Chris and Richard's Midterm Prep Packet

A Fine Selection of Premium, Handpicked Problems

ENCOURAGE MINT

Big OH - 12sp

1. (**12** points) Give a big-O bound on the worst-case running time of each code fragment below in terms of n.

   - Assume integer arithmetic (division rounds down).
   - Each bound should be as "tight" and "simple" as possible.

(a)
```
for(i = 0; i < n; i++) {
   for(j = 0; j < n; j++) {
     for(k = 0; k < i + j - 4; k++) {
        sum++;
     }
   }
}
```

(b)
```
x = n;
while (x > 0) {
  sum++;
  x = x / 2;
}
```

(c)
```
sum = 0;
i = 0;
j = 7n;
while(i < j) {
  sum++; i++; j--;
}
```

(d)
```
x = n;
while (x > 0) {
  y = n;
  while (y > 0) {
    sum++;
    y = y / 2;
  }
  x = x / 2;
}
```

(e)
```
tree = new AVLTree();
for(i = 0; i < n; i++) {
  tree.insert(foo()); // foo produces some element in O(1) time
}
sum = 0;
for(i = 0; i < n; i++) {
  sum += tree.findMin();
  tree.insert(foo()); // foo produces some element in O(1) time
}
```

f)
```
for(int i = 0; i < n; i += 2) {
    for(int j = n; i < j; i++) {
        System.out.println("Sup.");
    }
}
```

```
g)
int i = 0;
while(i < n) {
    i++;
    for(int j = 0; j * 2 < n; j++) {
         sum++;
    }
}
```

Recurrence Relations - 13wi

4. (6 pts) Recurrence Relationships -

Suppose that the running time of an algorithm satisfies the recurrence relationship

$$T(1) = 10.$$

and

$$T(N) = T(N/2) + 3 \quad \text{for integers } N > 1$$

Find the closed form for T(N) ***and show your work step by step***. In other words express T(N) as a function of N. Your answer should *not* be in Big-Oh notation – show the relevant *exact* constants in your answer (e.g. don't use "C" in your answer).

4a. Identify the recurrence relationship of the following algorithm. Specifically find the recurrence of a call of `zapdos(n)`. You do NOT have to solve this recurrence.

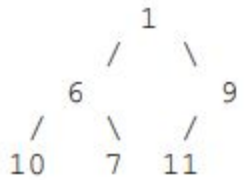
```
public static int zapdos(int n) {
    if(n < 2) {
        return 1;
    } else {
        for(int i = 0; i < 15; i++) {
            print("whoa.");
        }
        return articuno(n - 1);
    }
}

public static int articuno(int n) {
    for(int i = n; i > 0; i--) {
        print("Hrm.");
    }
    return zapdos(n - 1);
}
```

Heaps - 12wi

3) 10 Points

Consider this binary min-heap:



Perform the following operations in order, drawing the result after each operation and using it as the starting point for the next operation. You only need to show the result of the operation, but showing your work will allow partial credit in case of error.
If the space here is insufficient, use the back of this sheet (clearly labeling your work).
Circle the result of each operation so we can distinguish it from intermediate work.

a) DeleteMin

b) Insert 8

c) Insert 2

d) Draw an efficient array-based representation of your final heap from step *c*.

e) In your array-based representation, what is the index of:

the parent of the node at index *i*:

the left child of the node at index *i*:

the right child of the node at index *i*:

f) For the final heap from step *c*, identify the order to insert nodes that would result in the least number of percolates.

AVL Tree - 13sp

6. (8 pts) AVL Trees

a) (2 pts) What is the **minimum number of nodes** in an **AVL tree of height 4?** (Hint: the height of a tree consisting of a single node is 0) ***Give an exact number*** not a formula.

b) (2 pts) What is the **minimum number of nodes than must be examined in order to find the maximum value** in an **AVL tree of height 4?** ***Give an exact number*** not a formula.

c) (2 pts) What is the **maximum number of nodes than must be examined in order to find the minimum value** in an **AVL tree of height 4?** ***Give an exact number*** not a formula.

d) (2 pts) **Draw an example** of an **AVL tree of height 4** with the **minimum number of nodes**. You do not need to put values in the tree, just show the shape using dots for nodes.

e) Draw one 3-node AVL tree for each kind of rotation before having applied the rotation.

B-Tree - 12wi

Your brilliant teaching assistants have implemented a B-Tree in order to better track and correct your instructor's many mistakes. They give a name to each mistake and use the name as a key to store information about when the mistake was made, who is responsible for correcting it, and information about their progress. The parameters of the tree are:

Pointer Size = 8 bytes
Key Size = 12 bytes
Data Size = 52 bytes
M = 13
L = 4

c) Assuming these parameters were appropriately chosen to fit within a disk block, what is the likely size of disk blocks on the machine where this implementation is deployed? Give **a numeric answer** and **a short justification**. This justification should be based on one or more equations using the above parameter values.

d) Given these B-Tree parameters, what is the maximum number of items that can be stored in a tree of height 2? Give **a numeric answer**. Show your work for partial credit in case of an arithmetic error, but we do not expect a general equation.