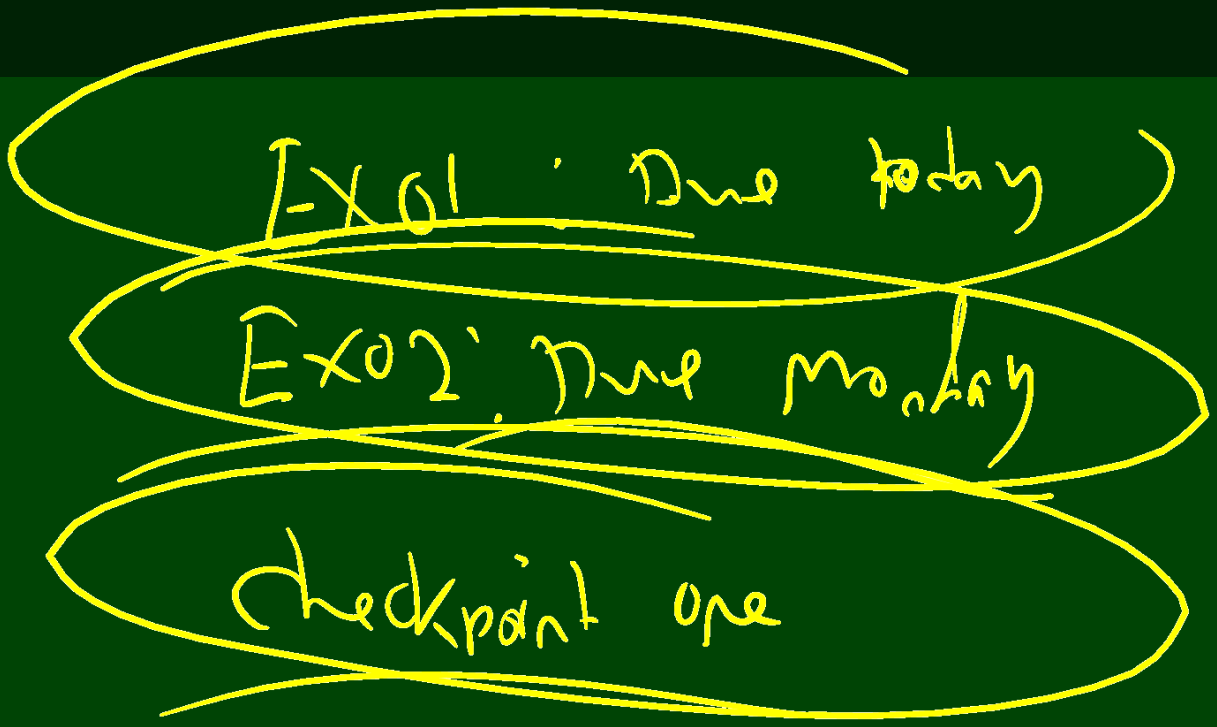
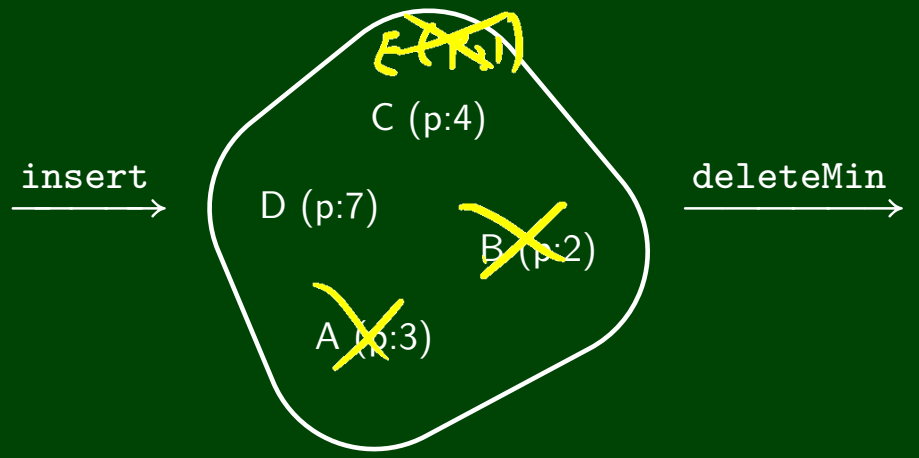
Outline



