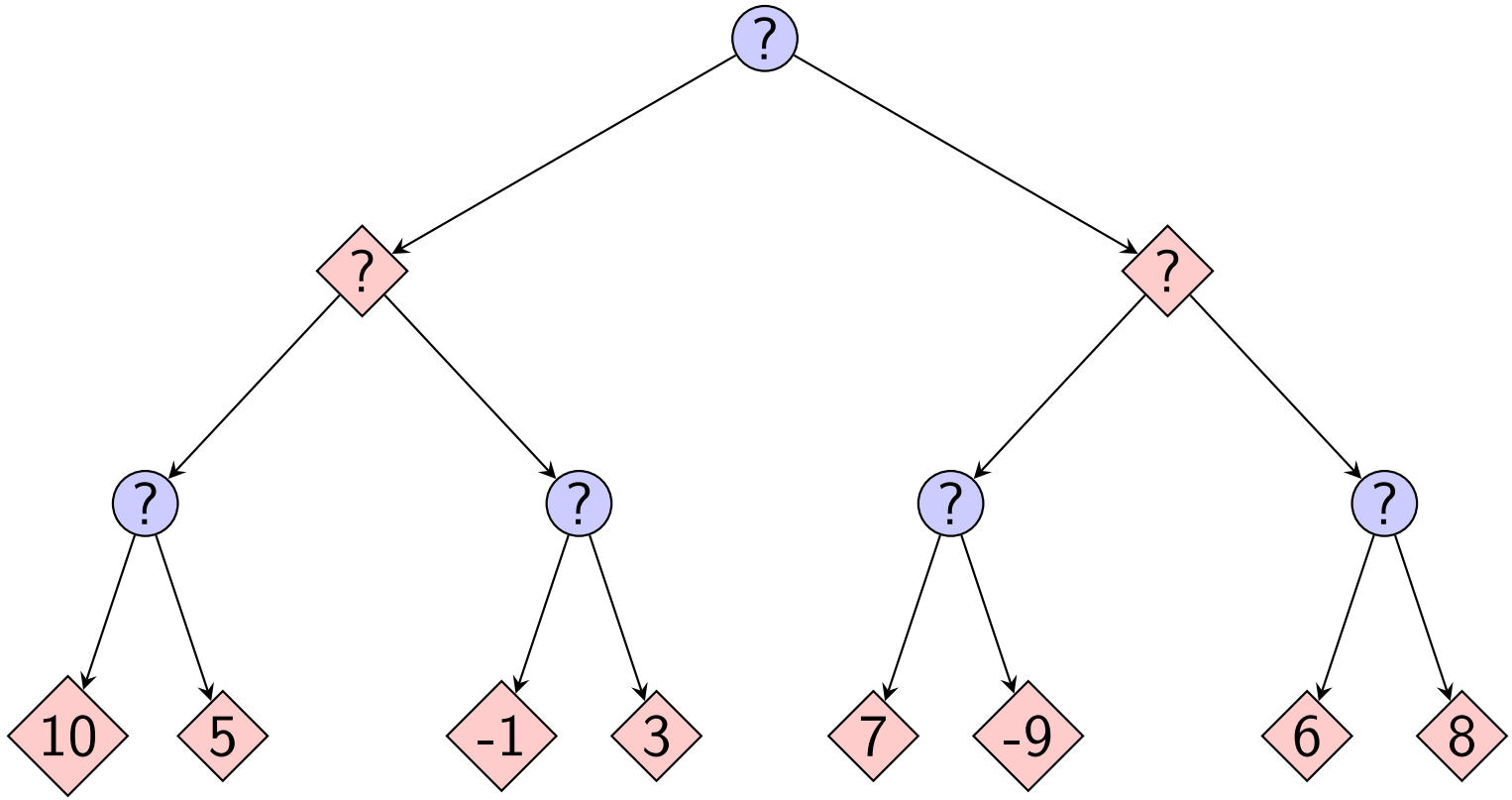
Then...

- For MAX, we are done
- For MIN, **max** the negation, then negate the return value to get the actual **min**.
- Now both players are maximizing, we can use the same piece of code.

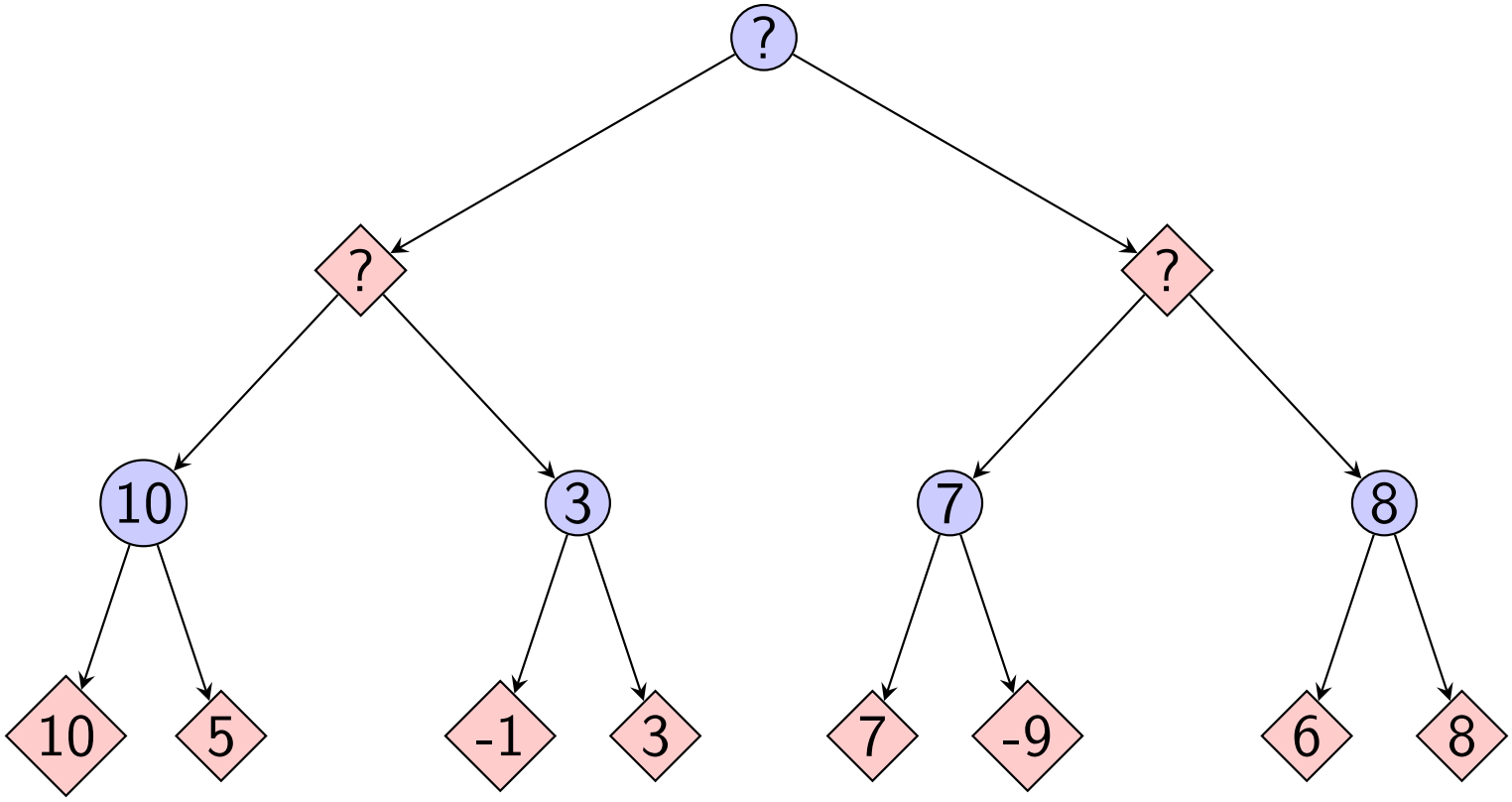
Code from your Game handout:

```
int minimax(Position p) {
  if (p is a leaf) {
    // position value of current player
    return p.evaluate();
  }
  int bestValue = −∞
  for (move in p.getMoves()) {
    p.applyMove();
    int value = -minimax(p);
    p.undoMove();
    if (value > bestValue) {
      bestValue = value;
    }
  }
}
```

