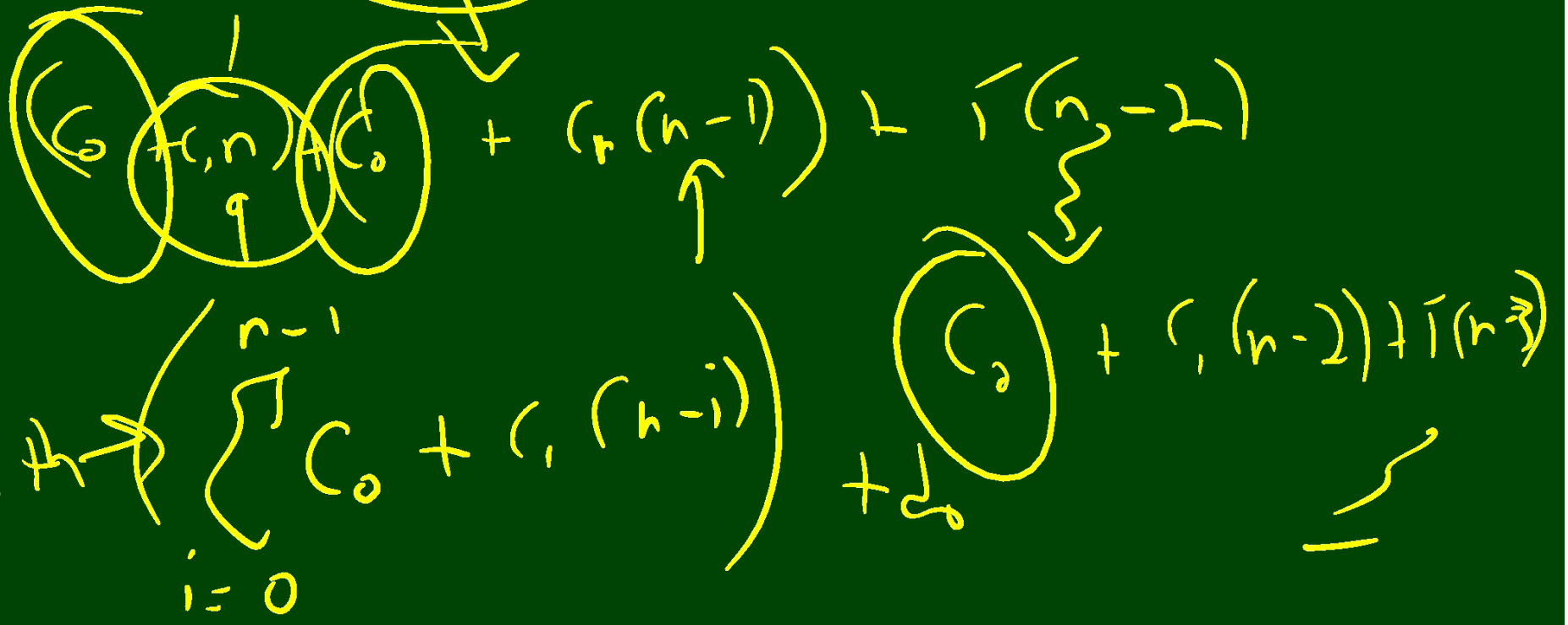
Solving the `reverse` Recurrence

$$T(n) = \begin{cases} d_0 & \text{if } n = 0 \\ c_0 + c_1 n + T(n-1) & \text{otherwise} \end{cases}$$

$$T(n) = (c_0 + c_1 n) + T(n-1)$$



Merge Sort

```
1  sort(L) {
2     if (L.size() < 2) {
3        return L;
4     }
5     else {
6        int mid = L.size() / 2;
7        return merge(
8           sort(L.subList(0, mid)),
9           sort(L.subList(mid, L.size())));
10       );
11    }
12 }
```

BC

NR

R

First, we need to find the recurrence:

