**CSE 332 Winter 2018 Final Exam**  
(closed book, closed notes, no calculators)

**Instructions:** Read the directions for each question carefully before answering. We may give partial credit based on the work you write down, so show your work! Use only the data structures and algorithms we have discussed in class so far. Writing after time has been called will result in a loss of points on your exam.

**Note:** For questions where you are drawing pictures, please circle your final answer.

You have 1 hour and 50 minutes, work quickly and good luck!

Total: Time: 1 hr and 50 minutes.

<table>
<thead>
<tr>
<th>Question</th>
<th>Max Points</th>
<th>Score</th>
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<tbody>
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<td>1</td>
<td>12</td>
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<td><strong>Total</strong></td>
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</tbody>
</table>
1) [12 points total] Hash Tables
For a) and b) below, insert the following elements in this order: 14, 22, 33, 44, 13, 73, 28. For each table, TableSize = 10, and you should use the primary hash function h(k) = k%10. If an item cannot be inserted into the table, please indicate this and continue inserting the remaining values.

a) Separate chaining hash table – use a sorted linked list for each bucket where the values are ordered by increasing value

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<td>5</td>
<td>6</td>
<td>7</td>
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</tr>
</tbody>
</table>

b) Quadratic probing hash table

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
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<td>5</td>
<td>6</td>
<td>7</td>
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</tbody>
</table>

c) What is the load factor in Table b)?

d) In a sentence or two, describe **double hashing**.

e) What is **re-hashing**? Would it be beneficial to implement **re-hashing** on a separate chaining hash table like the one shown above in part a)? Why or Why not?

f) What is one **disadvantage** quadratic probing has compared to linear probing?
2) [9 points total] Graphs!
   a) [6 points] Find a minimum spanning tree with Prim’s algorithm using vertex G as the starting node (mark, circle, or highlight edges below to indicate they are in your minimum spanning tree). You must show your steps in the table below for full credit. Show your steps by crossing through values that are replaced by a new value. Break ties by choosing the lowest letter first; ex. if B and C were tied, you would explore B first. Note that the next question asks you to recall what order vertices were declared known.

   ![Graph Diagram]

<table>
<thead>
<tr>
<th>Cost</th>
<th>Prev</th>
<th>Known?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
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<td>D</td>
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<td></td>
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<tr>
<td>E</td>
<td></td>
<td></td>
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<tr>
<td>F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td></td>
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</tr>
</tbody>
</table>

   b) [1 point] List the order the vertices are added to the known set:

   c) [1 point] Pick a node you could start at to get a different minimum spanning tree than the one you found in part a). Which edge would be in this new tree that is not in your tree above? **DO NOT DRAW THE WHOLE TREE.**

   **Starting node** (for example, “Z”): _______ **Edge** (for example, “(X, Y)”): ______________

   d) [1 point] Will Prim’s starting at vertex G find a correct minimum spanning tree if the weight of edge (A,F) is set to be -6? (circle one)

   YES / NO

3 of 12
3) [11 points total] More Graphs!

a) [2 points] If you needed to calculate the out-degree of all vertices in a graph, which representation would you prefer (circle one):

- adjacency matrix
- adjacency list

In a couple of sentences describe WHY?

b) [2 points] Give an example of a directed graph with exactly two topological orderings and one node of in-degree zero.

c) [1 point] Let G be a connected, undirected, weighted graph. Convert G to a directed graph as follows: replace every undirected edge (u, v) with directed edges (u -> v) and (v -> u). The resulting graph is strongly connected.

- TRUE
- FALSE

d) [4 points] What is the worst case running time of Dijkstra’s algorithm described in lecture that:

i. Does NOT use a priority queue:

ii. Uses a priority queue:

e) [2 points] Give an EXACT number (in terms of V) for:

i. Maximum number of edges in an undirected graph without self-loops:

ii. Minimum number of edges in a weakly connected directed graph
4) [10 points] Parallel “Suffix Max” (Like Prefix, but from the Right instead):
a) Given the following array as input, calculate the “suffix max” using an algorithm similar to
the parallel prefix algorithm discussed in lecture. Fill the output array with the max of the
values contained in all of the cells to the right (including the value contained in that cell) in
the input array. The first pass of the algorithm is similar to the first pass of the parallel prefix
code you have seen before. Fill in the values for max and var in the tree below. The output
array has been filled in for you. Do not use a sequential cutoff. You can assume that the
array contains only positive integers.

```
Index        0  1  2  3  4  5  6  7
Input        14 50 38 29  6 12  3  5
Output       50 50 38 29 12 12  5  5
```

b) How is the var value computed for the left and right children of a node in the tree. Give
exact code (not just an English description) where p is a reference to the current tree node.

```c
p.left.var =
```

```c
p.right.var =
```

c) How is output[i] computed? Give exact code assuming leaves[i] refers to the leaf
node in the tree visible just above the corresponding location in the input and output
arrays in the picture above.

```c
output[i] =
```
5) [14 points] In Java using the ForkJoin Framework, write code to solve the following problem:

- **Input**: An array of positive ints
- **Output**: an array of 10 ints containing a count of the ones place digits of the values in the Input array. The count of digit i will be in Output[i].

For example, if the input array is {2007, 13, 17, 24, 5, 17, 38, 407, 0, 7, 4, 17}, the output array (always containing exactly 10 ints) would be {1, 0, 0, 1, 2, 1, 0, 6, 1, 0}.

