## CSE 332 Summer 18 Exercise 12B

## DJ Kistra

Due Date: Tuesday August 14, 11:59 PM

If you do not use a token on exercise 12, do ONLY ONE of 12A and 12B (of your choice). If you do use a token on exercise 12, submit both 12A and 12B, and your score will be the max of the two submissions.

Due to time constraints, you will not be able to redo exercise 12.

Submit as a pdf to gradescope.

Each of the following problems will ask you to describe an algorithm. In all cases, for full credit the algorithm should be as efficient as possible (in O() terms).

a. [5 points] Your disk jockeying career has taken off, and DJ Kistra (that's you) has been invited to perform at k different clubs across Seattle. The gigs all pay the same, so you decide to play at the club which will allow you to be back home the earliest after the show.<sup>1</sup> The clubs would have you start at different times, so you can leave from some venues earlier than others.

You have a directed graph representing drive times for all Seattle roadways you need. The graph contains specially marked vertices for each club location and your home. You also know the times you would leave each club. You may assume traffic doesn't change throughout the night, so the map is accurate regardless of the time.

- Describe an algorithm to solve this problem. Your algorithm must have the same big-O as Dijkstra's algorithm, even if k is very large.
- Argue that your algorithm actually has the required running time.
- b. [10 points] During your show you're playing "Shake It Off," and decide you want to slow things down with "Wildest Dreams." But you know that if you play two songs whose tempos differ by more than 10 beats per minute or if you play only a portion of a song, that the crowd will be disappointed. Instead you'll need to find a list of songs to play to gradually get you to "Wildest Dreams." Your goal is to transition to "Wildest Dreams" as quickly as possible (in terms of seconds).

You have a list of all the songs you can play, their speeds in beats per minute, and the length of the songs in seconds.

 $<sup>^1 \</sup>rm You$  need your sleep, after all!

- (i) Describe a graph you could construct to help you solve the problem. At the very least you'll want to mention what the vertices and edges are, and whether the edges are weighted or unweighted and directed or undirected.
- (ii) Describe an algorithm to construct your graph from the previous part. You may assume your songs are stored in whatever data structure makes this part easiest.
- (iii) Describe an algorithm you could run on the graph you just constructed to find the list of songs you can play to get to "Wildest Dreams" the fastest without disappointing the crowd.
- (iv) What is the running time of your plan to find the list of songs? You should include the time it would take to construct your graph and to find the list of songs. Give a simplified big-O running time in terms of whatever variables you need.
- c. [5 points] After the show, a group of adoring fans want your autograph. The fans form a graph around you. You know that if you sign an autograph, that person will leave the crowd, and you can take over their spot in the graph, but that you can only sign an autograph of someone standing in a vertex adjacent to yours. You'd like to get home soon, so you want to get to the edge of the crowd as quickly as possible. Signing any of these autographs will take the same amount of time. You're given a graph of your fans with you as a marked vertex and every vertex on the "edge" of the crowd (from which you can successfully escape) also marked.
  - Describe an algorithm to determine the fastest route to leave the crowd.
  - What is the running time of your algorithm?