Name: _____

UWNetID: _____

Lecture Section: A

CSE 332 : Kat and Zhe's Midterm Exam

(closed book, closed notes, no calculators)

Instructions: Read the directions for each question carefully before answering. We will give partial credit based on the work you **write down**, so show your work! Use only the data structures and algorithms we have discussed in class so far.

You are forbidden to communicate with anyone about the exam until after 1:30pm.

Note: For questions where you are drawing pictures, please circle your final answer.

Good Luck!

Total: 100 points. Time: 50 minutes.

Question	Max Points	Score
1	18	
2	8	
3	10	
4	8	
5	10	
6	9	
7	11	
8	16	
9	10	
Total	100	

1. (20 pts) Big-Oh

(2 pts each) For each of the operations/functions given below, indicate the tightest bound possible (in other words, giving $O(2^N)$ as the answer to every question is not likely to result in many points). Unless otherwise specified, all logs are base 2. **Your answer should be as "tight" and "simple" as possible.** For questions that ask about running time of operations, assume that the most efficient implementation is used. For array-based structures, assume that the underlying array is large enough.

You do not need to explain your answer.

a) *Post-order traversal of all elements in a **AVL tree** containing N elements. (worst case).* _____

b) $f(N) = \log_3 (2^N) + (\log N)^2$ _____

c) *Combining 2 **binary min heaps,** each containing N elements, into one binary min heap (worst case).* _____

d) $T(N) = T(N/2) + 7$ _____

e) $f(N) = \log \log (N^3) + \log^2 N$ _____

*f) Finding the 2^{nd} largest value in a **AVL tree** containing N elements. (worst case)* _____

g) $T(N) = 2\ T(N/2) + 6$ _____

h) *Pop in a **stack** containing N elements implemented as an array (worst case)* _____

i) *Finding the maximum value in an **binary search tree** containing N elements (worst case)* _____

j) decreaseKey*(k, amount) on a **binary min heap** containing N elements. Assume you have a reference to the key k that should be updated. (worst case)* _____

2. **(12 pts) Big-Oh and Run Time Analysis:** Describe the worst case running time of the following pseudocode functions in Big-Oh notation in terms of the variable n. Your answer should be as "tight" and "simple" as possible. ***Showing your work is not required***

I.
```
void haunted(int n, int sum) {
    for (int i = 1; i < n * n; i++) {
        for (int j = 0; j < n; j += 3) {
            sum++;
        }
    }
    for (int k = 0; k < n * n; k++) {
        sum++;
    }
}
```
Runtime:

II.
```
int sweet(int n, int sum) {
    if (n < 5) {
        return sum;
    } else {
        for (int i = 0; i < n; i++) {
            sum++;
        }
    }
    return sweet(n-2, sum);
}
```

III.
```
int treat(int n) {
    if (n < 10) {
        return n;
    } else if (n < 100) {
        return n + 1;
    }
    return n * treat(n / 2) + treat(n / 2);
}
```

IV.
```
void spooky(int n, int sum) {
    int k = n;
    while (k > 0) {
        for (int i = 0; i < n * n; i++) {
            if (i % 2 == 0) {
                for (int j = 0; j < i; j++) {
                    sum++
                }
            }
        }
        k--;
    }
}
```

3. (9 pts) Big-O, Big Ω, Big Θ

(3 pts each) For parts (a) – (c) circle **ALL** of the items (if any) that are TRUE. You do not need to show any work or give an explanation.

a) $7 \log^2 N + N \log N$ is:

$\Omega(N \log N)$ $O(\log^2 N)$ $\Omega(N)$ $O(N)$

b) $2^{3^* N}$ is:

$O(N^6)$ $\Omega(2^N)$ $\Theta(2^N)$ $\Omega(N^N)$

c) $\log \log (N^2)$ is:

$\Omega(N)$ $O(\log N)$ $\Omega(\log N)$ $\Theta(\log^2 N)$

4. (6 pts) D-Heaps

a) Give a tight big-O running time of the worst case of a single deletemin operation in terms of N and D. KEEP all D and N terms in your answer, do not simplify your answer by removing D terms.

b) Increasing D will decrease the height of our heap. One idea is to set D to be equal to N. In a couple of sentences explain why this is or is not a good idea.