PriorityQueue ADT

`insert(`**val**`)`	Adds **val** to the queue.
`deleteMin()`	Returns the **highest priority** item not already returned by a `deleteMin`. (Errors if empty.)
`findMin()`	Returns the **highest priority** item not already returned by a `deleteMin`. (Errors if empty.)
`isEmpty()`	Returns true if all inserted elements have been returned by a `deleteMin`.

- Data in `PriorityQueues` **must be comparable** (by priority)!
- Highest Priority = Lowest Priority Value
- The ADT **does not specify how to deal with ties**!



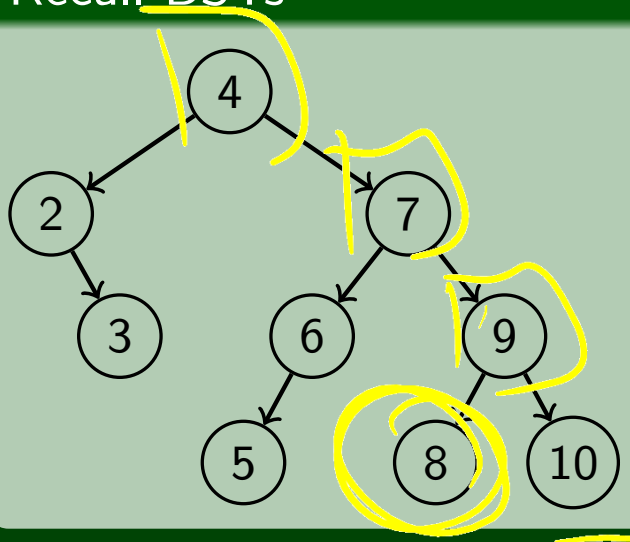
- `findMin` → B
- `deleteMin` → B
- `insert(E (p:1))` ✓
- `deleteMin` → E
- `deleteMin` → A

For each of the following potential implementations, what is the worst case runtime for `insert` and `deleteMin`? Assume all arrays do not need to resize.

	insert	deleteMin
Unsorted Array	$\Theta(1)$	$\Theta(n)$
Unsorted Linked List	$\Theta(1)$	$\Theta(n)$
Sorted Circular Array List	$\Theta(\log n)$ 或 $\Theta(n)$	$\Theta(1)$
Sorted Linked List	$\Theta(n)$	$\Theta(1)$
Binary Search Tree	$\Theta(n)$ (?)	$\Theta(n)$ (?)