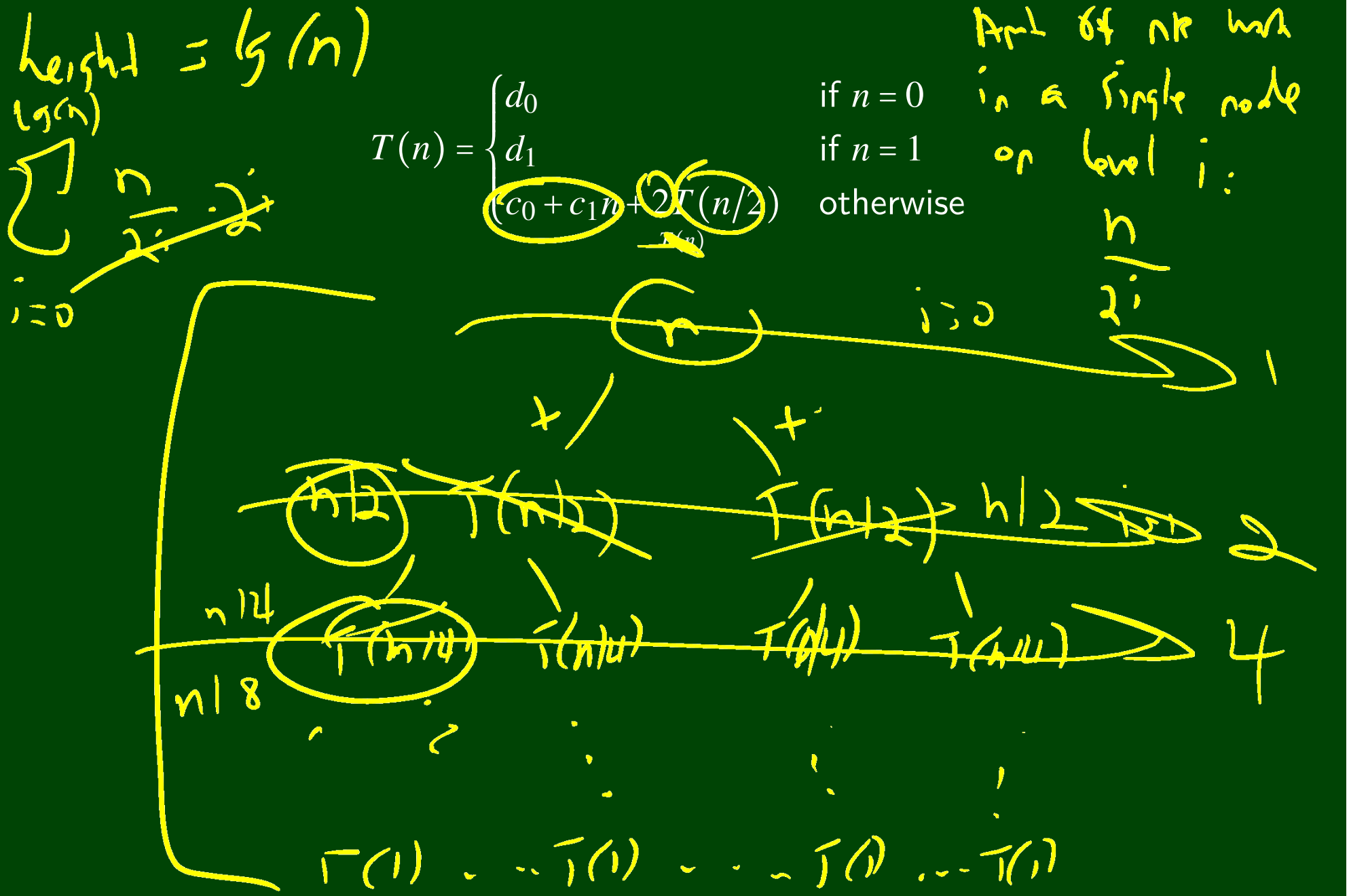
$T(n) =$ $2T(n/2) + n$

$BC = \Theta(1)$

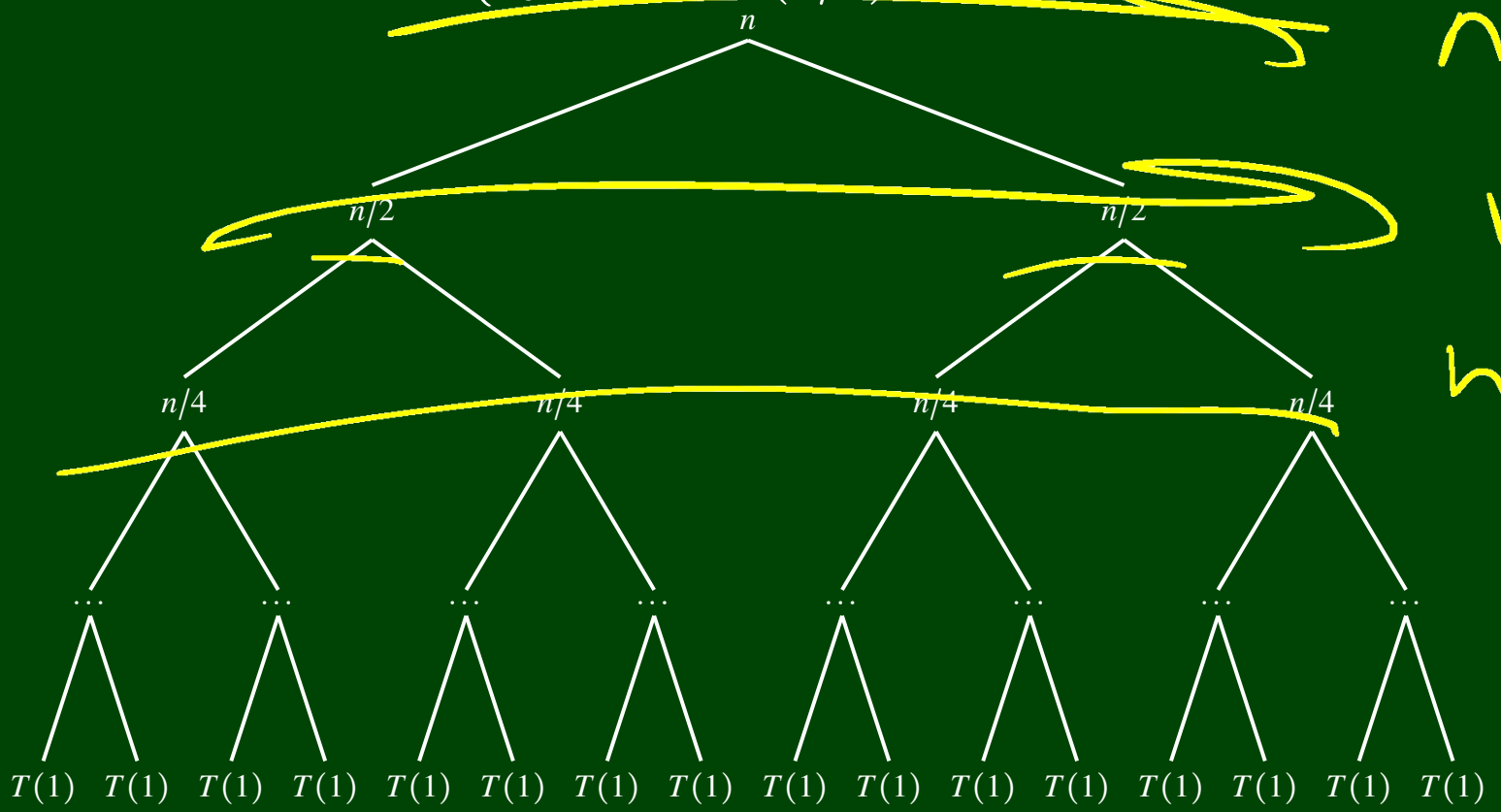
$NR = \Theta(n)$

$R = T(n/2) + T(n/2)$

