CSE 332 Summer 18
Exercise 12A

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Due Date: Tuesday August 14, 11:59 PM
If you do not use a token on exercise 12, do ONLY ONE of 12A and 12B (of your choice).
If you do use a token on exercise 12, submit both 12A and 12B, and your score will be the max of the two submissions.
Due to time constraints, you will not be able to redo exercise 12.

Submit as a pdf to gradescope.

a. [7 points] If there is more than one minimum cost path from x to y, will Dijkstra's Algorithm always find the path with the fewest edges? If not, explain in a few sentences how to modify Dijkstra's algorithm so that if there is more than one minimum cost path from x to y, a path with the fewest edges is chosen. Assume no negative-cost edges.

b. [6 points] Give a concrete example (show the graph and the starting vertex) where Dijkstra's Algorithm gives the wrong answer in the presence of a negative-cost edge but no negative-cost cycles. Explain briefly why Dijkstra's Algorithm fails on your example. The example need not be complex; it is possible to demonstrate the point using as few as 3 vertices.

c. [7 points] Suppose you are given a weighted graph that has at least one negative-cost edge but no negative-cost cycles. We want to find the shortest paths, where the cost of a path is the sum of the weights of edges along the path. Consider the following strategy to find shortest paths in this graph: Uniformly add a constant k to the cost of every edge, so that all costs become non-negative, then run Dijkstra's Algorithm and return that result with the edge costs reverted back to their original values (i.e., with k subtracted from each edge).

   - Give a concrete example in the form of a graph and a starting vertex, where this technique fails (Dijkstra's would not find what is actually the shortest path) and explain why it fails.
   - Also, give a general explanation as to why this technique does not work. Think about your example and why the original least cost path is no longer the least cost path after adding k.