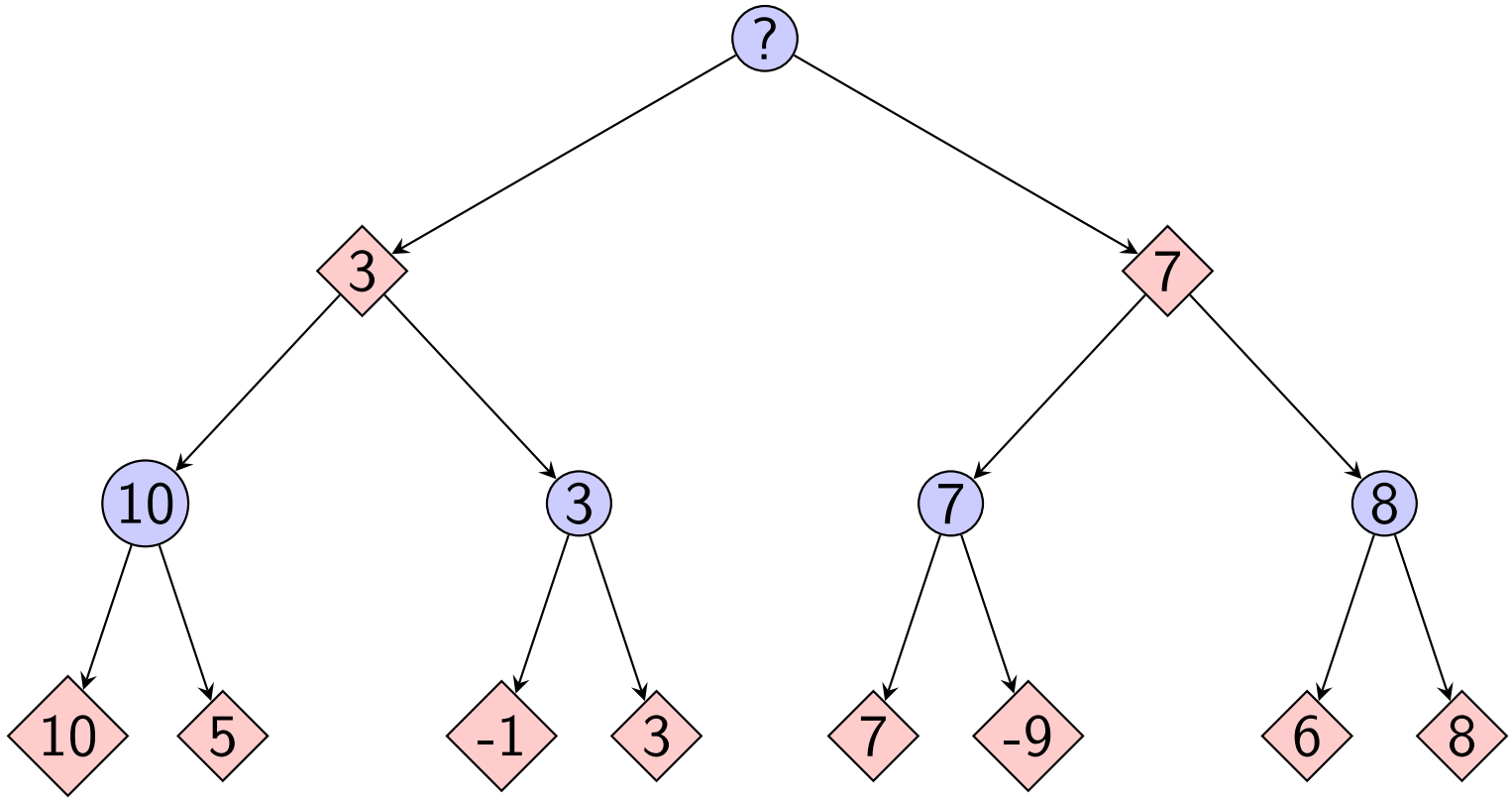
Minimax Example



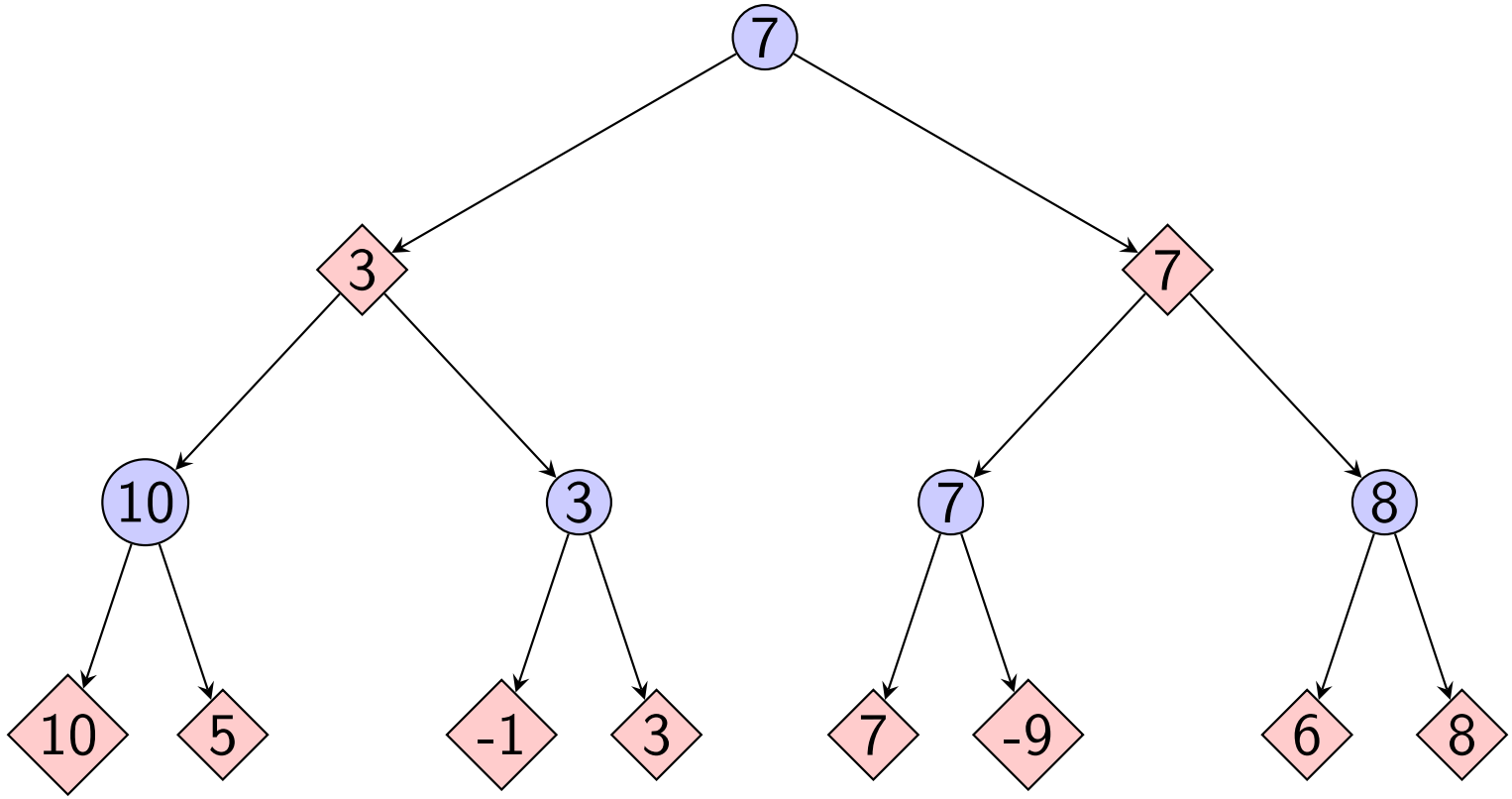
Minimax Example