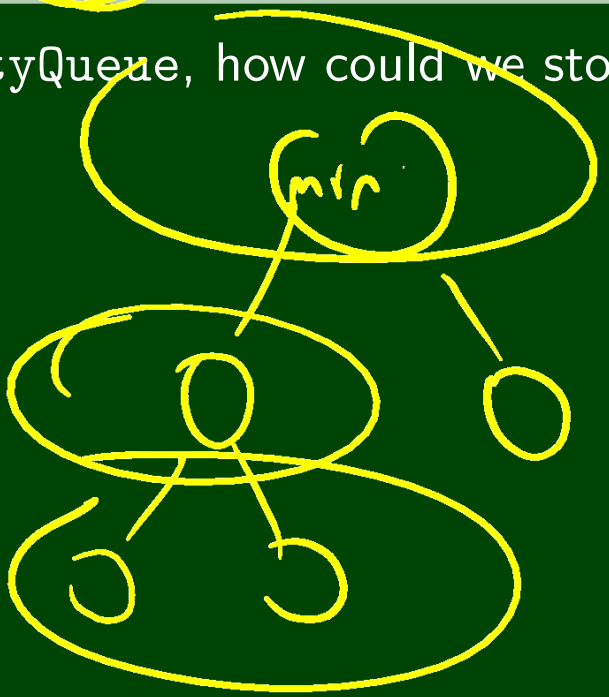
A New Data Structure: Heap

Recall BSTs

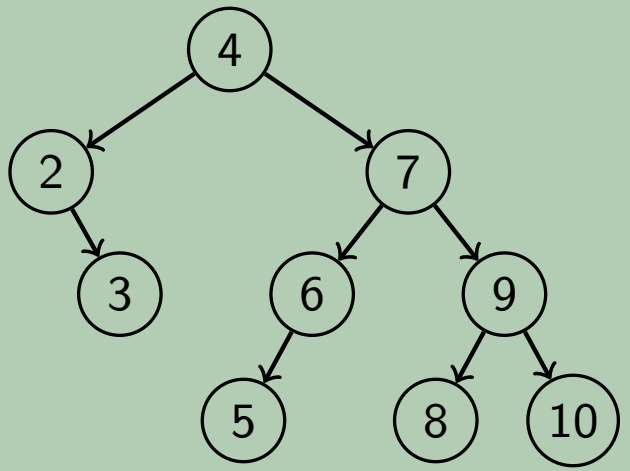


BST Property:
Left Children are smaller
Right Children are larger

For a `PriorityQueue`, how could we store the items in a tree?



Recall BSTs



BST Property:
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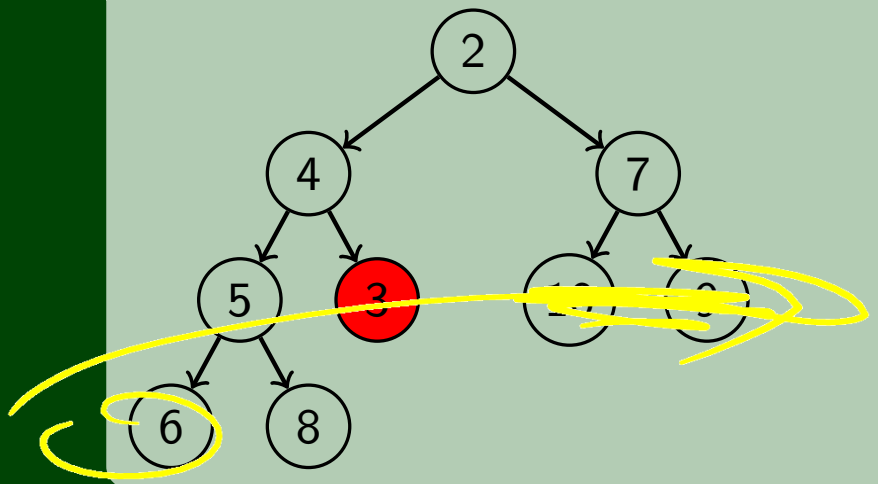
For a `PriorityQueue`, how could we store the items in a tree?