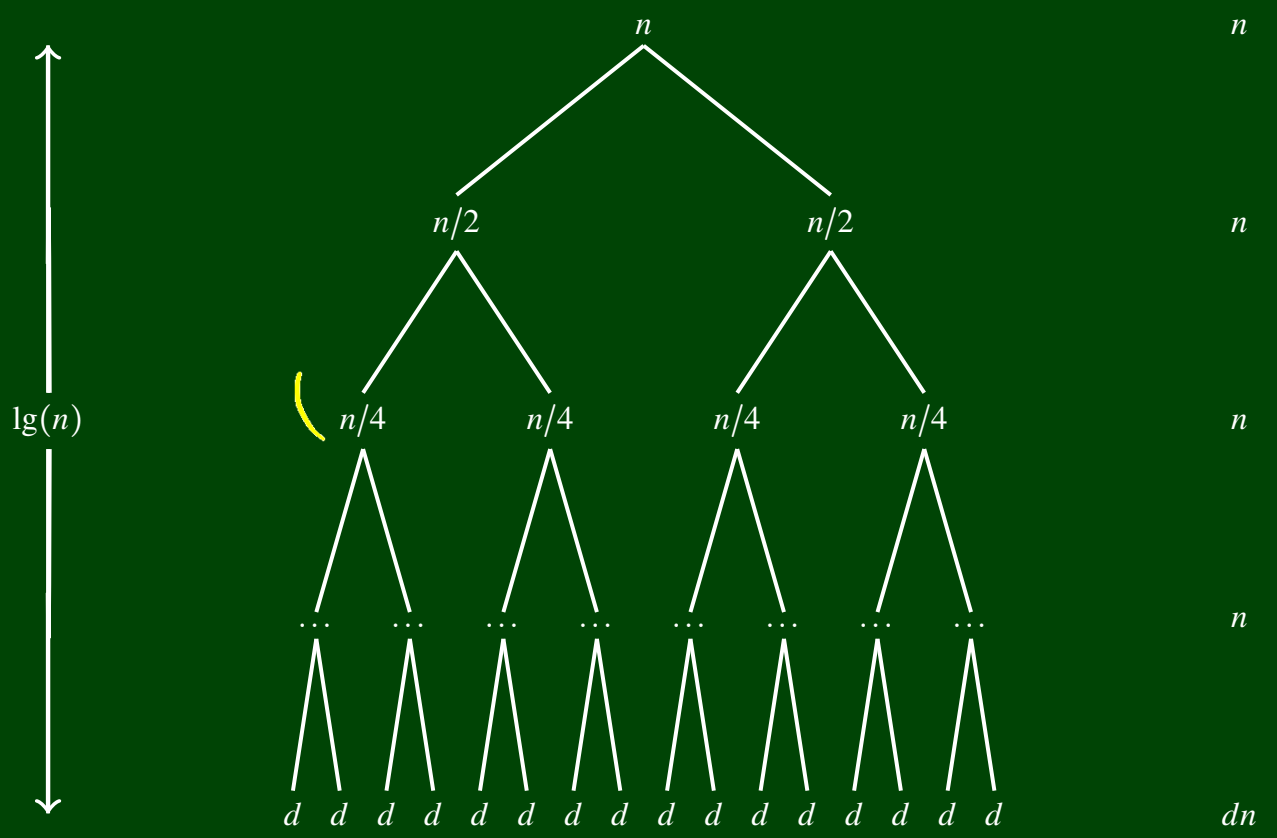
$$T(n) = \begin{cases} d_0 & \text{if } n = 0 \\ d_1 & \text{if } n = 1 \\ c_0 + c_1 n + 2T(n/2) & \text{otherwise} \end{cases}$$