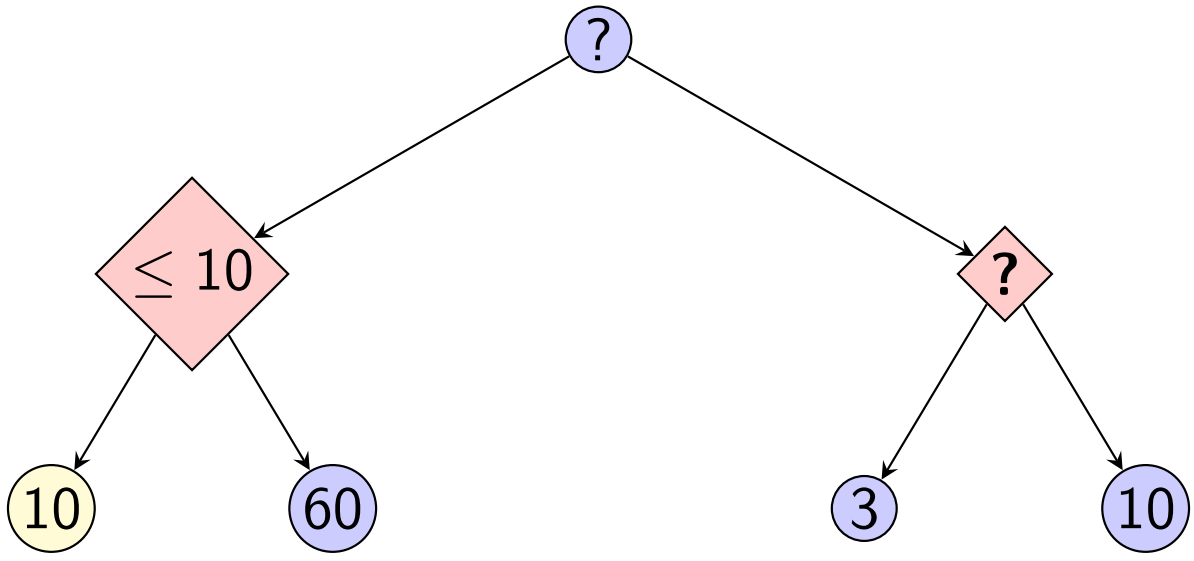
Minimax Example



Minimax Example



Do we need to look at every branch?