And Now, Heaps

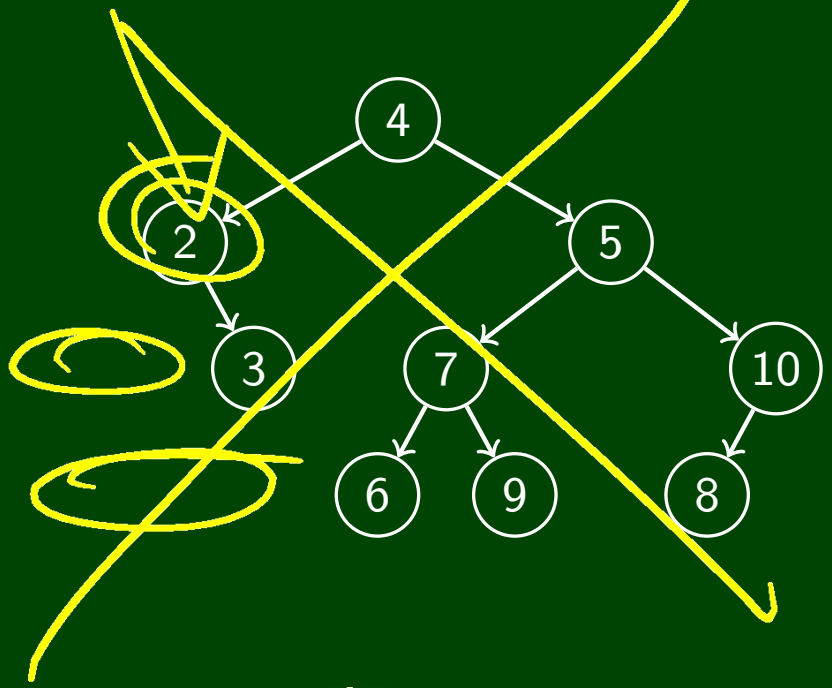


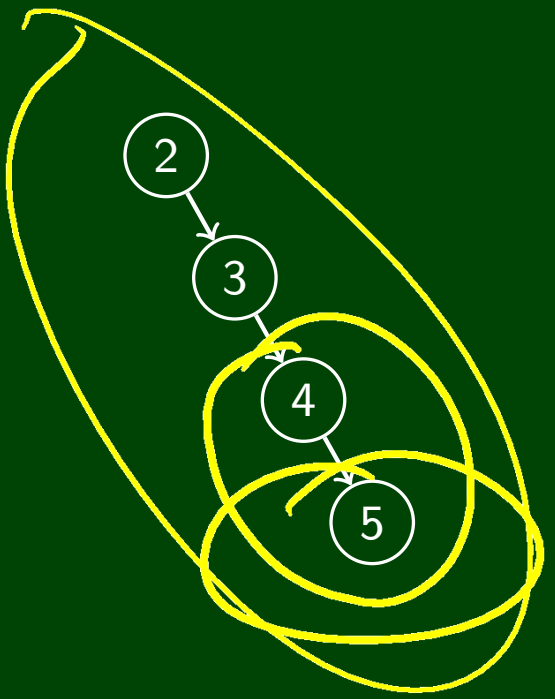
Heap Property:
All Children are larger

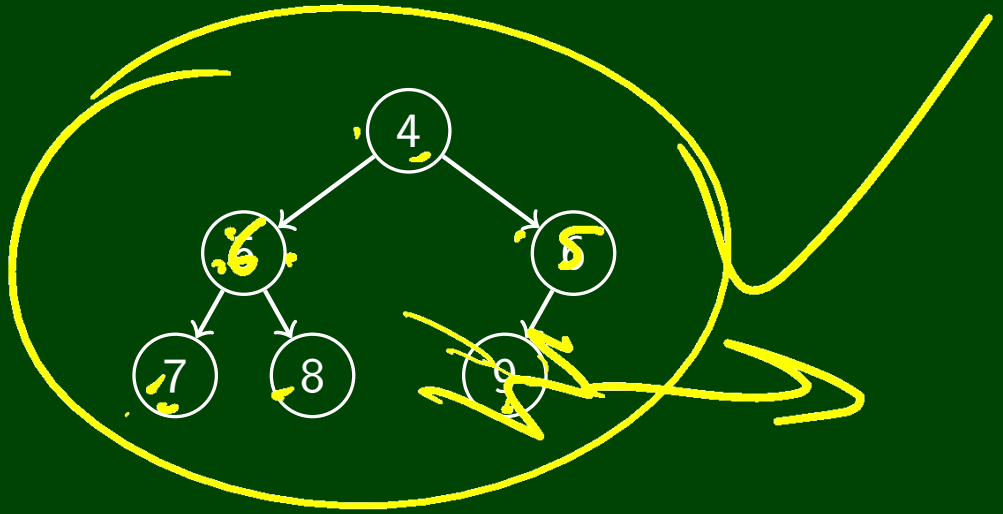
Structure Property:
Insist the tree has no "gaps"

