CSE 332 Summer 2018: Final Exam Part 1
(closed book, closed notes, no calculators)

Instructions: Read the directions for each question carefully. We can only give partial credit based on the work you write down, so show your work.

For questions where you are drawing pictures, please circle your final answer.

Unless otherwise noted, any algorithms or code you write should be as efficient as possible, both in $O()$ terms and with respect to constant factors.

Take a deep breath.
Every tree is a forest.
You got this.

Good Luck!

Total: 75 points. Time: 60 minutes.

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1 Amdhal’s Law

[5 points]

Use Amdahl’s law to answer the following question. You must show your work for any credit. Your company has a program which is $1/5$ sequential and $4/5$ parallelized. At a minimum, how many processors do you need for a 4x speedup? For full credit, your answer must be a simplified fraction or an integer.
2 Parallel Code

[15 points]

Complete java code using the fork-join framework for the following problem:

Input: an array of strings
Output: the index of the leftmost String of even length
For example, if your input is \{ "a", "abc", "ab", "cd", "e", "defg" \} the output would be 2 (the index of "ab"). If there are no even length strings, -1 should be returned. null is not considered to have even length.

- Do not use a sequential cutoff. Your base case must process one element.
- Give a class definition LeftmostEvenString, along with any other code or classes you need.
- Fill in the function findLeftmostEvenString below.

You may not use any global data structures (except the input array) or synchronization primitives.

```java
import java.util.concurrent.ForkJoinPool;
import java.util.concurrent.RecursiveTask;
import java.util.concurrent.RecursiveAction;

public class Main{

    //Returns index of leftmost String of even length
    // in the array input. Returns -1 if no strings have
    // even length. null does not have even length.
    public static int findLeftmostEvenString(String[] input){

    }
}
```

Fill in the function above, and write your class definition on the next page.
Write your class definition on this page.
3 Parallel Patterns

[12 points]

Explain the steps you would use to perform the following tasks in parallel. Your algorithm should have the best possible $O(\cdot)$ span, but you need not worry about constant factors in this problem. You may assume you have access to already allocated auxiliary arrays as needed, and may alter the input array. Use the following parallel code patterns discussed in class:

- **out = map(f, arr)** Applies $f$ to every element of $arr$, storing the results in $out$.

- **out = reduce(baseFn, combineFn, arr)** Given `baseFn` which produces a value for a single element and `combineFn` which takes in the values for two subarrays and produces a value for the combined array, `reduce` stores the value for the full array in $out$.

- **out = parallelPrefixSum(arr)** Runs `ParallelPrefixSum` on $arr$, storing the results in $out$.

- **out = pack(condition, arr)** Given `condition`, performs a pack/filter on $arr$, storing the results in $out$.

When describing $f$, `baseFn`, `combineFn`, or `condition` you may assume you have access to whatever data you could access if you wrote out a full fork-join function for the associated function.

You can use whatever combination of sentences and pseudocode you prefer.

1. **Input** $arr$, an array of integers
   
   **Output** the sum of only the even numbers of the array.
2. Input \( \text{arr} \), an array of integers
   Output an array containing exactly the indices of \( \text{arr} \) such that \( \text{arr}[i] > \sum_{j=0}^{i-1} \text{arr}[j] \).

3. Input \( \text{arr} \), a sorted array of integers
   Output an array containing exactly the elements that appear only once in \( \text{arr} \) in sorted order.
4 Concurrency
[16 points]

Dumbledore needs your help again! Having heard the wonders of concurrency, he has altered
the discipline review system you helped with on the midterm. Now he has one priority heap,
and intends that both Professors McGonagall and Snape insert their disciplinary actions into
the same heap concurrently.

He shows you his current code.

```java
public class ConcurrentHeap {
    private Discipline[] arr;
    private int size; // current number of elements
    private int capacity; // size of arr

    /**
     * inserts new disciplinary action d into the heap
     */
    public void insert(Discipline d) {
        if (size == capacity)
            resize();
        arr[size++] = d;
        percolateUp(size - 1);
    }

    // details omitted. Methods work as discussed in class
    // and on earlier programming projects.
    private void percolateUp(int ind) { /*...*/ }
    private void percolateDown(int ind) { /*...*/ }
    public Discipline removeMin() { /*...*/ }
    private void resize(/*doubles size of arr and copies over elements*/)
    public Discipline peek() { /*...*/ }
    public int getSize() { /*...*/ }
}
```

1. Show Dumbledore a bad interleaving of the code above.
"Ahh, I think I see the problem," Dumbledore replies. "I've heard of these things called re-entrant locks. Let me add some of them. One per professor should do the trick."