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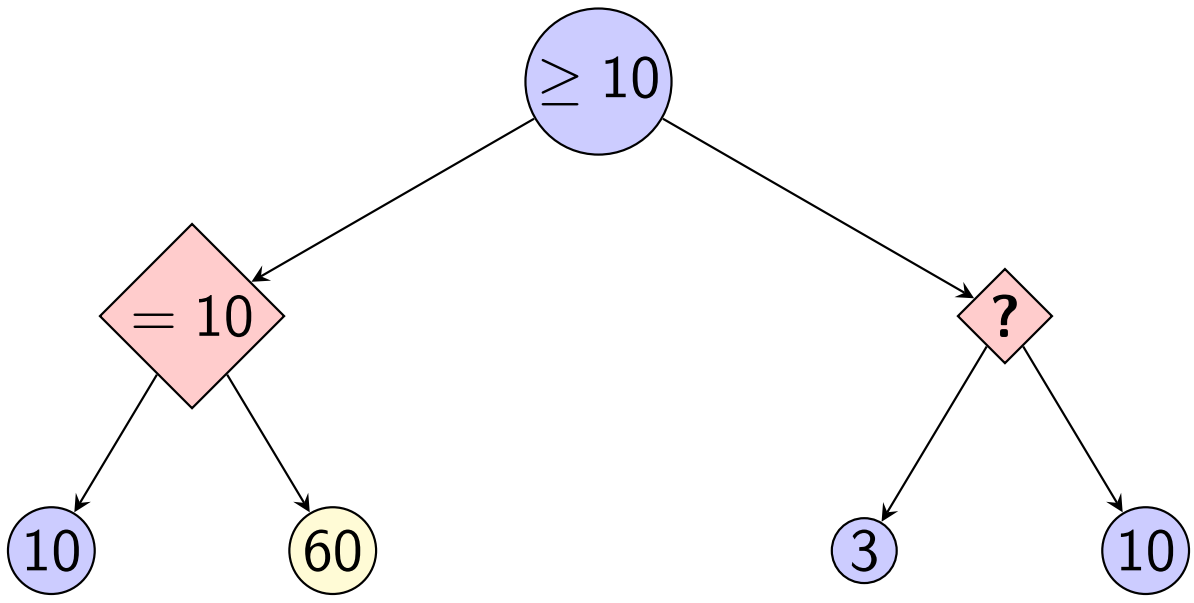

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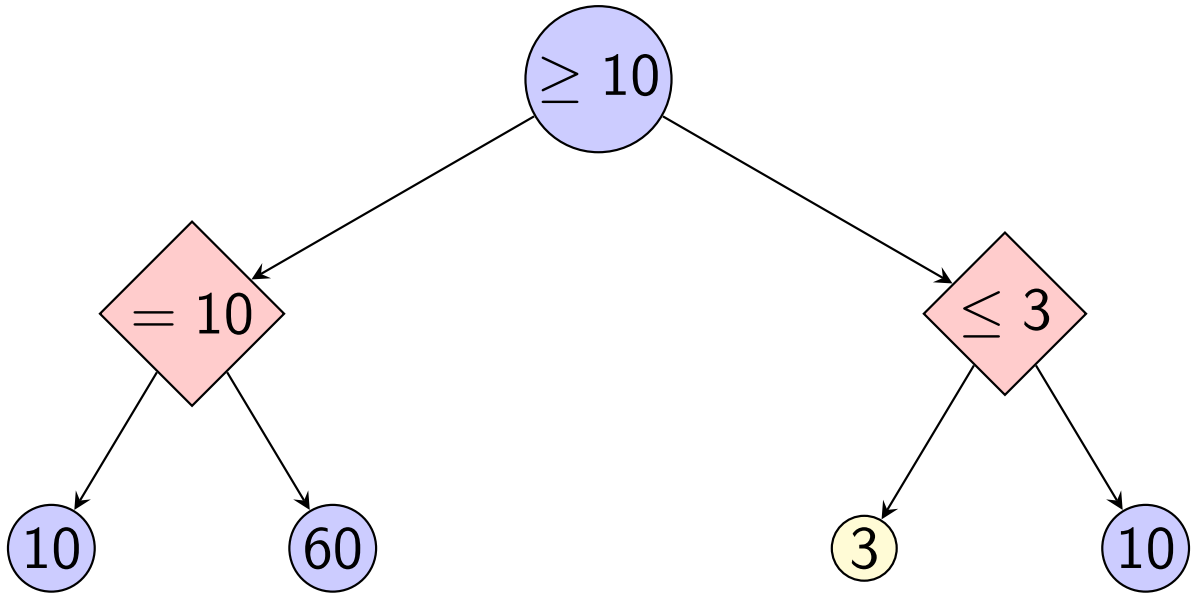
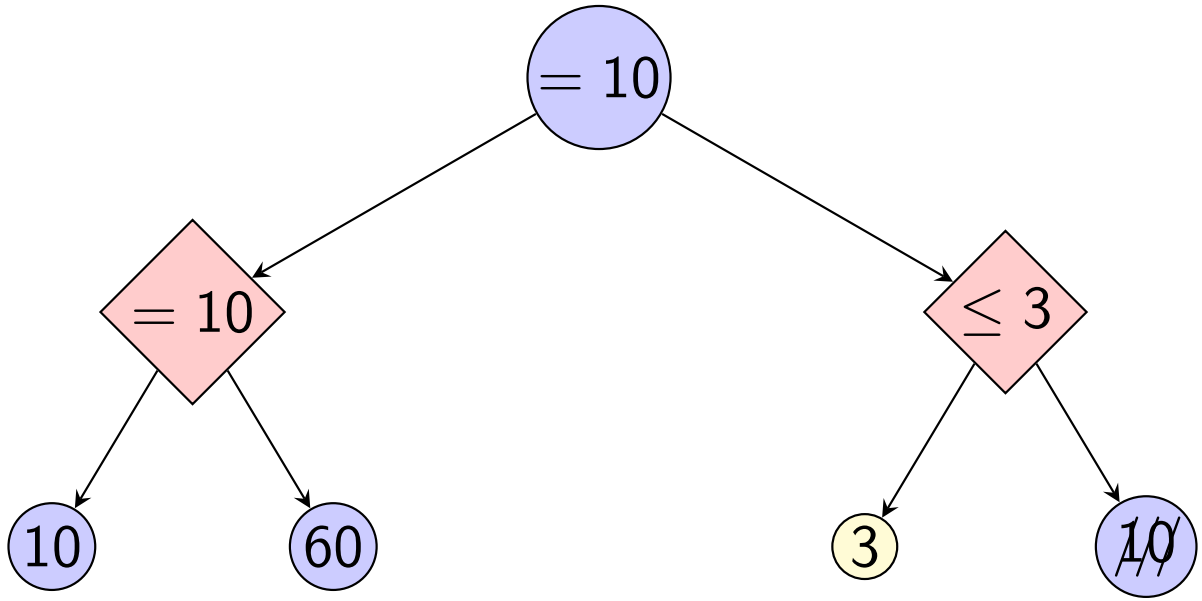


Now, without looking at 10, we know that the minimizer will only give a value ≤ 3, so no possible value bigger than 10 at root

Do we need to look at every branch?