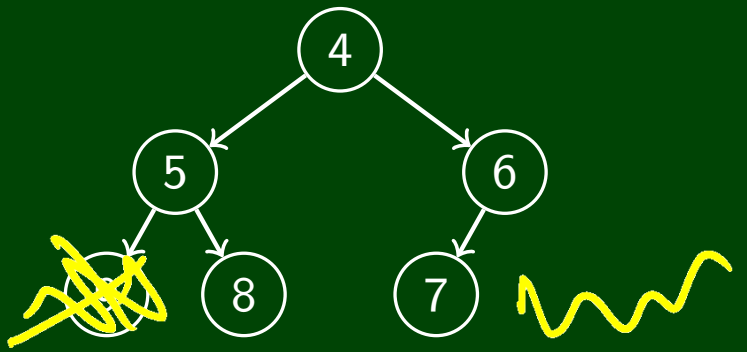
Is It A Heap?

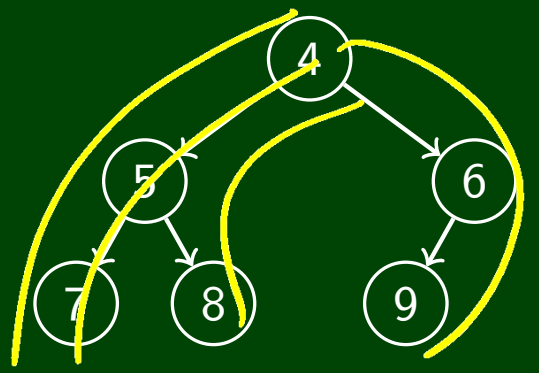
For each of the following, is it a heap?











- Where is the minimum item in a heap?

  It's at the top!

- What is the height of a heap with n items?

