CSE 332 Summer 18
Section 02

$\mathcal{O}$, Ω, and Θ oh my!

Give a formal proof of each of the following statements, along with the scratch-work to find the c and n_0.

(a) $5n + 7$ is $O(n)$

Solution: Scratch work:
$5n \leq 5 \cdot n$ for all n
$7 \leq n$ for $n \geq 7$
So $5n + 7 \leq 6n$ for $n \geq 7$.

Proof. We take $c = 6$ and $n_0 = 7$. For $n \geq n_0$ we have both of the following inequalities:

$$5n \leq 5n \text{ and } 7 \leq n$$

Adding together the two inequailities we have: $5n + 7 \leq 6n$ as long as $n \geq n_0$, which is what we needed to show. □

(b) $3n^2 - 17n$ is $O(n^2)$

Solution: Scratch work:
$3n^2 \leq 3 \cdot n^2$ for all n
$-17n \leq 0 \cdot n^2$ if $n \geq 1$ Take $c = 3 + 0 = 3$ and $n_0 = 1$.

Proof. We take $c = 3$ and $n_0 = 1$. For n at least 1, $-17n$ is negative, so it is certainly at most $0 = 0n^2$, and $3n^2$ is always at most $3n^2$. Adding together these inequalities we get $3n^2 - 17n \leq 3n^2$ for $n \geq 1$, which is what we wanted to show. □

(c) $\log_5(n)$ is $\Omega(\log_3(n))$

Solution: Scratch work:
Applying the change of base formula, $\log_5(n) = \frac{\log_3(n)}{\log_3(5)}$.

Proof. We take $c = \frac{1}{\log_3(5)}$, and $n_0 = 1$. Applying the change-of-base formula:

$$\log_5(n) = \frac{\log_3(n)}{\log_3(5)} \geq c \cdot \log_3(n)$$

for all $n \geq 1$. □

(d) $2n^3 + 3$ is $\Theta(n^3)$

Solution: This is basically two proofs in one.
Scratch work for O:
$2n^3 \leq 2n^3$
$3 \leq 3n^3$ for $n \geq 1$.
Scratch work for Ω:
$2n^3 \geq 2n^3$
$3 \geq 0n^3$

Proof. To show $2n^3 + 3$ is $O(n^3)$, we take $c = 5$ and $n_0 = 1$. We have the following inequalities for $n \geq 1$:

$$2n^3 \leq 2n^3 \text{ and } 3 \leq 3n^3$$

Adding these inequalities together gives: $2n^3 + 3 \leq 5n^3$, as required. Thus $2n^3 + 3$ is $O(n^3)$.
To show $2n^3 + 3$ is $\Omega(n^3)$, we take $c = 2$ and $n_0 = 1$. We have $2n^3 + 3 \geq 2n^3 = c \cdot n^3$, which is what we needed to show to conclude $2n^3 + 3$ is $\Omega(n^3)$.
Combining these two statements we have $2n^3 + 3$ is $\Theta(n^3)$. □