Circle one: This IS a good idea This IS NOT a good idea

Explanation:

5. (8 pts) Draw the AVL tree that results from inserting the keys 7, 1, 2, 5, 8, 9, 4, 3 in that order into an initially empty AVL tree. You are only required to show the final tree, although if you draw intermediate trees, ***please circle your final result for ANY credit.***

6. (6 pts) Recurrences

Give a base case and a recurrence for the runtime of the following function. Use variables appropriately for constants (e.g. c1, c2, etc.) in your recurrence (you do not need to attempt to count the exact number of operations). **YOU DO NOT NEED TO SOLVE** this recurrence.

```
int spooky(int n) {
    if (n <= 10)
       return n + n;
    else {
       for (int i = n; i > 0; i--) {
          print i;
       }
       return 5 + spooky(n - 2) + n * spooky(n - 3);
    }
}
```

T(n) = _____ For n <= 10

T(n) = _____ For n > 10

Yipee!!!! YOU DO NOT NEED TO SOLVE *this* recurrence…

7. (11 pts) Solving Recurrences

Suppose that the running time of an algorithm satisfies the recurrence relationship:

$$T(1) = 9.$$

and

$$T(N) = 2 * T(N/2) + 7N \qquad \text{for integers } N > 1$$

Find the closed form for T(N). **You may assume that N is a power of 2.** Your answer should *not* be in Big-Oh notation – show the relevant ***exact*** constants in your answer (e.g. don't use "c1, c2" in your answer). You should not have any summation symbols in your answer. The list of summations on the last page of the exam may be useful. ***Show your work***.

8. (10 pts) B-Trees

a) (5 pts) Given M=7 and L=20, what is the minimum number of data items in a B-Tree (as defined in lecture and in Weiss) of height 4? Give a single number for your answer, not a formula. **Explain briefly how you got your answer.**

b) (5 pts) Given the following parameters for a B-tree with M= 22 and L = 5:

Key Size = 8 bytes

Pointer Size = 4 bytes

Data Size = 50 bytes per record (***includes*** the key)

Assuming that M and L were chosen appropriately, **what is the likely size of a disk block** on the machine where this implementation will be deployed? Give a numeric answer and **a short justification *based on two equations*** using the parameter values above.

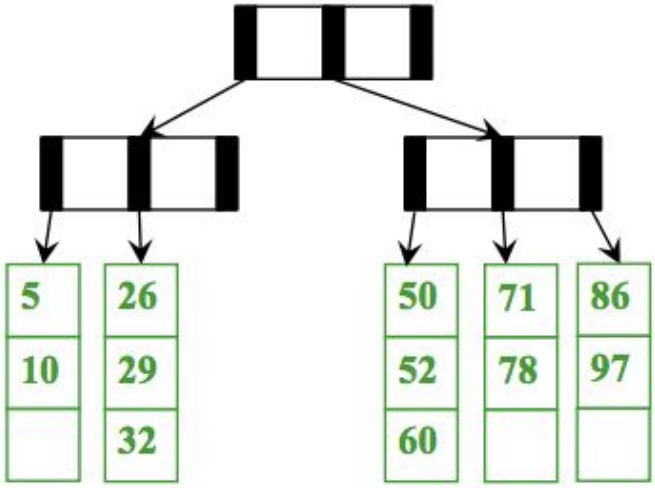
9. (8 pts) B-trees

a) (2pts) In the B-Tree shown below, please **write in the appropriate values for the interior nodes**.

b) (2 pts) Based on the picture below, what are the values for M and L?

M = L =

c) (4 pts) Starting with the B-tree shown below, insert **61**. Draw and circle the resulting tree (*including values for interior nodes*) below. Use the method for insertion described in lecture and used on homework.



Where can I find the solution?

17 au(1)
16 au(2)
18 wi(3, 4)
14 sp(5)
17au(6)
17au(7)
16au(8)
15wi(9)