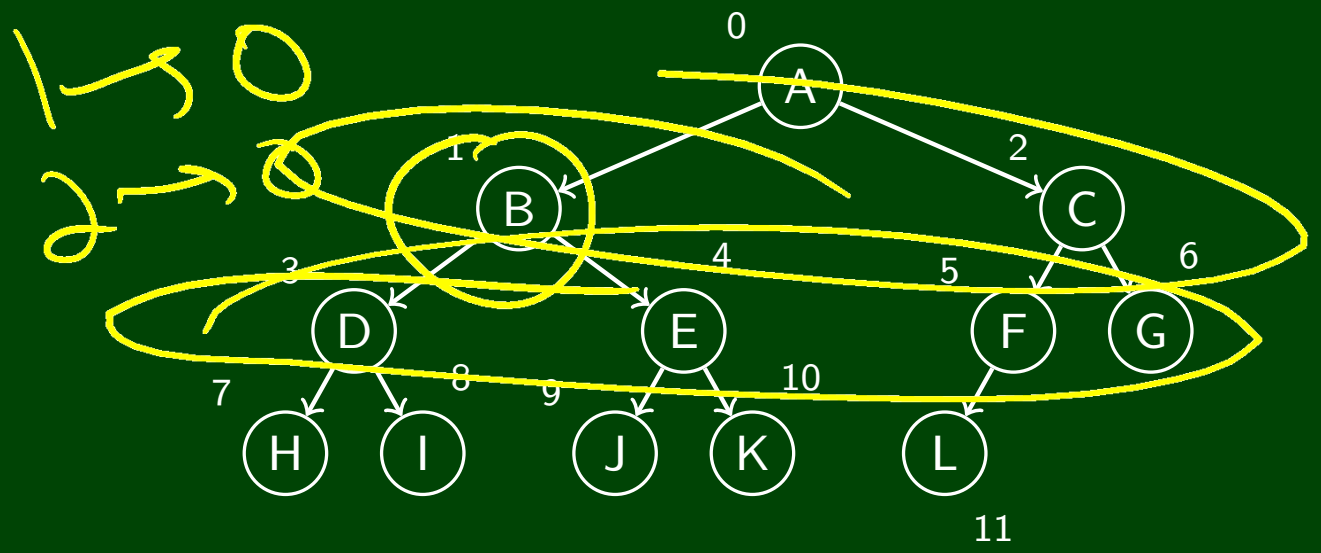
k levels

$2^0 + 2^1 + 2^2 + \cdots + 2^{k-1}$

$n = 2^k - 1$

And. . . how do we implement `Heap`?

We've insisted that the tree be complete to be a valid `Heap`. Why?



Fill in an array in **level-order** of the tree:

`heap`:	A	B	C	D	E	F	G	H	I	J	K	L	0	0	0
	`h[0]`	`h[1]`	`h[2]`	`h[3]`	`h[4]`	`h[5]`	`h[6]`	`h[7]`	`h[8]`	`h[9]`	`h[10]`	`h[11]`	`h[12]`	`h[13]`	`h[14]`

If I have the node at index i, how do I get its:

And. . . how do we implement `Heap`?

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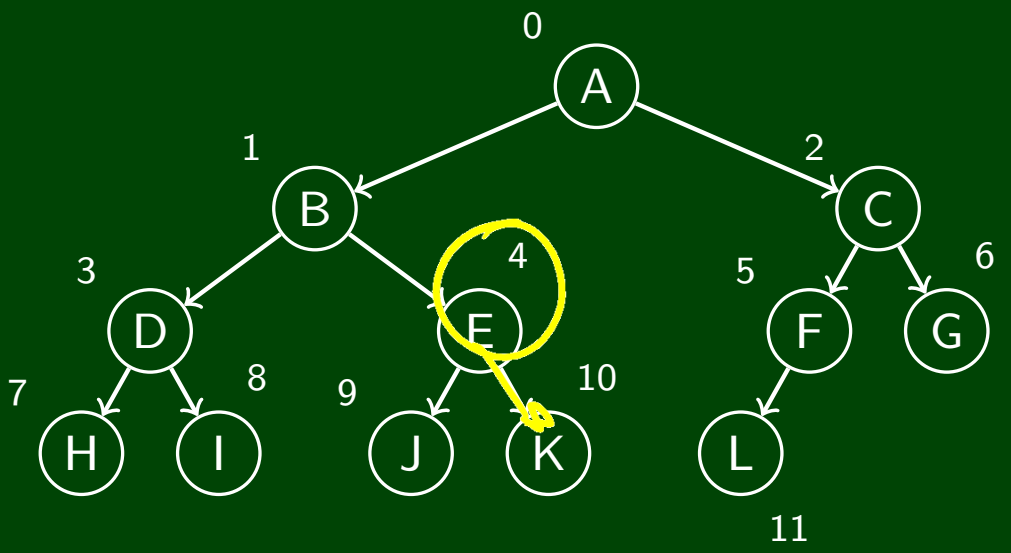

Fill in an array in **level-order** of the tree:

`heap`:	A	B	C	D	E	F	G	H	I	J	K	L	0	0	0
	`h[0]`	`h[1]`	`h[2]`	`h[3]`	`h[4]`	`h[5]`	`h[6]`	`h[7]`	`h[8]`	`h[9]`	`h[10]`	`h[11]`	`h[12]`	`h[13]`	`h[14]`

If I have the node at index i, how do I get its:

- Parent? $3 \to 1$, $4 \to 1$, $10 \to 4$, $9 \to 4$, $1 \to 0$

  This indicates that it's approximately $n/2$. In fact, it's $\frac{n-1}{2}$.

- Left Child? $2k+1$, $2k+2$