- Do not employ a sequential cut-off: the base case should process one element. (You can assume the input array will contain at least one element.)
- Give a class definition, CountOnesPlaceTask, along with any other code or classes needed.
- Fill in the function findOnesPlaces below.

You may not use any global data structures or synchronization primitives (locks).

```java
import java.util.concurrent.ForkJoinPool;
import java.util.concurrent.RecursiveTask;
import java.util.concurrent.RecursiveAction;

class Main{
    public static final ForkJoinPool fjPool = new ForkJoinPool();

    // Returns an array of 10 ints. Where the ith element contains a count of the number of times i appears in the ones place in the values in input.
    public static int[] findOnesPlaces (int[] input) {
        // Please fill in the function above and write your class on the next page.
    }
}
```

a) Write the code.
b) Answer this: Is this a map or a reduction (circle one)? Why?
5) (Continued) Write your class on this page.

Don’t forget to answer b) on the previous page!
6) [13 points] Concurrency: The following class implements a Bank account class that keeps track of multiple Bank Accounts a user might have. Multiple threads could be accessing the same BankAccounts object.

```java
public class BankAccounts {
    private Map<String, Double> acctsMap = new HashMap<>();

    // Returns null if account does not exist
    public Double getBalance(String acctName) {
        return acctsMap.get(acctName);
    }

    public Double withdraw(String acctName, Double amount) {
        Double acctBalance = getBalance(acctName);
        if (acctBalance == null || acctBalance < amount) {
            throw new InvalidTransactionException();
        }
        acctBalance -= amount;
        return amount;
    }

    // Deposit amount in acctName
    // Creates acctName if it does not already exist
    public void deposit(String acctName, Double amount) {
        Double acctBalance = getBalance(acctName);
        if (acctBalance == null) {
            acctBalance = 0.0;
        }
        acctBalance += amount;
    }
}
```
6) (Continued)
a) Does the BankAccounts class above have (circle all that apply):

a race condition, potential for deadlock, a data race, none of these

If there are any problems, describe them in 1-2 sentences.

b) Suppose we made the withdraw method synchronized, and changed nothing else in the code. Does this modified BankAccounts class above have (circle all that apply):

a race condition, potential for deadlock, a data race, none of these

If there are any FIXED problems, describe why they are FIXED in 1-2 sentences. If there are any NEW problems, describe them in 1-2 sentences.

c) Modify the code on the previous page to use locks to avoid any of the potential problems listed above. Create locks as needed. Use any reasonable names for the locking methods you use. **DO NOT use synchronized.** You should create re-entrant lock objects as needed as follows (place this in your code as needed):

```java
ReentrantLock lock = new ReentrantLock();
```

d) Clearly circle all of the critical sections in your code on the previous page.
7) [16 points] Sorting
a) [2 points] Give the recurrence for SEQUENTIAL Mergesort – worst case: (Note: We are NOT asking for the closed form.)

b) [3 points] Give the recurrence for Quicksort (parallel sort & parallel partition) – worst case span: (Note: We are NOT asking for the closed form.)

c) [5 points] Give the big-O runtimes requested below.

   ________   A) Quicksort (parallel sort & parallel partition) – best case span
   ________   B) Heapsort – worst case
   ________   C) Insertion Sort – worst case
   ________   D) Bucket Sort – best case
   ________   E) Mergesort (sequential) – worst case

d) [1 point] Is the version of Quicksort described in lecture a stable sort?

   YES       NO

e) [2 points] In 1-2 sentences, describe what it means for a sort to be stable?

f) [3 points] Radix Sort: Give a formula for the worst case big-O running time of radix sort. For full credit, your formula should include all of these variables:

   n         – the number of values to be sorted
   max_value – the values to be sorted range from 0 to max_value
   radix     – the radix or base to be used in the sort

   Answer:
8) [9 points] P, NP, NP-Complete

a) [2 points] “NP” stands for __________________________

b) [2 points] What does it mean for a problem to be in NP?

c) [5 points] For the following problems, circle **ALL** the sets they (most likely) belong to:

<table>
<thead>
<tr>
<th>Problem</th>
<th>P</th>
<th>NP</th>
<th>NP-complete</th>
<th>None of these</th>
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</thead>
<tbody>
<tr>
<td>Finding the shortest path from one vertex to another vertex in a weighted directed graph</td>
<td>NP</td>
<td>P</td>
<td>NP-complete</td>
<td>None of these</td>
</tr>
<tr>
<td>Finding a cycle that visits each edge in a graph exactly once</td>
<td>NP</td>
<td>P</td>
<td>NP-complete</td>
<td>None of these</td>
</tr>
<tr>
<td>Determining if a program will run forever</td>
<td>NP</td>
<td>P</td>
<td>NP-complete</td>
<td>None of these</td>
</tr>
<tr>
<td>Finding the prefix sum of an array in parallel using 10 processors</td>
<td>NP</td>
<td>P</td>
<td>NP-complete</td>
<td>None of these</td>
</tr>
<tr>
<td>Finding a path that starts and ends at the same vertex that visits every vertex exactly once</td>
<td>NP</td>
<td>P</td>
<td>NP-complete</td>
<td>None of these</td>
</tr>
</tbody>
</table>
9) [6 points] Speedup
Your boss wants 111x speedup on a program of which 9/10 is parallelizable. What do you tell them? At least how many processors would you need? 

Justify your answer with a computation. No credit given without an explanation.