Now, without looking at 10, we know that the minimizer will only give a value ≤ 3, so no possible value bigger than 10 at root

Alpha Beta Pruning (a.k.a What we just did)

First, when did we stop?

When we know MAX has range ≥ 10 but MIN will only give ≤ 3.

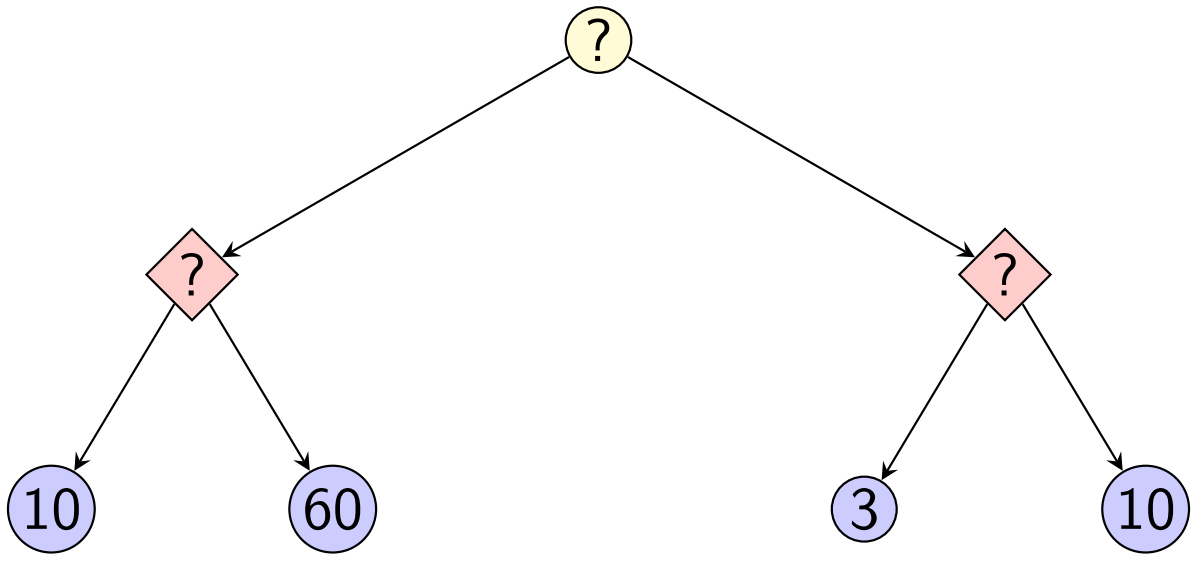
What is the 10?

- 10 is the best value for MAX we found **so far**.
- The branch with 3 will not give us a better value than 10. So we stop.
- 10 is the α value, 3 is the β value. We stop when $\alpha > \beta$.
- We return α for the result.

Alpha and Beta

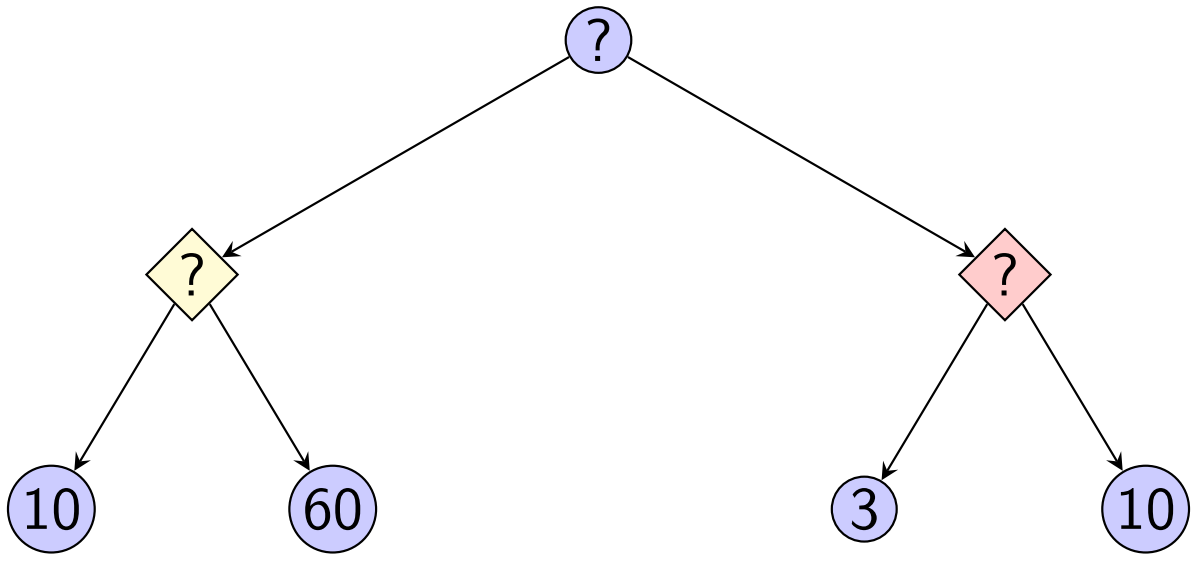
- α is the best MAX we found along the path to the root.
- β is the best MIN we found along the path to the root.

