

# CSE 332: Data Structures and Parallelism

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## Section 4: Balanced Trees

### 0. The *ABC's* of *AVL* Trees

What are the constraints on the data types you can store in an AVL tree? When is an AVL tree preferred over another dictionary implementation, such as a HashMap?

### 1. Let's Plant an *AVL* Tree.

Insert 10, 4, 5, 8, 9, 6, 11, 3, 2, 1, 14 into an initially empty AVL Tree.

## 2. MinVL Trees

Draw an AVL tree of height 4 that contains the minimum possible number of nodes.

## 3. AVL Trees

Insert 6, 5, 4, 3, 2, 1, 10, 9, 8, 7 into an initially empty AVL Tree.

#### 4. The ABC's of B-Trees

- (a) What properties must a B-tree of  $n$  values have with given values for  $M$  and  $L$ ?
- (b) Give an example of a situation that would be a good job for a B-tree. Furthermore, are there any constraints on the data that B-trees can store?

#### 5. Implement a B-Tree? Nah, Let's Analyze!

Given the following parameters for a B-Tree with a page size of 256 bytes:

- Key Size = 8 bytes
- Pointer Size = 2 bytes
- Data Size = 14 bytes per record (includes the key)

Assuming that  $M$  and  $L$  were chosen appropriately, what are  $M$  and  $L$ ? Recall that  $M$  is defined as the maximum number of pointers in an internal node, and  $L$  is defined as the maximum number of values in a leaf node. Give a numeric answer and a short justification based on two equations using the parameter values above.

## 6. Oh, B-Trees

Find a tight upper bound on the *worst case runtime* of these operations on a B-tree. Your answers should be in terms of  $L$ ,  $M$ , and  $n$ .

- (a) Insert a key-value pair
- (b) Look up the value of a key
- (c) Delete a key-value pair

## 7. B-Trees

- (a) Insert the following into an empty B-Tree with  $M = 3$  and  $L = 3$ : 12, 24, 36, 17, 18, 5, 22, 20.
- (b) Delete 17, 12, 22, 5, 36
- (c) Given the following parameters for a B-Tree with  $M = 11$  and  $L = 8$ 
  - Key Size = 10 bytes
  - Pointer Size = 2 bytes
  - Data Size = 16 bytes per record (includes the key)

Assuming that  $M$  and  $L$  were chosen appropriately, what is the likely page size on the machine where this implementation will be deployed? Give a numeric answer and a short justification based on two equations using the parameter values above.

## 8. It's Fun to B-Trees!

- (a) Insert the following into an empty B-Tree with  $M = 3$  and  $L = 3$ : 3, 18, 14, 30, 32, 36, 15, 16, 12, 40, 45, 38.
- (b) Delete 45, 14, 15, 36, 32, 18, 38, 40, 12