He alters the code to look like this.

```java
public class ConcurrentHeap{
private Discipline[] arr;
private int size; //current number of elements
private int capacity; //size of arr
private Lock SnapeLock;
private Lock McGonaLock;

/**<n
* inserts new disciplinary action d into the heap
*/

public void insert(Discipline d){
    SnapeLock.acquire();
    McGonaLock.acquire();
    if(size == capacity)
        resize();
    arr[size++] = d;
    percolateUp(size-1);
    McGonaLock.release();
    SnapeLock.release();
}

//details omitted.
//All other methods acquire SnapeLock then McGonaLock at the top of the method
//And release both at the bottom, in the same way insert does.
private void percolateUp(int ind){ /*...*/}
private void percolateDown(int ind) { /*...*/}
private Discipline removeMin() { /*...*/}
public Discipline peek() { /*...*/}
private void resize("doubles size of arr and copies over elements")
public int getSize(){ /*...*/ }
}

2. Does this code have a bad interleaving to produce a race condition? If so describe one. If not briefly describe why it doesn't.
3. Does this code have potential for deadlock? If so, describe an interleaving to cause it. If not briefly describe why it can't happen.

4. Tell Dumbledore a way to improve his code. If you found errors in the previous parts, your alterations should fix them. If you did not find errors, you should still find a way to improve his use of synchronization.
5 Sorting
[15 points; 3 points each]

For each of the following scenarios, choose one of the sorting algorithms discussed in class, and describe why you believe it is best-suited for the task.

1. You’re writing code to sort numbers on the next NASA rover. Since it’s going to space, you can’t afford much memory, and because of limited communication windows with Earth, you want the code to run consistently quickly.

2. You just finished sorting a large array, when another thread changes the values of a few entries in the array. You acquire a lock on the entire array and decide to sort again.

3. At your new job working for Nanofluff, you’re asked to write a sorting algorithm to be used by their spreadsheet software Collective. Users of Collective enter data spanning multiple columns, and wish to sort their data in arbitrary combinations of those columns.
4. Briefly explain what it means for a sort to be stable.

5. Caitlin tells you she’s invented a new comparison-based sorting algorithm that takes $\Theta(n)$ time on lists that have many repeated elements. Should you believe her, or is such an algorithm impossible? Briefly justify your answer (1-2 sentences).
6 True/False

[12 points] For each of the following statements, say whether the statement is true or false. If it is false, explain how to make the statement correct. There may be more than one error in a false statement.

1. Regardless of algorithm, creating an AVL tree with \( n \) elements requires \( \Omega(n \log n) \) time in the worst case, because most of the AVL tree nodes are in the bottom few levels, and therefore take \( \Omega(\log n) \) time to put them in the tree.

2. Building a heap can be done faster than building an AVL tree because there is only one possible AVL tree for a set of elements, but there are many possible heaps.

3. Parallel Quicksort (where we parallelize both the recursion and partitioning) has span \( \Theta(\log^2 n) \) in the best case.

4. Each step of radix sort must be in-place for the sort to produce the correct output.
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Some Useful Facts

When we're using the tree method to solve a recurrence, we usually use the following steps:

0. Draw a few levels of the tree.
1. Let the root node be at level 0. Give a formula for the size of the input at level i.
2. What is the number of nodes at level i?
3. What is the work done at the \(i^{th}\) recursive level?
4. What is the last level of the tree?
5. What is the work done at the base case?
6. Write an expression for the total work done.
7. Simplify until you have a "closed form" (i.e. no summations or recursion).

Geometric series identities:

\[
\sum_{i=0}^{k} c^i = \frac{c^{k+1} - 1}{c - 1} \quad \sum_{i=0}^{\infty} c^i = \frac{1}{1 - c} \text{ if } |c| < 1
\]

Common Summations:

\[
\sum_{i=0}^{n} i = \frac{n(n + 1)}{2} \quad \sum_{i=0}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6} \quad \sum_{i=0}^{n} i^3 = \frac{n^2(n + 1)^2}{4}
\]

Log identities:

\[
a^{\log_b c} = c^{\log_b a} \quad \log_b(a) = \frac{\log_d(a)}{\log_d(b)}
\]

Master Theorem:
Given a recurrence of the following form:

\[
T(n) = \begin{cases} 
  d & \text{if } n \leq \text{ some constant} \\
  aT(n/b) + n^c & \text{otherwise}
\end{cases}
\]

with \(a, b, c\) are constants.
If \(\log_b(a) < c\) then \(T(n) = \Theta(n^c)\)
If \(\log_b(a) = c\) then \(T(n) = \Theta(n^c \log n)\)
If \(\log_b(a) > c\) then \(T(n) = \Theta(n^{\log_b(a)})\)