

CSE 332: Data Structures and Parallelism

Exercises (Graphs)

Directions: *Submit your solutions on Gradescope. You must submit a pdf file.*

EX16. Dijkstra? (20 points)

Please use **directed** graphs in your examples for this problem.

- (a) [7 Points] If there is more than one minimum cost path from x to y , will Dijkstra's Algorithm always find the path with the fewest edges? If not, explain in a few sentences how to modify Dijkstra's algorithm so that if there is more than one minimum cost path from x to y , a path with the fewest edges is chosen. Assume no negative-cost edges.
- (b) [6 Points] Give a concrete example (show the graph and the starting vertex) where Dijkstra's Algorithm gives the wrong answer in the presence of a negative-cost edge but no negative-cost cycles. Explain briefly why Dijkstra's Algorithm fails on your example. The example need not be complex; it is possible to demonstrate the point using as few as 3 vertices.
- (c) [7 Points] Suppose you are given a weighted graph that has at least one negative-cost edge but no negative-cost cycles. We want to find the shortest paths, where the cost of a path is the sum of the weights of edges along the path. Consider the following strategy to find shortest paths in this graph: Uniformly add a constant k to the cost of every edge, so that all costs become non-negative, then run Dijkstra's Algorithm and return that result with the edge costs reverted back to their original values (i.e., with k subtracted).
- Give a concrete example in the form of a graph and a starting vertex, where this technique fails (Dijkstra's would not find what is actually the shortest path) and explain why it fails.
 - Also, give a general explanation as to why this technique does not work. Think about your example and why the original least cost path is no longer the least cost path after adding k .