Example



$$\alpha = -\infty, \beta = \infty$$

Example



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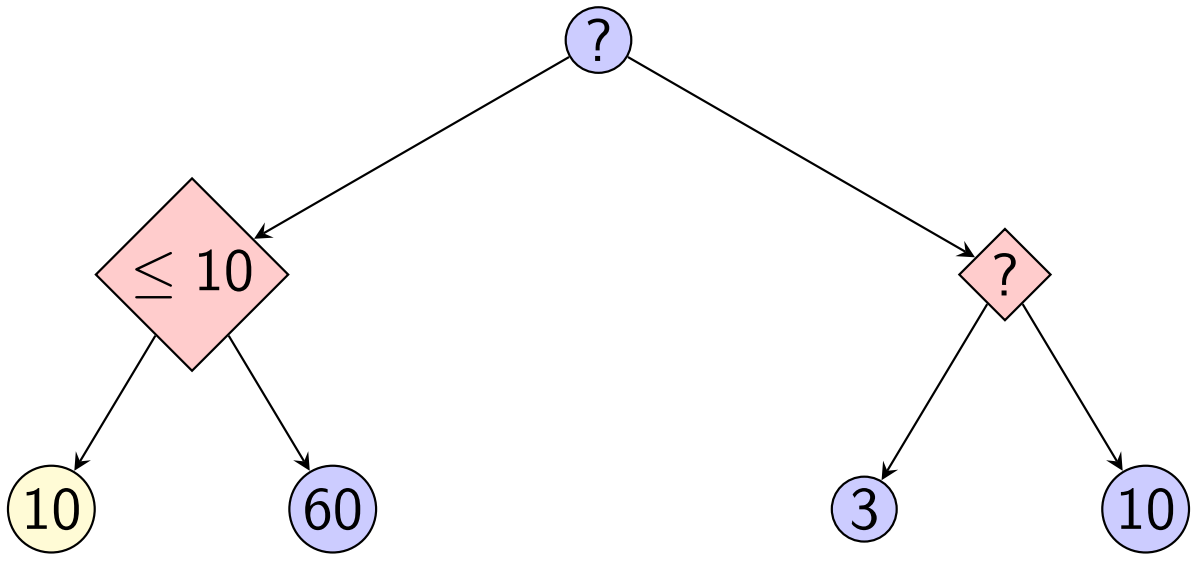
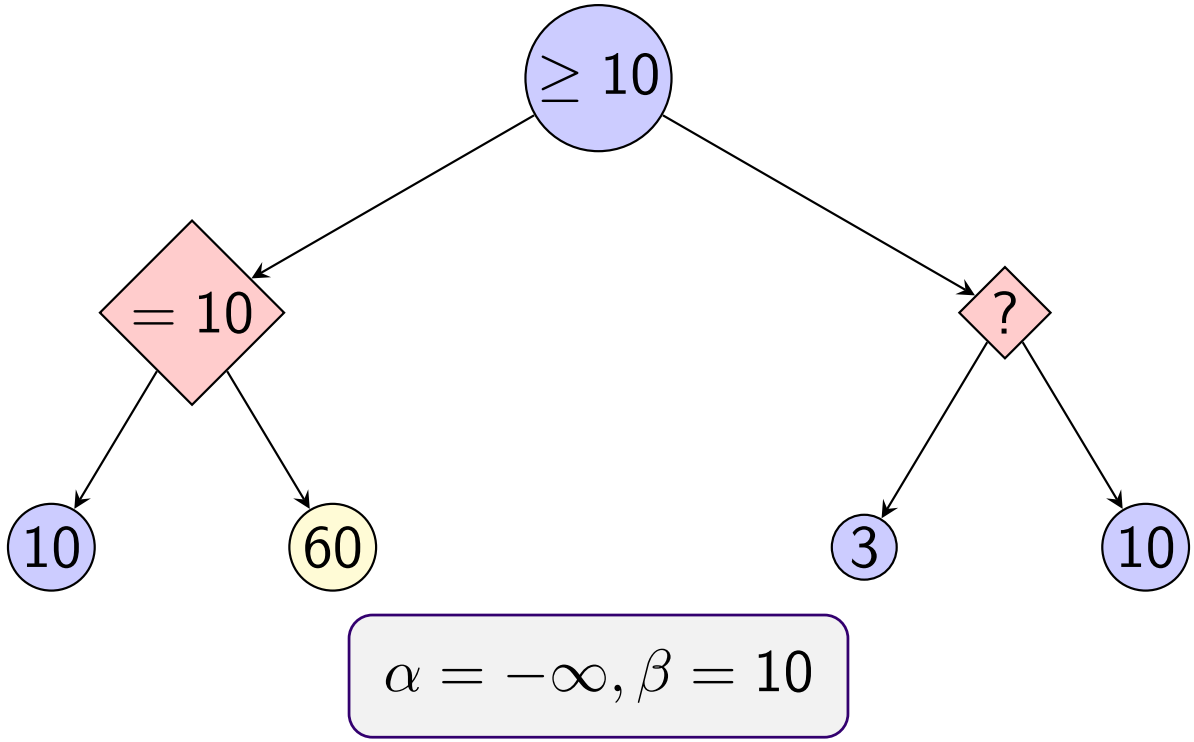
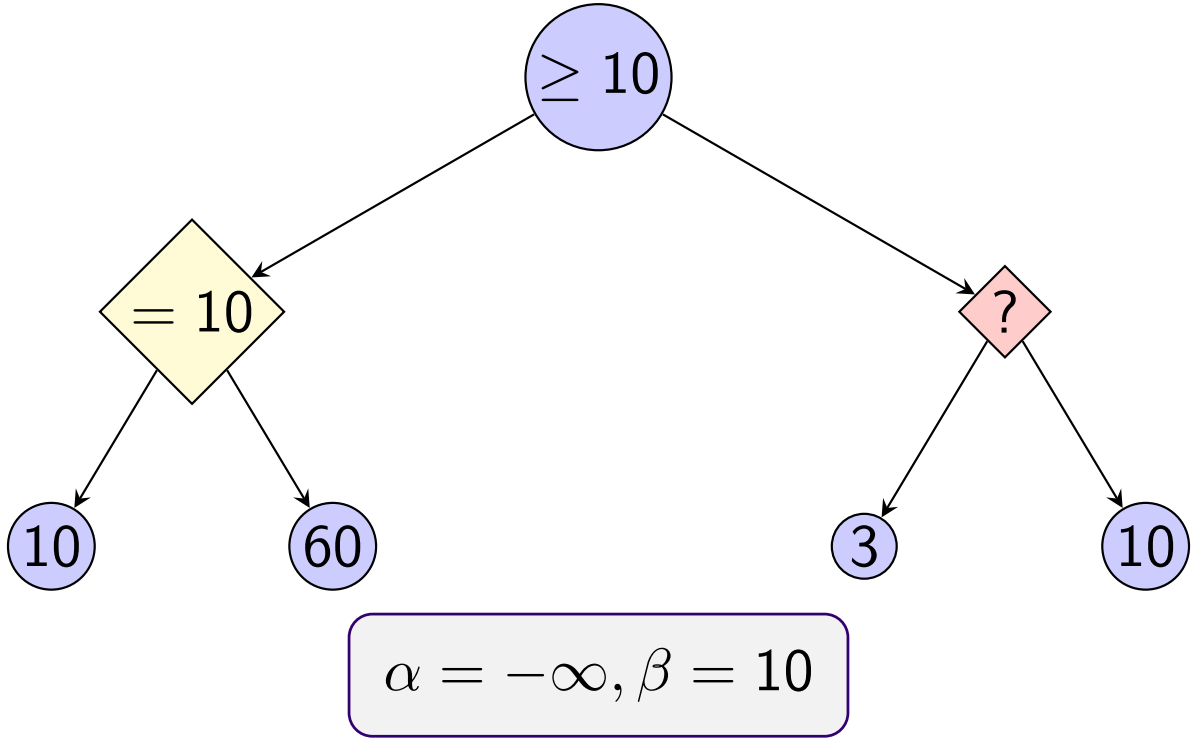


