QuickSort complete parallel

iteration

$i=0$

$$T(n) = \underbrace{T\left(\tfrac{n}{2}\right)} + \log(n) \qquad i=1$$

$$\hookrightarrow T\left(\tfrac{n}{2}\right) = T\left(\tfrac{n}{4}\right) + \log\left(\tfrac{n}{2}\right)$$

1. Expansion

$$= \underbrace{T\left(\tfrac{n}{4}\right)} + \log\left(\tfrac{n}{2}\right) + \log(n) \qquad i=2$$

$$\hookrightarrow T\left(\tfrac{n}{4}\right) = T\left(\tfrac{n}{8}\right) + \log\left(\tfrac{n}{4}\right)$$

$$= T\left(\tfrac{n}{8}\right) + \log\left(\tfrac{n}{4}\right) + \log\left(\tfrac{n}{2}\right) + \log(n) \qquad i=3$$

$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$ expand logs

$$= T\left(\tfrac{n}{8}\right) + \left(\log(n) - \underbrace{\log(4)}_{2}\right) + \left(\log(n) - \underbrace{\log(2)}_{1}\right) + \log n$$

simplify logs b/c $\log_2 2^i = i$

$$= T\left(\tfrac{n}{8}\right) + 3\log(n) - \underbrace{(2+1)}_{\text{summation}}$$

2. Find pattern

$$= T\left(\tfrac{n}{2^i}\right) + i\log(n) - \sum_{j=0}^{i-1} j$$

$\hookrightarrow$ Solve $T(1) = T\left(\frac{n}{2^i}\right) = c_0 \rightarrow \frac{n}{2^i} = 1$

3. Solve for i $\rightarrow i = \log n$

4. substitute in i & simplify

$$= c_0 + \log(n)\cdot\log(n) - \underbrace{\sum_{j=0}^{\log n - 1} j}$$

$\hookrightarrow$ solve Gauss's Summation (see summation handout)

$$= \frac{(\log n - 1)(\log n)}{2}$$

$$= c_0 + \log^2(n) - \tfrac{1}{2}\left(\log(n)\log(n) - \log(n)\right)$$

$$= \underbrace{\tfrac{1}{2}\log^2(n)}_{\text{worse runtime}} + \tfrac{1}{2}\log(n)$$

$$= O\left(\log^2(n)\right)$$