$$T(n) = \begin{cases} d_0 & \text{if } n = 0 \\ d_1 & \text{if } n = 1 \\ c_0 + c_1 n + 2T(n/2) & \text{otherwise} \end{cases}$$



$$T(n) = \begin{cases} d_0 & \text{if } n = 0 \\ d_1 & \text{if } n = 1 \\ c_0 + c_1 n + 2T(n/2) & \text{otherwise} \end{cases}$$



Find A Big-Oh Bound For The Worst Case Runtime

```
1 sum(n) {
2     if (n < 2) {
3         return n;
4     }
5     return 2 + sum(n - 2);
6 }
```

$$T(n) = \begin{cases} 1 & \text{if } n < 2 \\ 1 + T(n-2) & \text{otherwise} \end{cases}$$

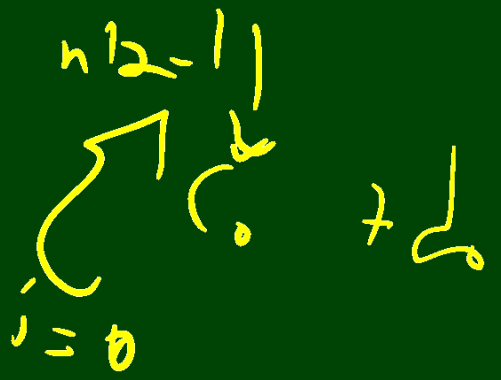
$$T(n) = \underbrace{1 + 1 + \cdots + 1}_{n/2}$$

Find A Big-Oh Bound For The Worst Case Runtime

```
1 sum(n) {
2    if (n < 2) {
3       return n;
4    }
5    return 2 + sum(n - 2);
6 }
```

$$T(n) = \begin{cases} d_0 & \text{if } n = 0 \\ d_0 & \text{if } n = 1 \\ c_0 + T(n-2) & \text{otherwise} \end{cases}$$

$$\begin{aligned} T(n) &= c_0 + c_0 + \cdots + c_0 + d_0 \\ &= c_0\left(\frac{n}{2}\right) + d_0 \\ &= \mathcal{O}(n) \end{aligned}$$



Find A Big-Oh Bound For The Worst Case Runtime

```
1  binarysearch(L, value) {
2     if (L.size() == 0) {
3        return false;
4     }
5     else if (L.size() == 1) {
6        return L[0] == value;
7     }
8     else {
9        int mid = L.size() / 2;
10       if (L[mid] < value) {
11          return binarysearch(L.subList(mid + 1, L.size()), value);
12       }
13       else {
14          return binarysearch(L.subList(0, mid), value);
15       }
16    }
17 }
```

n/2

n/2

$$T(n) = \begin{cases} 1 & \text{if } n < 2 \\ 1 + \max(T(n/2), T(n/2)) \end{cases}$$