Example

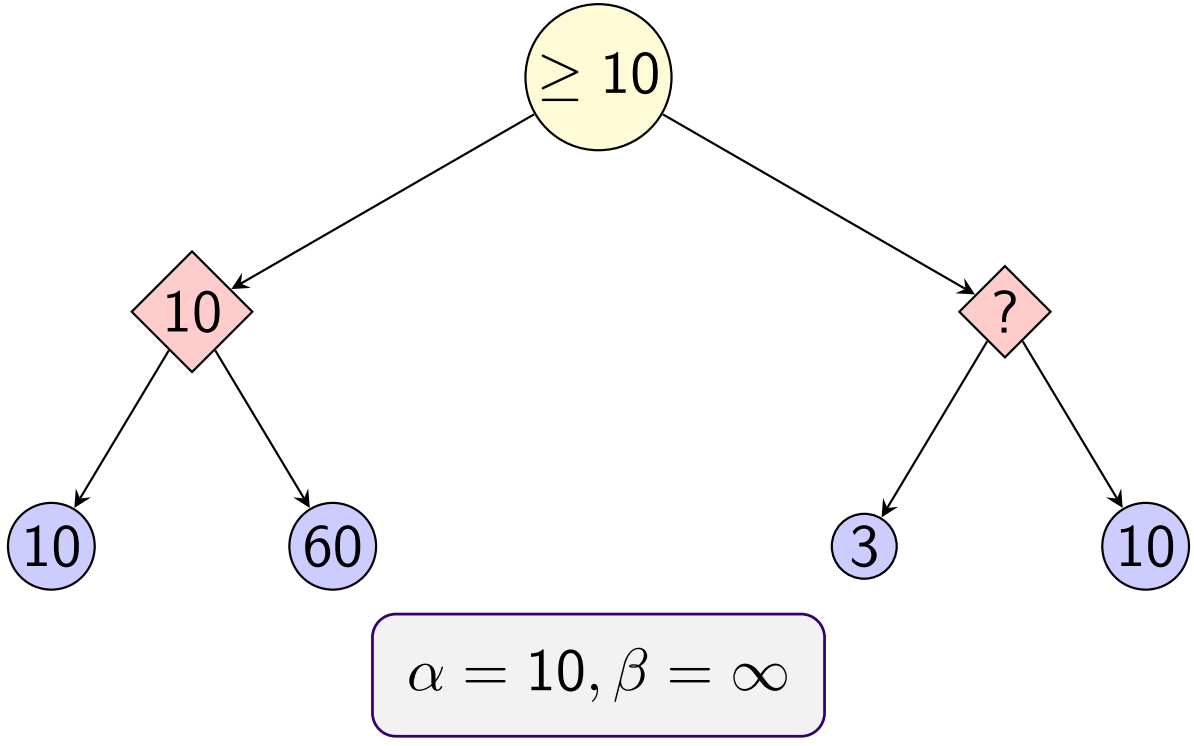
Do we check 60? Yes: $\alpha < 10$.



Example



Example



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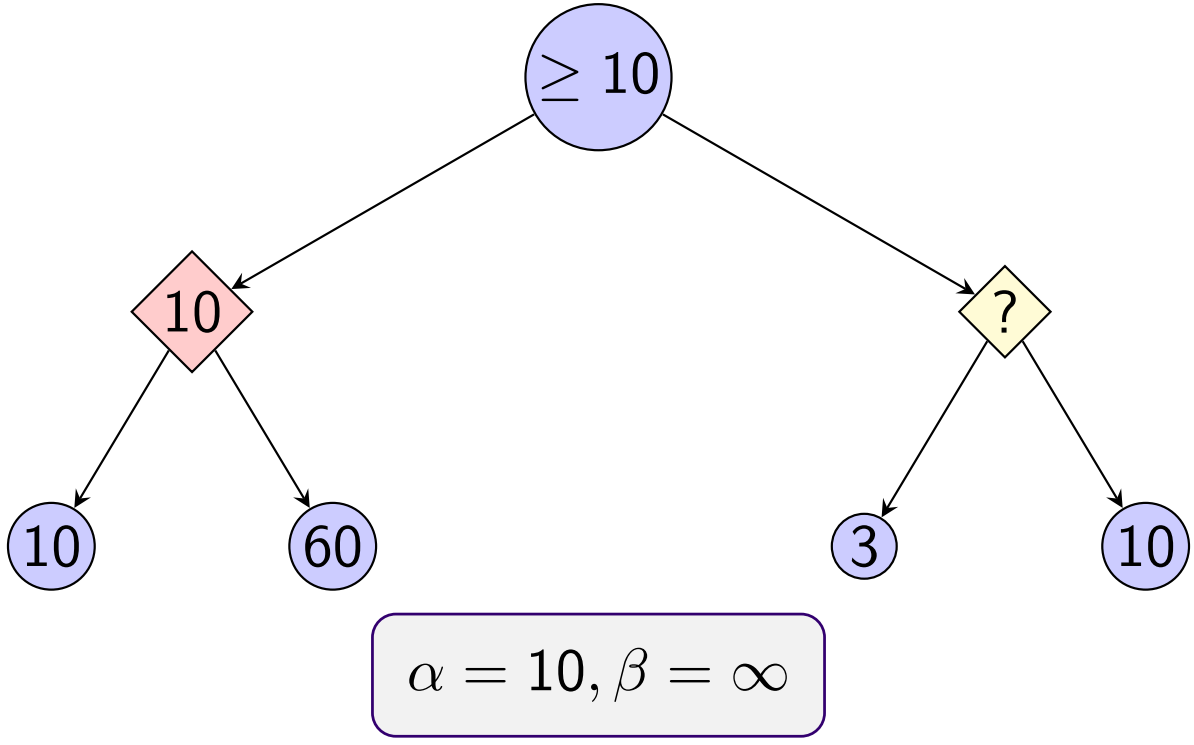
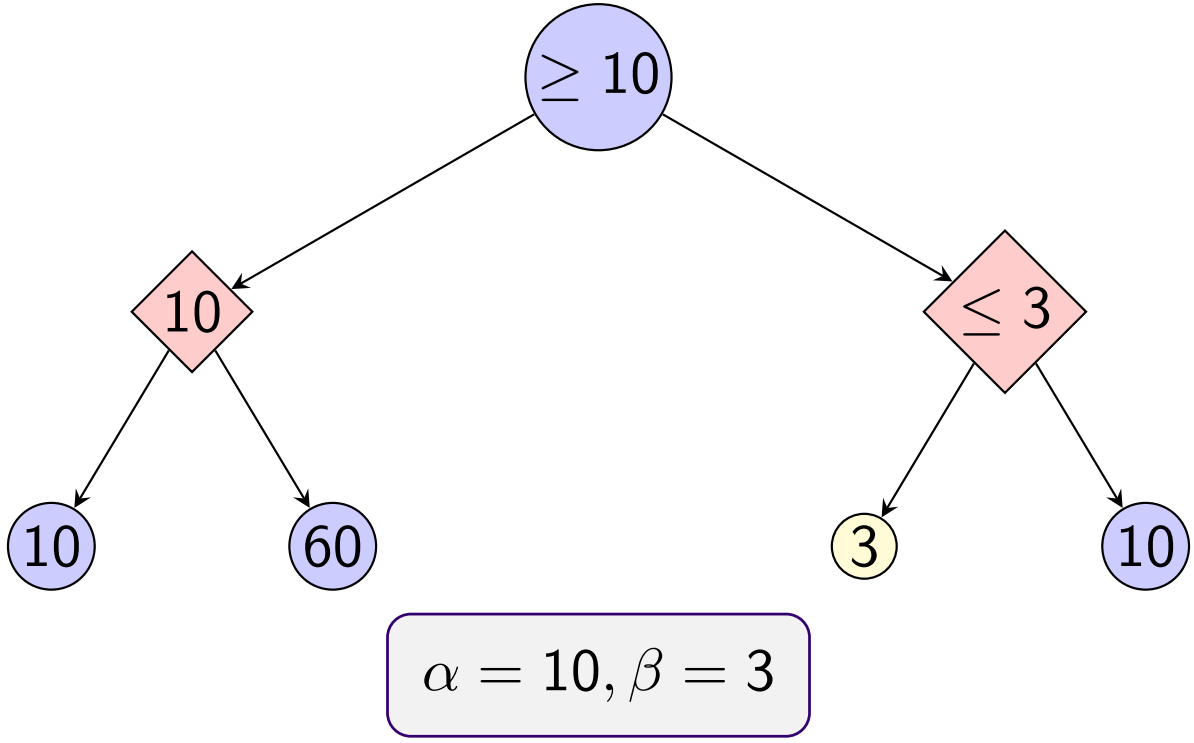
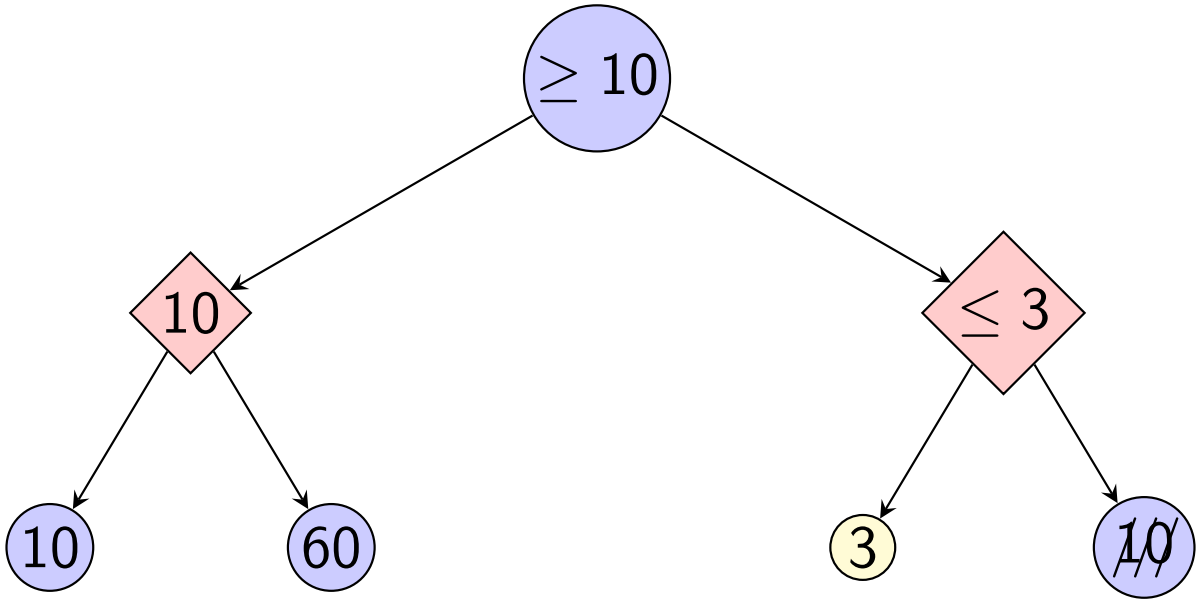


Example



Example

Do we check 10? No: $\alpha = 10 > 3$



$\alpha = 10, \beta = 3$, we stop!